New Theoretical Analysis for the Discrete-Time Feedback Error Learning Method

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Abstract

In this study, a new theoretical foundation for the Discrete-Time Feedback Error Learning (DTFEL) method is proposed. This method is analogous to the original Continuous-Time Feedback Error Learning (FEL) originally proposed as a control model of the cerebellum in the field of computational neuroscience. DTFEL is superior to FEL since it is applicable for digital controllers which are commonly used nowadays. Based on the strictly positive realness, the stability of DTFEL has been validated. However, from the viewpoint of adaptive control, this condition is restricted since some required assumptions cannot be satisfied in real systems, such as the direct input-output transmission gain is required to be large. Moreover, the stability of the system is guaranteed by selecting a sufficiently large positive constant feedback gain, which is a vague requirement from the viewpoint of control system design. This study proposes another scheme for the DTFEL method which does not require any positive real condition and overcomes the direct input-output transmission gain requirement. This can be achieved by utilizing the error signal more effectively.

Keywords: Discrete-time system, Feedback Error Learning, strictly positive realness, learning control, Adaptive control, feedback and feedforward control

1. Introduction

One of the great challenges of science today is to understand the human brain and the biological basis of perceiving, learning, action, and memory. The main obstacle is the complexity of structure and subtlety of design of the brain. At low levels, the biochemical mechanisms underlying the operation of individual neuron cells, and the operation and interactions of various simple neural circuits in the brain are well understood. However, our understanding becomes less about how the brain works at a "systems level". That is to understand how its various parts interact to produce coordinated, adaptive, and intelligent behaviors. The biological motor system is acceptably considered as an ideal realization of control. Similar to other control systems, it consists of actuators, sensors, and controllers. Unlike artificial control systems, however, it exhibits performance with much greater flexibility and versatility in spite of the nonlinearity, uncertainties, and large degrees of freedom of animal bodies.

Doya et al. [1] introduced some examples of converging efforts from both sides towards understanding and building adaptive autonomous systems, and aimed to promote future collaborations between the neuroscience and the systems theory communities. Kawato et al. [2] proposed a novel architecture of the brain motor control called the Feedback Error

Learning (FEL) method, which is a two-degrees-of-freedom control system consisting of an adaptive feedforward controller and a fixed feedback controller. For continuous-time linear time-invariant systems, the stability of this scheme has been theoretically discussed by many researchers [1-5]. Nevertheless, the theoretical results were based on an assumption of continuous-time systems. As a result, they must be modified prior to implementation in real systems since virtually all today's controllers are computer-based, which requires a transformation from a continuous-time system to the discrete-time before applying. This transformation may create some problems in the stability of the system or may lead to poor tracking performance. This obstacle is motivated by development of new theoretical control knowledge to facilitate a direct implementation of the Feedback Error Learning method in real systems.

Recently, Wongsura and Kongprawechnon [6,7] proposed a discrete-time version of FEL called "Discrete-Time Feedback Error Learning (DTFEL)". They considered the stability of the DTFEL system and established a control theoretical validity. The formulation of the problem is in the framework of the adaptive control theory. The stability of the DTFEL algorithm was proved under an assumption of strictly positive realness. More specifically, the strictly positive realness is imposed on a certain transfer function which contains the feedback controller and unknown parameters of the inverse model. The condition can be satisfied if the feedback controller is chosen as a sufficiently large positive constant. However, from the viewpoint of control system design, it is not clear how large the feedback gain should be in order to guarantee the positive realness since such largeness depends on the unknown parameters.

This study intends to establish a new theoretical ground and improvement of the Discrete-Time Feedback Error Learning (DTFEL) method by removing the positive real conditions from the DTFEL scheme. To accomplish this, a more effective way to utilize the error signal between the reference and output signals, as well as the feedback input, for learning the inverse model is proposed. Using the error signal, an adaptive law without recourse to the positive realness is derived. [The notation is written to express the input-output relation by the transfer function $y(k) = G(z)u(k)$.] Also, the term "(z)" representing a z-domain variable can be skipped without notation throughout this paper. That is, for example, P and $P(z)$ can be written interchangeably.

Notation: Throughout this study, a fairly standard notation is used. The overview is as follows:

$\gamma_{\min}[P]$	The smallest eigenvalue of P.
$\|A\|$	$=\sqrt{tr(A^T A)} = \sqrt{\sum_{i,j} a_{ij}^2}$ Frobenius norm.
$(A,B,C,D) = D + C(zI - A)^{-1} B$	Minimal realization.
$H^*(z)$	the complex conjugate transpose of $H(z)$
p.r.	positive real.
s.p.r.	strictly positive real.
PE	Persistently Exciting.

The paper is organized as follows. In the first part, the mathematical knowledge which is useful for analyzing the DTFEL system is briefly discussed. Then, the proposed DTFEL method is summarized. Later, simulation results are presented. Finally, the conclusion is given.

