Tolerance Design via Inverse Analysis and Cost-of-Quality Minimization in a Straight Flanging Process

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Abstract

Tolerance plays an important role in modern manufacturing processes and directly impacts the product quality and cost. The decision on tolerance management in manufacturing processes is an important task for the business to make products competitively. In this paper, the tolerance design is studied by two different approaches: inverse analysis and cost-of-quality minimization. For the inverse analysis, the tolerance allocation is obtained by setting a desired tolerance of the product and then determining an optimal combination of input tolerances resulting in the desired product tolerance. For the cost-of-quality minimization, the tolerance allocation is obtained by minimizing the cost of quality through setting an illustrative cost function. In this paper, we use a straight flanging process to demonstrate the tolerance design schemes. The computational cost of the uncertainty analysis of springback prediction in the straight flanging process is reduced by using response surface techniques. An optimization formulation is used to create mathematical models. The computational results for making a decision on tolerance management for the sample straight flanging process are presented. Numerical problems on non-uniqueness of solutions in the inverse analysis are presented. Remarks on solving these classes of problems are also discussed.

Keywords: tolerance design; inverse analysis; cost of quality; uncertainty propagation;

1. Introduction

In the currently competitive market, balancing the product quality and cost is a challenging problem that every business nowadays needs to solve. Generally, to achieve a higher product quality requires proportionally a higher cost. In every manufacturing process, tolerance is a parameter used to control quality due to the existence of variations in real systems [Abdel-Maleck and Asadathorn, 1994; Groover, 2001; Jordaan and Ungerer, 2002]. Tolerance has been an important factor in manufacturing processes since the idea of interchangeable parts was practically implemented. To achieve a high quality, small tolerance is usually enforced; as a result, the operating cost increases. On the other hand, selecting broad tolerance ranges could violate the functionality requirement and lead to a future significant loss. Therefore, a systematic approach for tolerance design is important in today manufacturing systems.

Tolerance design is a means to manage the combination of the tolerance of each design parameter to achieve a criterion. Two schemes for the tolerance design are considered. The first scheme is to determine an optimal combination of the input tolerances to meet a specifically desired tolerance, also known as inverse analysis. The second scheme is to determine an optimal combination of the input tolerances to minimize the cost of quality. In this paper, a combination of the standard deviation of design variables given their nominal values is designed to illustrate the methodologies.

In this paper, a straight flanging process in Buranathiti and Cao (2004) is used to demonstrate the tolerance design. In sheet metal forming operations, a flanging process, one of the most common processes, is used to either create a mating surface or increase the stiffness of a sheet panel. One of the major concerns in flanging processes is springback, a shape discrepancy between the fully loaded and unloaded configurations. Therefore, the final configuration of the flanged sheet after springback is used as the output variable.

By incorporating the uncertainty of the system into the model, we can obtain statistical descriptions of the system. However, uncertainty analysis is typically a challenging task due to a significantly higher computational effort. Its deterministic nature is considerably large due to the use of finite element simulations [Liu *et al.*, 1986; Buranathiti 2004; Buranathiti *et al.*, 2006]. For a probabilistic nature being static, which time has no substantive role, we use Monte Carlo simulation to obtain the statistical descriptions of the system for the tolerance design.

The paper is organized as follows: two approaches for the tolerance design will be presented in Section 2 followed by, details on the sample flanging process in Section 3, the uncertainty analysis in Section 4, examples and results in Section 5, and, finally, discussions and remarks in Section 6.

2. Tolerance design

Tolerance is a parameter widely used to control manufacturing processes. Typically, tolerance is to create an acceptable region consisting of upper and lower limits because of unavoidable variation due to uncertainty. It is generally considered to be unacceptable if a variable is outside its tolerance limits. In the literature, tolerance and variance (standard deviation σ) have a blured relationship. However, some papers consider these terms separately. In this paper, it is considered that the nature of a normally functional process should stay within the tolerance limits. Thus, a system that properly follows the tolerance limits should have characteristics (variance or standard deviation) that correspond to the tolerance limits, i.e., these two terms are related and interchangeable. In this paper, all design variables presumably are normally distributed. Using a standard approach in engineering quality control relating to process capability, a range of 6σ or 99.74% is used to describe the tolerance as follows:

$$\sigma = \frac{\text{tolerance}}{6}. \quad (1)$$

This range is also known as the natural tolerance limit.

Basic methods for estimating tolerance are the worst-case scenarios and the root sum square. Both methods are typically used in a linear assembly and for analytical purpose. In this paper, the tolerance design is conducted by using two systematic schemes following the flow chart shown in Fig. 1.