2. Mathematical Preliminaries

In this section, the mathematical requirement to analyze the DTFEL in the next section is discussed. The main and most important area is to study the strictly positive real system.

Definition 1. [8] A square matrix $H(z)$ of real rational functions is a positive real (p.r.) matrix if (d1) $H(z)$ has elements analytic in $|z|>1$.

(d2) $H^*(z)+H(z)$ is positive, semidefinite and Hermitian for $|z|>1$.

Condition (d2) can be replaced by

(d3) The poles of the elements of $H(z)$ on $|z|=1$ are simple and the associated residue matrixes of $H(z)$ at these poles are 0.

(d4) $H(e^{j\theta})+H^T(e^{-j\theta})$ is a positive semidefinite Hermitian matrix for all real θ for which $H(e^{j\theta})$ exists.

Definition 2. [8] A rational transfer matrix $H(z)$ is a strictly positive real (s.p.r.) matrix if $H(\rho z)$ is p.r. for some $0<\rho<1$.

Given Definition 2, a necessary and sufficient condition in the frequency domain for s.p.r. transfer matrices in the class H can be defined as in the following.

Definition 3. [8] An $n\times n$ rational matrix $H(z)$ is said to belong to class H if $H(z)+H^T(z^{-1})$ has rank n almost everywhere in the complex z-plane.

Theorem 1. [8] Consider the $n\times n$ rational matrix $H(z)\in H$ given in Definition 3. Then, $H(z)$ is a s.p.r. matrix if and only if:

(a) All elements of $H(z)$ are analytic in $|z|\geq 1$,

(b) $H(e^{j\theta})+H^T(e^{-j\theta})>0, \forall\theta\in[0,2\pi]$.

Lemma 1. (Discrete-time version of Kalman-Yakubovich-Popov Lemma) [8] Assume that the rational transfer matrix $H(z)$ has poles that lie in $|z|<\gamma$, where $0<\gamma<1$ and (A,B,C,D) is a minimal realization of $H(z)$. Then, $H(\gamma z)$ is strictly positive real (s.p.r.), if and only if real matrices $P=P^T>0$, Q and K exist such that

$$A^TPB=C^T-QK,$$
$$A^TPA-P=-QQ^T-(1-\gamma^2)P,$$
$$K^TK=D+D^T-B^TPB.$$

For the formal case, that is, the transfer matrix is stable or $\gamma=1$, the following lemma is applied.

Lemma 2. (Discrete-time version of Kalman-Yakubovich-Popov Lemma)[8] Assume that the rational transfer matrix $H(z)$ has poles that lie in $|z|<1$, and (A,B,C,D) is a minimal realization of $H(z)$. Then, $H(z)$ is s.p.r., if and only if real matrices $P=P^T>0$, q ε and v exist such that

$$A^TPA-P=-qq^T-\varepsilon L,$$
$$A^TPB=\frac{C}{2}+vq,$$
$$B^TPB=D-v^2.$$

Remark:

If $L(z)$ is a stable transfer function, for a given constant α, there exists sufficiently large K such that $\frac{\alpha}{K}\left(L(z)+K\right)^{-1}$ is s.p.r..

Consider the linear discrete-time varying system given by:

$$x(k+1)=A(k)x(k)+B(k)u(k), \quad (1)$$
$$y(k)=C(k)x(k), \quad (2)$$

with $A(k),B(k),C(k)$ are appropriately dimensioned matrices.

Lemma 3. [9] Define $\psi(k_1,k_0)$ as the state-transition matrix corresponding to $A(k)$ for the system (1), that is,

$$\psi(k_1,k_0)=\prod_{k=k_0}^{k_1-1}A(k). \quad (3)$$

Then, if $\|\psi(k_1,k_0)\|\leq 1$, $\forall k_1,k_0\geq 0$, the system (1) is exponentially stable.

Lemma 4. [9] If $A(k)=I-\beta\phi(k)\phi^T(k)$ in the system (1), where $0<\beta<2$ and $\phi(k)$ is a regressor vector of past inputs and outputs, $\|\phi(k_1,k_0)\|<1$ is guaranteed. If there is an $L>0$ such that $\sum_{k=k_0}^{k_1+L-1}\phi(k)\phi^T(k)>0$ for all k. Then, Lemma 3 guarantees the exponential stability of the system (1).

Definition 4. [9] An input sequence $x(k)$ is said to be persistently exciting(PE) if there exist $\gamma>0$ and an integer $k_1\geq 1$ such that

$$\gamma_{\min}\left[\sum_{k=k_0}^{k_1+L-1}\phi(k)\phi^T(k)\right]>\gamma, \forall k_0\geq 0. \quad (4)$$

Note: PE is exactly the stability condition needed in Lemma 4.

Theorem 2. A difference equation

$$z(k+1)=\left(I-\xi(k)L(z)\xi^T(k)\right)z(k) \quad (5)$$

is asymptotically stable for any time-varying vector $\xi(k)$ which satisfies the PE condition, if

$L(z)$ is s.p.r..

Note that a special case of Theorem 2, where $L(z)=1$, corresponds to Lemma 4.

3. Discrete-Time Feedback Error Learning

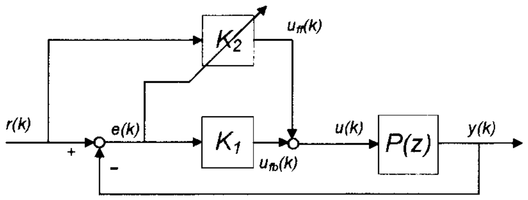

Fig 1: Original DTFEL Scheme

This section is devoted to summarize the DTFEL scheme proposed in [6] and [7]. Fig. 1 shows the block diagram of the DTFEL system. The objective of this system is to minimize the error $e(k)$ between the command signal $r(k)$ and the plant output $y(k)$. The input $u(k)$ to the plant P is composed of the output $u_{ff}(k)$ of feedforward controller K_2 and $u_{fb}(k)$ of the feedback controller K_1. In the DTFEL scheme, K_2 is tuned to be P^{-1} which is the inverse model of P. In [6] and [7], the plant P is assumed to be linear time-invariant and satisfy the following assumptions.

(A1) The plant, P, is stable and has stable inverse P^{-1}.

(A2) The upper bound of the order of P is known.

(A3) $l_0=\lim_{z\to\infty}P(z)$ is assumed to be positive.

(A4) Input signal is bounded and satisfies the PE condition.

The parameterization of the unknown P^{-1} is expressed as:

$$\eta_1(k+1)=F\eta_1(k)+gr(k), \tag{6}$$

$$\eta_2(k+1)=F\eta_2(k)+gu_0(k), \tag{7}$$

$$u_0(k)=c_0^T\eta_1(k)+d_0^T\eta_2(k)+l_0 r(k),$$
$$=\theta_0^T\eta(k). \tag{8}$$

$$\theta_0:=[c_0^T \quad d_0^T \quad l_0]^T, \tag{9}$$

$$\eta(k):=[\eta_1(k)^T \quad \eta_2(k)^T \quad r(k)]^T, \tag{10}$$

where F is any stable matrix , g is any vector with $\{F,g\}$ being controllable and $u_0(k)$ is the desired output of this system. It is easy to see that appropriate selection of parameters c_0, d_0 and l_0 can yield an arbitrary transfer function from $r(k)$ to $u_0(k)$.

To see this, let the matrix F and vector g be in a controllable canonical form:

$$F=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & 1 \\ -f_1 & -f_2 & -f_3 & \cdots & -f_n \end{bmatrix}, g=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \tag{11}$$

From Eqs. (6)–(8), the transfer function from $r(k)$ to $u_0(k)$ is given by:

$$\frac{U_0(z)}{R(z)}=\left(\frac{U_0-d^T\eta_2}{R}\right)\left(\frac{U_0}{U_0-d^T\eta_2}\right) \tag{12}$$

$$=\left(\frac{U_0-d^T\eta_2}{R}\right)\left(\frac{1}{1-d^T\frac{\eta_2}{U_0}}\right)$$

$$=\frac{l_0+c_0^T(zI-F)^{-1}g}{1-d_0^T(zI-F)^{-1}g}$$

$$=\frac{l_o z^n+(f_n l_o+c_n)z^{n-1}+\ldots+(f_1 l_o+c_1)}{z^n+(f_n-d_n)z^{n-1}+\ldots+(f_1-d_1)}. \tag{13}$$

Therefore, any transfer function of degree less than or equal to n can be constructed by selecting parameters c_0, d_0 and l_0 appropriately. The advantage of the parameterization described by Eqs. (6)–(10) is that the unknown parameters enter linearly in the system description. The continuous version of this parameterization was firstly used in an adaptive observer[10].

Adaptation law: The same parameterization of the adaptive feedforward controller K_2 as in Eqs. (6)–(7) is taken. Instead of $u_0(k)$ in Eqn. (8), the plant input $u_{ff}(k)$ is used:

$$\xi_1(k+1)=F\xi_1(k)+gr(k), \tag{14}$$

$$\xi_2(k+1) = F\xi_2(k) + gu_0(k), \quad (15)$$

$$u_{ff}(k) = c^T(k)\xi_1(k) + d^T(k)\xi_2(k) + l(k)r(k), \quad (16)$$

$$u(k) = u_{ff}(k) + K_1 e(k), \quad (17)$$

where F is stable and $\{F, g\}$ is controllable. In Eqs. (16) and (17), $c(k), d(k)$ and $l(k)$ are unknown.