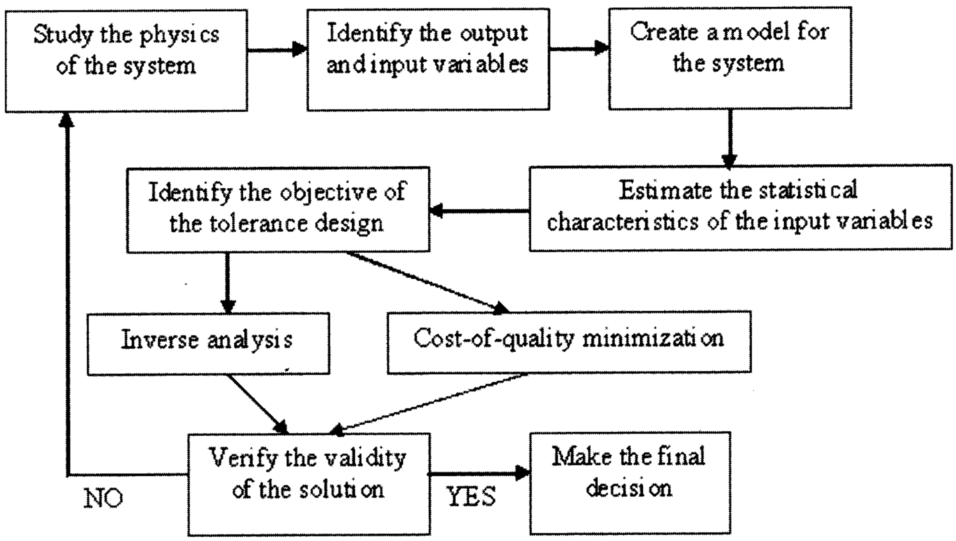

Figure 1. The flow chart for the tolerance design.

2.1. Inverse Analysis

For the tolerance design, an inverse problem of the flanging process is mathematically modeled as an optimization model, where the objective is to determine an optimal combination of the statistical descriptions of the design/input variables that produce a specified value of the final product. The optimization model for the inverse problem is constructed by minimizing the squared error between the desired and computed values as follows:

$$\begin{aligned} &\text{Minimize} && (\Theta - \theta(\mathbf{x}))^2, \\ &\text{Subject to} && \mathbf{x} \in \Omega, \end{aligned} \qquad (2)$$

where Θ is a desired value, $\mathbf{x}$ is a design variable vector, $\theta(\mathbf{x})$ is a computed value, and Ω is a feasible set.

Strictly speaking, this class of problem has two objective functions, i.e., the expected value and the variance of the final configuration, since the expected value of an output function is, in general, not equal to the function substituted with the expected values of each individual variable as shown in Eq. (3).

$$\mathrm{E}[y(x)] \neq y(\mathrm{E}[x]). \qquad (3)$$

However, in this particular problem, the expected value of the output function is insignificantly sensitive to the variation of the standard deviation of the design variables (see Fig. 5 in Section 5). We can see that by varying the standard deviation of input variables, the nominal value of the output y changes insignificantly (less than one percent). Therefore, the model is reduced to only the variance of the final configuration as the objective function.

The mathematical model is rewritten to the specific purpose in this paper as follows:

$$\begin{aligned} &\text{Minimize} && (\Sigma_y - \sigma_y)^2, \\ &\text{Subject to} && \boldsymbol{\sigma}_x \in \Omega, \end{aligned} \qquad (3)$$

where Σ_y is a desired value, σ_y is a computed value, $\boldsymbol{\sigma}_x$ is a vector of the standard deviation of design variables, and Ω is a feasible set.

2.2. Cost-of-Quality Minimization

Cost of quality (also known as quality cost) is a measure that goes beyond the traditional cost, e.g., labor cost, material cost, etc. The hidden presence of the cost of quality is usually analogous to an iceberg, i.e., the tip of the iceberg (the visible part) is the traditional cost. The cost of quality comprises the cost of control (i.e., prevention cost and appraisal cost) and the cost of failure of control (i.e., internal failure cost and external failure cost) in Feigenbaum (1983). These four parts of the cost of quality are also known as the costs of poor quality in Juran and Gryna (1988). In this paper, these four parts are simply grouped into two categories for the mathematical modeling, i.e., tolerance-controlled cost and penalty cost.

The cost-of-quality (COQ) formulation is a combination of the tolerance-controlled cost (TCC) and the penalty cost (PC) as follows:

$$\mathrm{COQ} = \sum_{i=1}^{N} \mathrm{TCC}_{x_i} + \mathrm{PC}_y, \qquad (4)$$

where N is the number of design variables. The tolerance-controlled cost (TCC_{x_i}) associated with the effort to control tolerance of the variable x_i within a given value σ_{x_i} is defined in this paper as follows:

$$\mathrm{TCC}_{x_i} = \frac{a_i}{\sqrt{\sigma_{x_i}}}, \qquad (5)$$

where a_i is a constant, and σ_{x_i} is the standard deviation of the design variable x_i. The penalty cost (PC_y) associated with the effort to liability, rework, and so forth as a result of the standard deviation (σ_y) of the output y is defined by using the Taguchi loss function in a quadratic function as follows:

$$\mathrm{PC}_y = a_y \cdot (\sigma_y)^2, \qquad (6)$$

where a_y is a constant, and σ_y is the standard deviation of the output y. Schematics of both costs are shown in Fig. 2.