In the ideal situation, K_2 is identical to P^{-1}. In that case, $e(k) = 0$, $u(k) =$ $u_{ff}(k) =$ $u_0(k) = P^{-1}(z)r(k)$. The true values c_0, d_0 and l_0 of $c(k)$, $d(k)$ and $l(k)$, respectively, satisfy:

$$\frac{l_0 + c_0^T(zI - F)^{-1}g}{1 - d_0^T(zI - F)^{-1}g} = P^{-1}(z) \quad (18)$$

as given in Eqn. (13). The error signal $e(k)$ is defined as:

$$e(k) = r(k) - y(k). \quad (19)$$

The cost function for adaptation is defined as:

$$J(k) = \frac{1}{2}\sum_{i=0}^{k} e^2(i). \quad (20)$$

The unknown parameters $c(k), d(k), l(k)$ must be updated so that the error signal $e(k)$ decreases. Let define $\xi(k)$ as:

$$\xi(k) := [\xi_1(k)^T \quad \xi_2(k)^T \quad r(k)]^T. \quad (21)$$

The usual gradient method gives rise to the updating rule. Then, the adaptation law of parameters is obtained as:

$$\theta(k) := [c(k)^T \quad d(k)^T \quad l(k)]^T, \quad (22)$$

$$\theta(k+1) = \theta(k) + \frac{\alpha}{K_1} e(k)\xi(k), \quad (23)$$

where α is an adaptive gain. Note that this is adapted from the continuous-time adaptation algorithm by using the gradient method presented by Miyamura[11].

In this case, the desired output $u_0(k)$ must be identical to $P^{-1}(z)r(k)$ because the feedforward controller is assumed to be an inverse of the plant P. Since P is unknown, $u_0(k)$ is not available. In the Kawato scheme, $u(k)$ is used instead of $u_0(k)$. An essential feature of DTFEL lies in the justification of this attempt. Using this adaptive law, we can state the following theorem.

Theorem 3. Under the assumptions (A1)–(A4), the feedback error learning method described by Eqs. (14) – (17) is converging, that is, the controller K_2 converges to $P^{-1}(z)$.

Proof : The proof of this theorem was done in [6] and [7].

4. DTFEL Theory Without Recourse To Positive Realness

In [6] and [7], to satisfy the positive realness of relation, a choice of the feedback controller $K_1(z)$ is proposed, that is $K_1(z) = K_1$ where a constant K_1 is a sufficiently large positive constant. There always exists such K_1 for any stable $G(z)$. However, since $G(z)$ includes the unknown parameters of P^{-1} , one cannot determine how large K_1 should be at the stage of control system design. This implies that the strictly positive realness causes disadvantages for practical use of the DTFEL method.

This study suggests a new theoretical analysis of DTFEL where the necessity of the positive realness condition is removed. The feedforward controller $K_2(z)$ in Fig. 1 is described by Eqs. (14)– (17), and the control input $u(k)$ is given by:

$$u(k) = u_{ff}(k) + K_1(z)e(k)$$
$$= \theta^T(k)\xi(k) + K_1(z)e(k), \quad (24)$$

which is the same form as that in [6] and [7]. The crucial part of our scheme is a tuning rule for K_2, which is derived by using $e(k)$ more effectively than the system described by Eqs. (14)–(17). In order to obtain the inverse model adaptively, we employ the feedback error signal $e(k)$, as well as the feedback input $u_{fb}(k)$. The use of $e(k)$ is based on the idea that $e(k) \to 0$ is achieved when $K_2(z) \to P^{-1}$.

To derive the parameter tuning rule, we consider expressing the control input $u(k)$ as a linear parameterization by θ_0 . By using the relation of:

relation of:

$$u_0(k)-u(k)=P^{-1}r(k)-P^{-1}y(k)=P^{-1}e(k) \quad (25)$$

together with Eqn. (10), $u(k)$ can be firstly described as:

$$u(k)=u_0(k)-P^{-1}e(k)=\theta_0^T\eta(k)-P^{-1}e(k), \quad (26)$$

where

$$\eta(k)=[\eta_1^T(k) \quad \eta_2^T(k) \quad r(k)]^T. \quad (27)$$

In the description of Eqn. (26), the signal $\eta_2(k)$ cannot be used for learning, since it is generated by Eqn. (7) with the unknown signal $u_0=P^{-1}r$. Transforming Eqn. (26) to another form with $\xi(k)$, which can be used for learning, is considered next. By comparing Eqn. (6) with Eqn. (14), it can be seen that

$$\eta_1(k)\to\xi_1(k) \quad (28)$$

since F is stable. From Eqs. (7) and (15), together with Eqn. (25), it is obviously seen that

$$\eta_2(k)\to\xi_2(k)+(zI-F)^{-1}gP^{-1}e(k). \quad (29)$$

Substituting Eqs. (28) and (29) into Eqn. (26) yields

$$\begin{aligned}u(k)&=\theta_0^T\xi(k)\\&+\{-1+d_0^T(zI-F)^{-1}g\}P^{-1}(z)e(k).\end{aligned} \quad (30)$$

From Eqn. (18), the second term of the right-hand side can be written as:

$$\begin{aligned}&\{-1+d_0^T(zI-F)^{-1}g\}P^{-1}(z)e(k)=\\&\{-l_0-c_0(zI-F)^{-1}g\}e(k).\end{aligned} \quad (31)$$

Let us introduce a vector $\xi_e(k)$, which is generated by $e(k)$ with:.

$$\xi_e(k+1):=F\xi_e(k)+ge(k). \quad (32)$$

Then, the relation (31) can be rewritten as:

$$\begin{aligned}&\{-1+d_0^T(zI-F)^{-1}g\}P^{-1}(z)e(k)=\\&-c_0^T\xi_e(k)-l_0e(k).\end{aligned} \quad (33)$$

By substituting this into Eqn. (30), and defining:

$$\tilde{\xi}(k)\triangleq\begin{bmatrix}\xi_1(k)-\xi_e(k)\\ \xi_2(k)\\ r(k)-e(k)\end{bmatrix} \quad (34)$$

we obtain

$$u(k)=\theta_0^T\tilde{\xi}(k), \quad (35)$$

which is linearly parameterized by θ_0. To derive an adaptive law for DTFEL, we define:

$$u(k)\triangleq\theta^T(k)\tilde{\xi}(k) \quad (36)$$

by replacing θ_0 in Eqn. (35) with $\theta(k)$, and define an error signal $\varepsilon(k)$ as:

$$\begin{aligned}\varepsilon(k)&\triangleq u(k)-\hat{u}(k)=\{\theta_0-\theta(k)\}^T\tilde{\xi}(k)\\&=-\psi^T(k)\tilde{\xi}(k),\end{aligned} \quad (37)$$

where

$$\psi(k)\triangleq\theta(k)-\theta_0. \quad (38)$$

Using these signals, we employ an adaptive law of:

$$\theta(k+1)-\theta(k)=\alpha\tilde{\xi}(k)\varepsilon(k), \quad (39)$$

where α is an adaptive gain. From Eqn. (37), this law is rewritten as:

$$\begin{aligned}\psi(k+1)-\psi(k)&=-\alpha\tilde{\xi}(k)\psi^T(k)\tilde{\xi}(k)\\&=-\alpha\tilde{\xi}(k)\tilde{\xi}^T(k)\psi(k).\end{aligned} \quad (40)$$

The stability of this difference equation is guaranteed by the following lemma, which is derived from Theorem 2 by setting $L(z)=1$.

Lemma 5. Let $\zeta(k)$ be an arbitrary time-varying vector. Then, the solution of the difference equation:

$$z(k+1)=\left(I-\zeta(k)\zeta^T(k)\right)z(k) \quad (41)$$

tends to a constant vector z_0 such that $\zeta^T(k)z_0\to 0$. If $\zeta(k)$ satisfies the PE condition, the previous z_0 is equal to 0.

Using this lemma, we can state the following theorem.

Theorem 4. Under assumptions (A1) and (A2), the adaptive law:

$$\begin{aligned}&\theta(k+1)-\theta(k)=\\&-\alpha\tilde{\xi}(k)\{u_{fb}(k)+c^T(k)\xi_e(k)+l(k)e(k)\}\end{aligned} \quad (42)$$

yields $e(k)\to 0$. If $\tilde{\xi}(k)$ satisfies the PE condition, the feedforward controller $K_2(z)$ described by Eqs. (14)–(17) tends to P^{-1}. The law of Eqn. (42) can be interpreted as a feedback error learning rule, since $\theta(k)$ is tuned by the feedback input $u_{fb}(k)$, the feedback error $e(k)$, and $\xi_e(k)$ which is generated by $e(k)$.

Proof: From Lemma 5, $\psi(k)$ in Eqn. (40) tends to a constant vector ψ_0 such that $\tilde{\xi}^T(k)\psi_0\to 0$. Thus, from Eqn. (38)

$$\theta(k) \to \text{constant} \tag{43}$$

and

$$\varepsilon(k) = -\tilde{\xi}^T(k)\psi(k) \to 0. \tag{44}$$

From Eqs. (38) and (44), if $\tilde{\xi}^T(k)$ satisfies the PE condition, $\theta(k) \to \theta_0$. By substituting Eqs. (24) and (36) into Eqn. (37), the signal $\varepsilon(k)$ is transformed to:

$$\varepsilon(k) = K_1(z)e(k) + c^T(k)\xi_e(k) + l(k)e(k) \tag{45}$$

Substituting this into Eqn. (39) yields Eqn. (42). From Eqn. (43), $c(k)$ and $l(k)$ in Eqn. (45) tend to constants $\bar{c}$ and $\bar{l}$ as $t \to \infty$. By this fact and the relation (32), Eqn. (45) can be asymptotically described as:

$$\varepsilon(k) = \{K_1(z) + \bar{c}^T(zI - F)^{-1}g + \bar{l}\}e(k). \tag{46}$$

From this and Eqn. (44), we obtain $e(k) \to 0$.