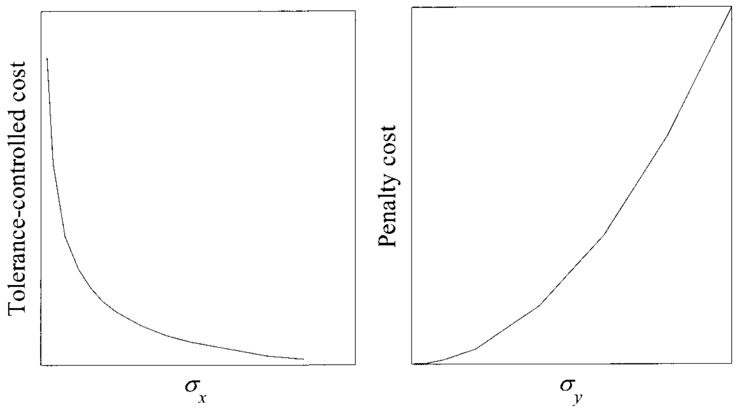

Figure. 2. Schematics of the tolerance-controlled cost and penalty cost.

A mathematical model for the tolerance design is similarly formulated into an optimization model to determine an optimal combination of the statistical descriptions (σ_{x_i}) of the design variables to minimize the cost of quality as follows:

$$\begin{aligned} &\text{Minimize} && COQ, \\ &\text{Subject to} && \boldsymbol{\sigma}_x \in \Omega, \end{aligned} \quad (7)$$

where $\boldsymbol{\sigma}_x$ is a vector of the standard deviation of the design variables, and Ω is a feasible set.

2.3. Variable Normalization

Due to potential numerical problems occurring from different scales of each variable x_i during regression analysis and optimization computation, all variables x_i are normalized into x_i' to avoid potential numerical problems as follows:

$$x_i' = \frac{x_i - \mathrm{Min}(x_i)}{\mathrm{Max}(x_i) - \mathrm{Min}(x_i)} = \frac{x_i - \mathrm{Min}(x_i)}{\mathrm{Range}(x_i)}, \quad (8)$$

where $x_i' \in [0,1]$ or x_i' is an order of one ($x_i' \in O(1)$). The variance of the normalized i^{th} input variable x_i' is defined as follows:

$$\mathrm{VAR}[x_i'] = \mathrm{VAR}\left[\frac{x_i - \mathrm{Min}(x_i)}{\mathrm{Range}(x_i)}\right] = \frac{\mathrm{VAR}[x_i]}{(\mathrm{Range}(x_i))^2} \quad (9)$$

where $\mathrm{VAR}[\cdot]$ is a statistic operator. The standard deviation ($\sigma_{x_i'}$) of the normalized i^{th} input variable x_i' is directly defined as:

$$\sigma_{x_i'} = \frac{\sigma_{x_i}}{\mathrm{Max}(x_i) - \mathrm{Min}(x_i)} = \frac{\sigma_{x_i}}{\mathrm{Range}(x_i)}, \quad (10)$$

where $\sigma_{x_i'} \in O(1)$. The minimum and maximum values are only supposedly given in a reasonable manner. They need not to be very precise because the purpose is only to reduce the different scales of variables to the same scale for numerical purposes. Similarly, for the computation involving the standard deviation (σ_{x_i}) of the i^{th} input variable x_i as the design variable, we also define a normalized standard deviation ($\tilde{\sigma}_{x_i}$) of the i^{th} input variable as follows:

$$\tilde{\sigma}_{x_i} = \frac{\sigma_{x_i} - \mathrm{Min}(\sigma_{x_i})}{\mathrm{Max}(\sigma_{x_i}) - \mathrm{Min}(\sigma_{x_i})}, \quad (11)$$

where $\tilde{\sigma}_{x_i} \in O(1)$. The variables in real scale are simply the inversion of the normalizing function. It should be noted that $\sigma_{x_i'}$ and $\tilde{\sigma}_{x_i}$ are different by their purpose. x_i' is used in a model involving x_i as the independent variable. $\tilde{\sigma}_{x_i}$ is used in a model involving σ_{x_i} as the independent variable.

3. A straight flanging process

A straight flanging process is used as an example system for the tolerance design in this paper. A setup for a conventional flanging process consists of a punch, a binder, a die and a blank. The straight flanging process is evaluated by considering the final configuration (θ_f) as a result of the fully loaded configuration (θ_0) and the springback angle ($\Delta\theta$) as shown in Fig. 3.