Remarks:

- The adaptive law of (42) utilizes $e(k)$ and $\xi_e(k)$. If $e(k)$ and $\xi_e(k)$ are removed from Eqs. (42) and (34), the law of (42) is identical to (23), which has been proposed in [6] and [7] requiring the strictly positive realness. Thus, we can observe that the adaptive law of (42) removes the positive real condition by utilizing $e(k)$ and $\xi_e(k)$ effectively.
- The DTFEL scheme in this study does not require the assumption (A3) which is related to the strictly positive realness in [6] and [7].
- Due to the removal of the positive realness, design of the feedback controller of the DTFEL control system is separated from adaptation of the feedforward controller. The careful selection of such gain is also eliminated. However such gain is still necessary.
- There are also many advantages of integrating DTFEL with another controller. A salient one is to improve the system robustness. This can be done easily by inserting the derivative gain into the feedback loop of DTFEL systems. This derivative term will act as the noise rejection.
- In the original DTFEL, the stability proof and some restricted requirements of the controlling plant are quite different from those of the continuous time FEL. It requires that the direct input-output transmission gain d has to be positive and sufficiently large. The removal of the positive realness property in the convergence proof overcomes these restrictions and makes DTFEL similar to FEL.

5. DTFEL Control of Noninvertible Plant

In the previous section, it is assumed that $P^{-1}(z)$ exists. This implies that the relative degree of $P(z)$ is zero, which does not always hold in practice. In this section, instead of assumption (A1), we make an assumption that all the finite zeros of $P(z)$ are stable, and the $P(z)$ has relative degree $k > 0$ which is known. The transfer function $P(z)$ is written as:

$$P(z) = \frac{b_k z^{n-k} + b_{k+1} z^{n-k-1} + \cdots + b_n}{z^n + a_1 z^{n-1} + \cdots + a_n}, \tag{47}$$

where a_i $(i = 1,\ldots,n)$ and b_i $(i = k,\ldots,n)$ are unknown parameters.

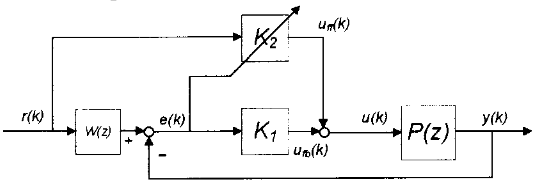

Fig 2. DTFEL Scheme for Noninvertible Plant

To cope with noninvertibility of $P(z)$, an approximate inverse $W(z)P^{-1}(z)$ is introduced. The $W(z)$ is a transfer function defined as:

$$W(z) = \frac{w_{k+1}}{z^k + w_1 z^{k-1} + \cdots + w_k}, \tag{48}$$

where w_i $(i = 1,\ldots,k+1)$ are known. The DTFEL control system for the noninvertible plant is illustrated by Fig. 2. If $K_2(z)$ is tuned to be the approximate inverse $W(z)P^{-1}(z)$, the transfer function from r to y becomes $W(z)$. This implies that the tracking performance can be characterized by $W(z)$. From Eqs. (47) and

$$\theta(k+1)-\theta(k)=\alpha\tilde{\xi}_w(k)\varepsilon_w(k). \qquad (71)$$

From Eqn. (69), this law is rewritten as:

$$\psi(k+1)-\psi(k)=-\alpha\tilde{\xi}_w(k)\psi^T(k)\tilde{\xi}_w(k)$$
$$=-\alpha\tilde{\xi}_w(k)\tilde{\xi}_w^T(k)\psi(k), \qquad (72)$$

where α is an adaptive gain. In the same manner as Theorem 4, we obtain the following theorem.

Theorem 5. The adaptive law

$$\theta(k+1)-\theta(k)=-\alpha\tilde{\xi}_w(k)\{W(z)u_{fb}(k) + c^T(k)\xi_e(k)+l(k)e(k)\} \qquad (73)$$

yields $e(k)\to 0$. If $\tilde{\xi}_w(k)$ satisfies the PE condition, the feedforward controller $K_2(z)$ described by Eqs. (49)–(51) tends to $W(z)P^{-1}$.

Proof: From Lemma 5, $\psi(k)$ in Eqn. (72) tends to a constant vector ψ_0 such that $\tilde{\xi}_w^T(k)\psi_0\to 0$. Thus, from Eqn. (70)

$$\theta(k)\to \text{constant} \qquad (74)$$

and

$$\varepsilon_w(k)=-\tilde{\xi}_w^T(k)\psi(k)\to 0. \qquad (75)$$

If $\tilde{\xi}_w^T(k)$ satisfies the PE condition, $\theta(k)\to\theta_0$ from Eqs. (70) and (75). From Eqs. (51), (67), and (68), we obtain:

$$\varepsilon_w(k)=W(z)\{u_{ff}(k)+u_{fb}(k)\}-\hat{u}_w(k)$$
$$=W(z)\theta^T(k)\xi_w(k)+W(z)u_{fb}(k)$$
$$-\theta^T(k)\tilde{\xi}_w(k). \qquad (76)$$