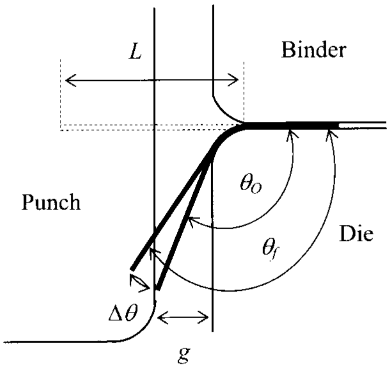

Figure 3. Schematic of a straight flanging process (from Buranathiti and Cao, 2004).

The straight flanging process in this paper is characterized by five variables, i.e., flanging length (L), sheet thickness (t), gap (g), Young's modulus (E), and yield stress (Y). Experiments in Buranathiti *et al.* (2006) show a relationship between the strength coefficient (K) and the hardening exponent (n) with the yield stress Y. Therefore, K and n are considered as functions of Y and reduce the number of design variables from seven to five.

In the literature (Taylor *et al.*, 1995; Cao *et al.*, 1999; Song *et al.*, 2000; Buranathiti and Cao, 2004), springback, a mostly elastic recovery process, is defined as a shape discrepancy between the fully loaded and unloaded configurations. Springback is a major concern in sheet metal forming processes. The springback angle $\Delta\theta$ is computed by using a commercial finite element code with nonlinear solver (ABAQUS™) in this paper. The finite element models for springback prediction consist of an implicit-time integration scheme, von Mises' yield criterion, power work hardening law, combined hardening law, and soft contact algorithm.

4. Uncertainty analysis

An important element in the tolerance design is the estimation of the system's statistical descriptions, i.e., the standard deviation. Therefore, the uncertainty analysis of the system needs to be extensively conducted to obtain the standard deviation of the output for the purpose of the tolerance design. However, the computational effort for uncertainty analysis generally is significantly higher than the deterministic model as follows:

$$\mathrm{E}[y] = \int_{-\infty}^{\infty} y \cdot f(y) dy, \tag{12}$$

$$\mathrm{VAR}[y] = \mathrm{E}\left[(y - \mathrm{E}[y])^2\right] = \int_{-\infty}^{\infty} (y - \mathrm{E}[y])^2 \cdot f(y) dy \tag{13}$$

where $\mathrm{E}[\cdot]$ is the expected value operator, and $f(y)$ is the probability density function of variable y.

Before the computation part is implemented, the specific problem in this paper can be assumed to be static, where the passage of time plays no substantial role. Therefore, Monte Carlo simulation, which is suitable and applicable for problems that are not analytically tractable, can be applied. Monte Carlo simulation basically is a statistical technique employing a random number $U(0,1)$. By considering an integration as follows:

$$\theta = \int_0^1 g(y) dy. \tag{14}$$

It can be rewritten as follows:

$$\theta = \mathrm{E}[g(U)], \tag{15}$$

where U is uniformly distributed over (0,1). By taking the strong law of large numbers, an expression can be obtained:

$$\lim_{k\to\infty}\sum_{i=1}^{k}\frac{g(U_i)}{k}\to \mathrm{E}[g(U)]=\theta. \qquad (16)$$

Using this concept, the statistical descriptions of the system can be obtained.

In this paper, all input/design variables are assumed to be normally distributed with parameters $\bar{x}_i$ and $\sigma^2_{x_i}$ as mean and variance, respectively. Two important issues in Monte Carlo simulation are the random number and the number of samples. For the random number, a random number generator from MATLAB™ is used. Further statistical details about random number generators can be consulted in Law and Kelton (1991). For the number of samples for Monte Carlo simulation, a simple rule of thumb is a large number that makes the statistical descriptions converge as shown in Eq. (16). The number of samples used in this paper is discussed in Section 5 and Fig. 4.

In this paper, response surface techniques [Jin *et al.*, 2001; Montgomery *et al.*, 2001] are utilized to reduce the computational cost of the uncertainty analysis. By taking advantage of a surrogate model representing the finite element simulations, the final configuration is represented by a polynomial function as follows:

$$\hat{y}=a+\sum_{i=1}^{N}b_i x_i+\sum_{i=1}^{N}c_i x_i^2+\sum_{i=1}^{N}\sum_{j=i}^{N}d_{ij}x_i x_j, \qquad (17)$$

where $\hat{y}$ is the estimated fully-unloaded configuration of the deformed sheet, a, b_i, c_i and d_{ij} are the corresponding constants, and N is the number of design variables. The effectiveness of the surrogate model $\hat{y}$ is measured by evaluating the mean of sum of squared error (MSE) as follows:

$$\mathrm{MSE}=\frac{\sum_{i=1}^{n}(y-\hat{y})^2}{n}, \qquad (18)$$

where n is the number of samples for the regression analysis, and y is the computational results from finite element methods. The linear regression analysis is used to create the surrogate model. For simplicity Eq. (17) is rewritten into a matrix form as follows:

$$\mathbf{Y}=\mathbf{XA}, \qquad (19)$$

where $\mathbf{Y}$ is the $n\times 1$ vector of the dependent variable, and $\mathbf{X}$ is the $n\times m$ matrix of the independent variables. The $m\times 1$ constant vector $\mathbf{A}$ in Eq. (19) is obtained by solving the normal equation as follows:

$$\mathbf{A}=\left(\mathbf{X}^{\mathrm{T}}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{Y}, \qquad (20)$$

where $\mathbf{X}^{\mathrm{T}}$ is a transpose of the matrix $\mathbf{X}$. In addition, the prediction of the standard deviation (σ_y) of the output variable y with respect to the combination of $\boldsymbol{\sigma}_x$ of the design variables can be further substituted as follows:

$$\hat{\sigma}_y=f(\boldsymbol{\sigma}_x), \qquad (21)$$

where $\hat{\sigma}_y$ is the estimated standard deviation of the product y, and $f(\cdot)$ is a surrogate function. In this paper, this estimation is called as the second response surface approximation.

5. Illustrative examples and results

The straight flanging process presented in Section 3 is used as an example for the tolerance design. Given the mean or nominal values of the design variables in Table 1, we use the data presented on both tolerance design schemes to obtain an optimal combination of tolerance or standard deviation of the design variables.

Table 1. The mean or nominal values of the design variables.

Variable	$x_1=L$ (in)	$x_2=t$ (mm)	$x_3=g$ (mm)	$x_4=E$ (MPa)	$x_5=K$ (MPa)
Mean value	4.06	1.55	19.98	1.97E5	375.9

For the effectiveness of the uncertainty analysis, the number of samples in Monte Carlo simulation is estimated by monitoring a convergence of the system's statistical descriptions, i.e., the expected value (E[y]) and the standard deviation (σ_y). From Fig. 4, it can be seen that the solutions seem to be steady after 100k samples. For being on the safe side, 200k samples are used for the Monte Carlo simulation.

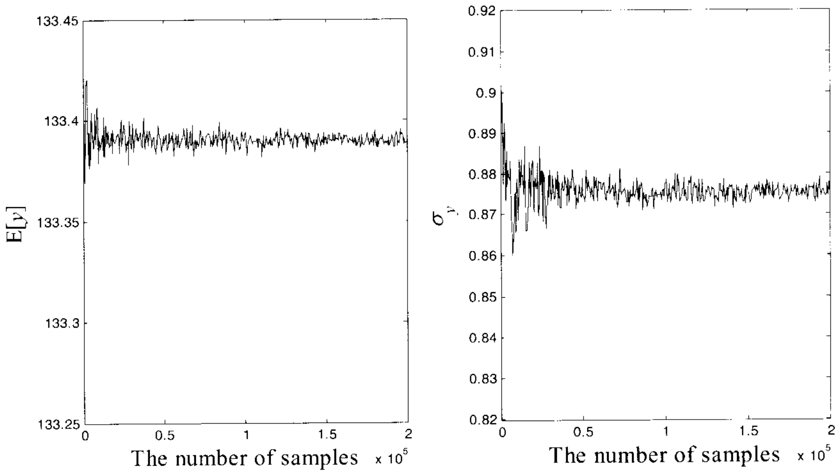

Figure 4. The effect of the number of samples in Monte Carlo Simulation.

An observation on the effect of different combinations of the standard deviation of the design variables on the expected value of (E[y]), of the output y in Fig. 5 is obtained by varying the combination of the standard deviation of the design variables. For this specific problem, it can be considered that different combinations of the standard deviation of the design variables have little effect on the expected value of the output. Therefore, the models have been reduced as previously discussed in Section 2.

The computational cost of the uncertainty analysis for the springback prediction from the finite element methods is reduced by utilizing the response surface techniques presented in Section 4. An indicator for the effectiveness of the model, the mean of the sum of squared error is 0.02827 or 0.15625% with respect to the average value of y. The response surface techniques are utilized to substitute the standard deviation of the substitute prediction from Monte Carlo simulation. The indicator for the effectiveness of the model, the mean of the sum of squared error, is 0.01022 or 0.52166% with respect to average value of σ_y. Note that a higher order polynomial function is used for this second response surface approximation.

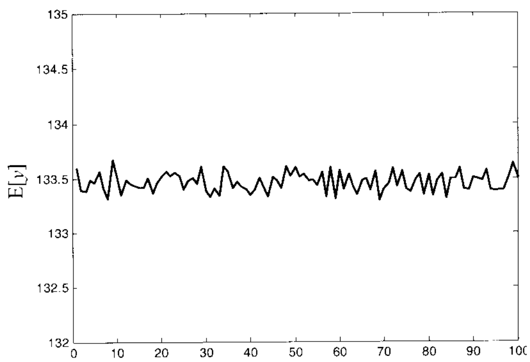

Figure 5. An observation on the effect of different combinations of σ_x on E[y].