From Eqs. (49), (50), (60), and (61), then:

$$W(z)\xi_{w1}(k)\to\xi_{wr}(k),$$
$$W(z)\xi_{w2}(k)\to\xi_{wu}(k). \qquad (77)$$

Then, from Eqs. (52) and (66), (76) is transformed to:

$$\varepsilon_w(k)=c^T(k)\xi_e(k)+l(k)e(k)+W(z)u_{fb}(k). \qquad (78)$$

Substituting this into Eqn. (71) yields Eqn. (73). From Eqn. (74), $c(k)$ and $l(k)$ in Eqn. (78) tend to constants c_0 and l_0 as $t\to\infty$. By this fact and Eqn. (32), the Eqn. (78) can be asymptotically described as:

$$\varepsilon_w(k)=\{\bar{c}_w^T(zI-F)^{-1}g+\bar{l}_w+W(z)K(z)\}e(k) \qquad (79)$$

From this and Eqn. (74), we obtain $e(k)\to 0$.

6. Simulation Results

In this section, the simulation results are illustrated to demonstrate the effectiveness of the theoretical results obtained in this study. Three main simulations have been done in order to guarantee the mathematical analysis.

In the first part, the simulation of the traditional DTFEL with stable and invertible stable plant is demonstrated. The pulse-transfer function of the plant is $P(z)=\dfrac{z+0.3}{z+0.5}$. Note that this plant is stable and has a stable inverse which satisfies the required assumption of DTFEL. A feedback controller is selected as $K_1(z)=\dfrac{z}{z+0.3}$.

In Fig. 3, the tracking performance between the input signal $r(k)$ and the output signal $y(k)$ is shown. It shows the convergence of the signal and the comparison of the tracking performance of the system when the adaptation began, from 0 sec, and after adaptation was successfully done, after 5.7 second.

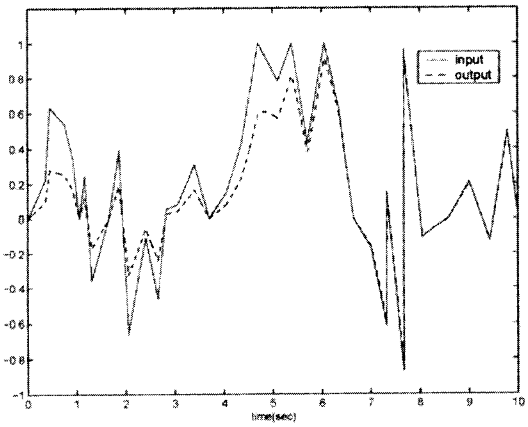

Fig 3. The simulation result of DTFEL control.

The tracking error $e(k)=y(k)-r(k)$ is also shown in Fig. 4. It should be noted that the error pulses between 7 and 8 seconds can be considered as the effect of unusual performance of the input.

It is interesting that the system is still stable. Note also that the learning rate is set to be very low to show the result clearly. In fact, the adaptation rate is very fast and the adaptation can be completely done within a millisecond.

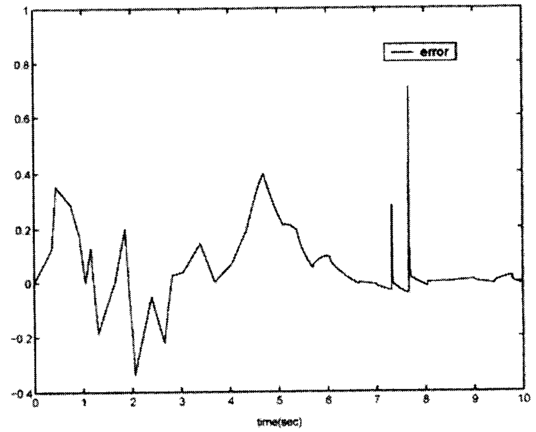

Fig 4. Error of DTFEL control.

For the second part, the DTFEL is extended to control the improper plant with a relative degree of 1. The pulse-transfer function of the controlling plant is $P(z)=\dfrac{1}{z+0.3}$, and the chosen pre-filter is $W(z)=\dfrac{1}{z+0.2}$.

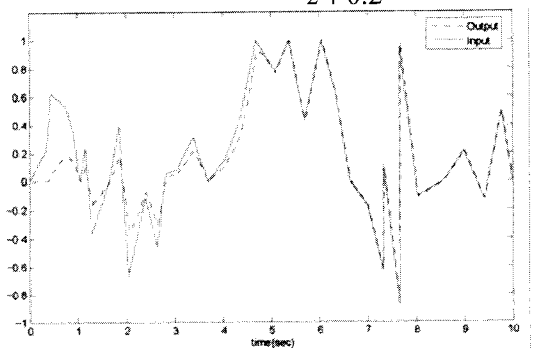

Fig 5. The simulation result of the DTFEL system when the relative degree of plant is 1.