To demonstrate the two tolerance design schemes, the presentation is divided into two groups, i.e., the inverse analysis and the cost-of-quality minimization. In each group, two uncertainty analysis techniques, i.e., σ_y from Monte Carlo simulation and the second response surface approximation, are used. To observe the influence of the initial values to the solution, the computations with two different sets of initial values shown in Table 2 are conducted for each approach.

Table 2. The initial values of the design variables σ_x (same units with Table 1).

Variable	$\sigma_L = \sigma_{x_1}$	$\sigma_t = \sigma_{x_2}$	$\sigma_g = \sigma_{x_3}$	$\sigma_E = \sigma_{x_4}$	$\sigma_K = \sigma_{x_5}$
#1	0.2000	0.0775	1.5000	9.861e+3	18.7500
#2	0.1200	0.0465	0.9000	5.917e+3	11.2500

5.1. Inverse Analysis

In this example, the desired value of the output tolerance (Σ_y) is set to 0.70 and the optimization model in Eq. (3) is numerically solved.

5.1.1. Uncertainty analysis with Monte Carlo simulation

By applying the first set of initial values, an optimal combination of σ_x is shown in Fig. 6. Similarly, by applying the second set of initial values, an optimal combination of σ_x is shown in Fig. 7.

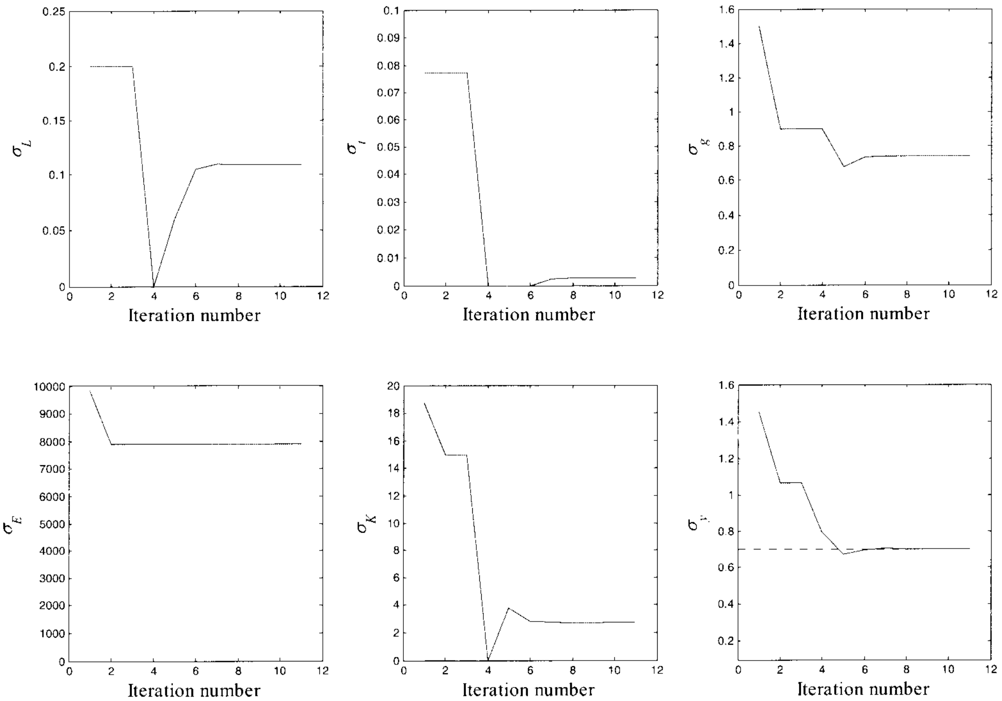

Figure 6. An optimal combination of σ_x from the first set of initial values.