In Fig. 5, the tracking performance between the input signal $r(k)$ and the output signal $y(k)$ is shown. This figure shows the convergence of the signal and the comparison of the the tracking performance of the system when the adaptation began, from 0 seconds, and after adaptation was successfully done, after 5.7 seconds. The learning rate is also set to be very low to show the result clearly. Finally, the simulation demonstrates the improved controlled system due to the freedom to select the feedback gain. The simulation results of DTFEL in two cases will be presented in this section; DTFEL with a constant feedback gain, and DTFEL with a PD feedback gain where the disturbance is present in both cases.

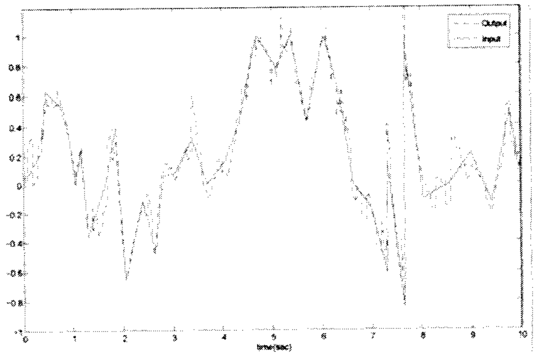

Fig 6. The simulation result of the DTFEL system with constant feedback gain with system disturbance.

Each simulation has the same input. In both cases, the pulse-transfer function of the controlling plant is $P(z)=\dfrac{z+0.2}{z+0.3}$.

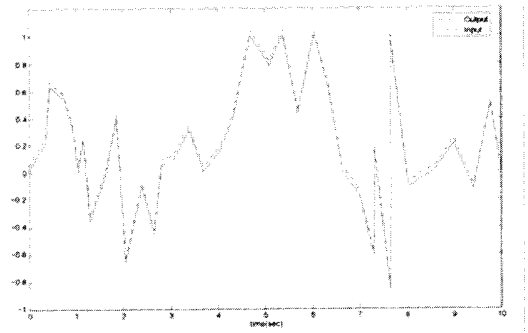

Fig 7. The simulation result of the DTFEL system with PD controller with system disturbance.

The result of DTFEL with a constant feedback gain in the presence of disturbance depicted by Fig. 6 confirms that DTFEL performs poorly under disturbance, as the fluctuated disturbance perturbs the input to the extent that the input is indistinguishable. Fig. 7 shows the result of the controlled system improved by replacing feedback gain with PD controller which is in the form of $K_1(z)=K_p+K_d\dfrac{z-1}{z}$, where K_p=1, K_d=0.1. The fluctuation is completely removed from the system output. The DTFEL with PD feedback controller gives a significant improvement in robustness. Specifically, the additional PD controller acts like a disturbance rejecter of the system. It shows the capability to apply another controller to the feedback path to improve the system performance. It is also clear that the DTFEL without recourse to positive realness eliminates many constraint assumptions from the original DTFEL presented in [6] and [7].

7. Conclusion

This study presents a new scheme for adaptive control based on DTFEL where the feedforward controller is tuned to be the inverse of the plant model. The necessity of a positive realness condition in [6] and [7] is removed from the design. This is the same as removing the strictly positive realness from [11] as shown in [5]. In [6] and [7], the adaptive law is derived from a control problem of finding $u_{ff}(k)$ such that $u_{ff}(k) \to u_0(k)$, which leads to the issue of the strictly positive realness. This necessity of the positive realness also occurred in "identification problem" [10,12,13], where the issue of strictly positive realness arises from the error equation based on a "control problem" of finding $u(k)$ such that $y(k) \to y_m(k)$ where $y_m(k)$ is a desired signal.

To avoid such an issue, an identification problem of obtaining the inverse model P^{-1} is considered instead. By introducing a linear parametric representation of the output signal of P^{-1} and estimating the parameter $\theta(k)$, the error equation for this problem is then calculated as $\varepsilon(k) = u(k) - \hat{u}(k) = (\theta_0 - \theta(k))^T \tilde{\xi}(k)$. This does not involve any transfer functions which should be strictly positive real. Although the adaptive law is derived from the identification problem, the control objective $e(k) \to 0$ is guaranteed. Furthermore, it has been revealed that the $\varepsilon(k)$ is expressed in terms of $e(k)$ and $u_{fb}(k)$. This implies that the adaptive law is consistent with the fundamental ideal of DTFEL; that is, the law is driven by the feedback error signals. Due to the removal of the positive realness of $L(z)$ depending on the choice of $K_1(z)$, the design of the feedback controller of the DTFEL control system is separated from adaptation of the feedforward controller. This separation is in accord with the property which the standard type of two-degrees-of-freedom control will possess. These results allow DTFEL in brain motor control to be more applicable to practical control systems. This leads to the notion of bio-mimetic control.

An integrated controller for DTFEL systems and the stability of DTFEL for the plant with time-delay could be future work. New algorithms for solving the control problems and improving the system performance are also open-problems in this research field.

8. References

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