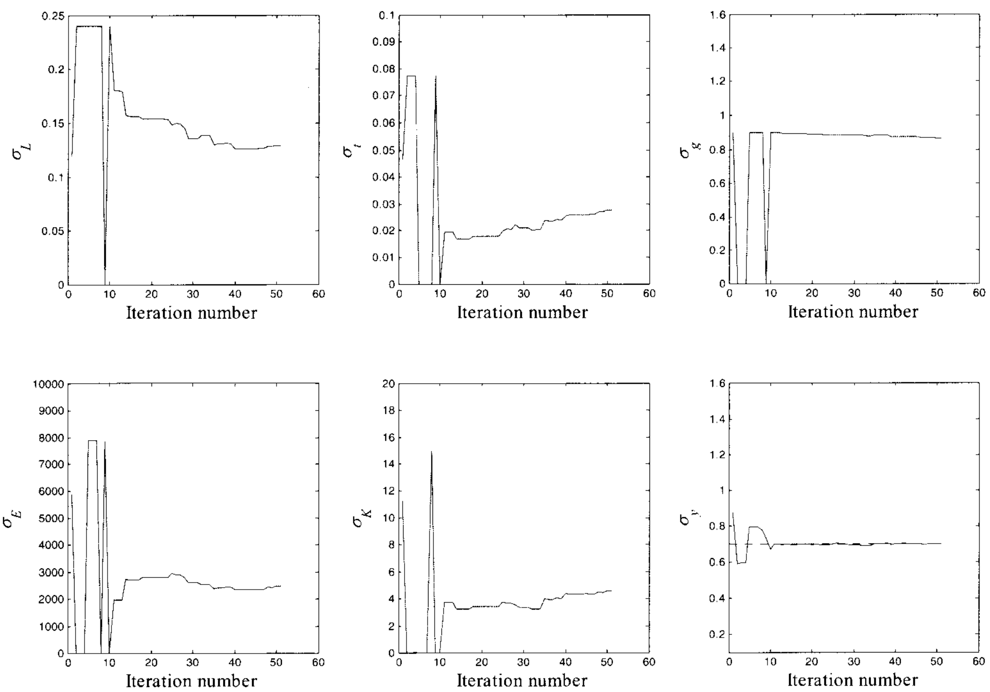

Figure 7. An optimal combination of σ_x from the second set of initial values.

5.1.2. Uncertainty analysis with the second response surface approximation

By applying the first set of initial values, an optimal combination of σ_x is shown in Fig. 8. Similarly, by applying the second set of initial values, an optimal combination of σ_x is shown in Fig. 9.

5.2. Cost-of-Quality Minimization

The summary of the parameters for the cost-of-quality formulation in Eqs. (4)-(6) is presented in Table 3. It should be noted that normalized variables are used here. The optimization model is numerically solved in Eq. (7) to minimize the cost of quality for the tolerance design.

5.2.1. Uncertainty analysis with Monte Carlo simulation

By applying the first set of initial values, an optimal combination of σ_x and the cost of quality are shown in Figs. 10 and 12, respectively. Similarly, by applying the second set of initial values, an optimal combination of σ_x and the cost of quality are shown in Figs. 11 and 13, respectively.

Table 3. The constants for the cost-of-quality function.

Variable	$\tilde{\sigma}_L$	$\tilde{\sigma}_t$	$\tilde{\sigma}_g$	$\tilde{\sigma}_E$	$\tilde{\sigma}_K$	$\tilde{\sigma}_y$
a_i	1.0	0.622	2.739	222.05	9.682	6.741

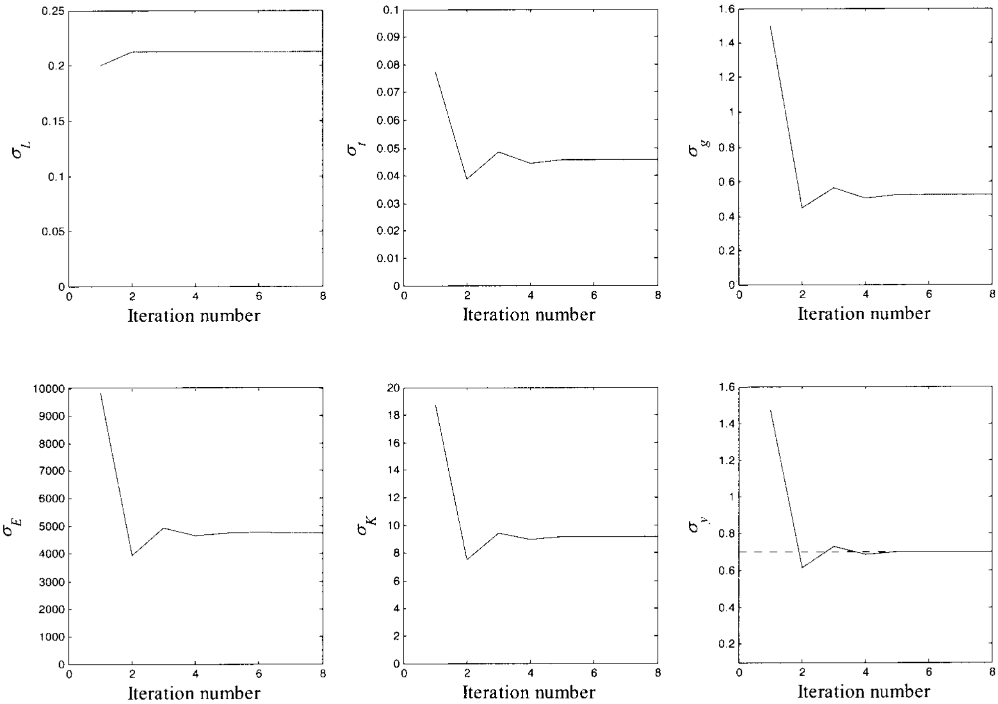

Figure 8. An optimal combination of σ_x from the first set of initial values.

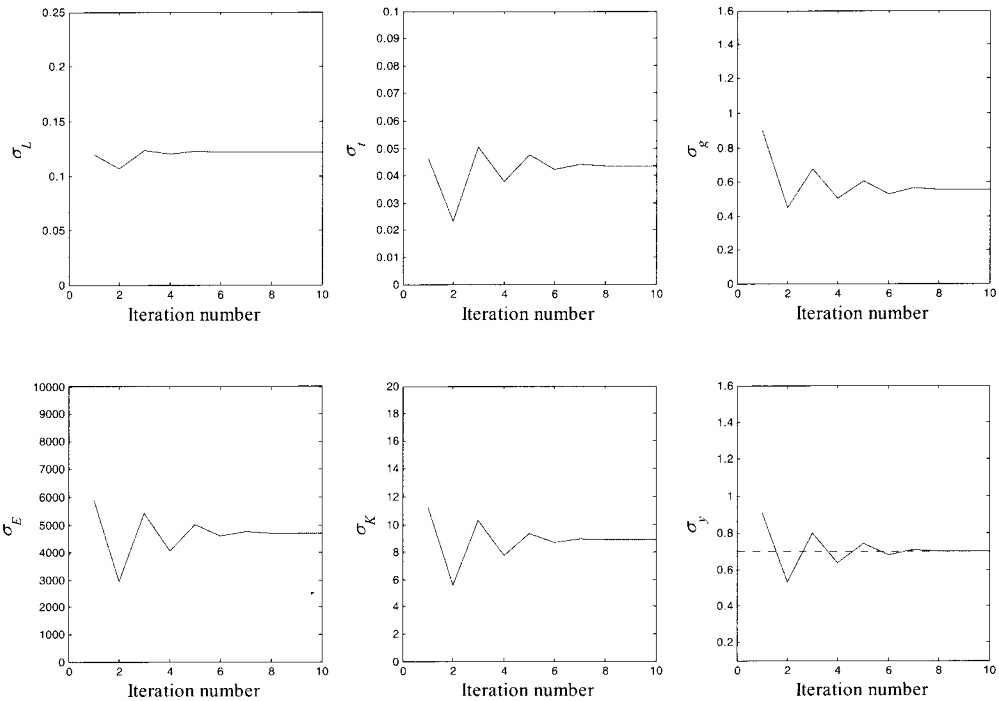

Figure 9. An optimal combination of σ_x from the second set of initial values.

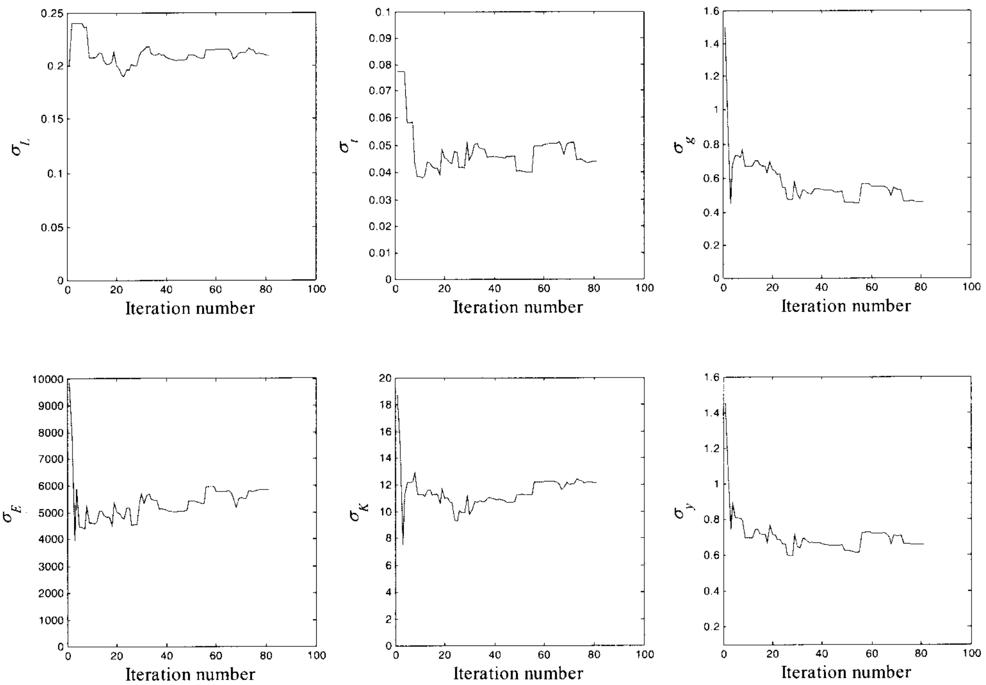

Figure 10. An optimal combination of σ_x from the first set of initial values.

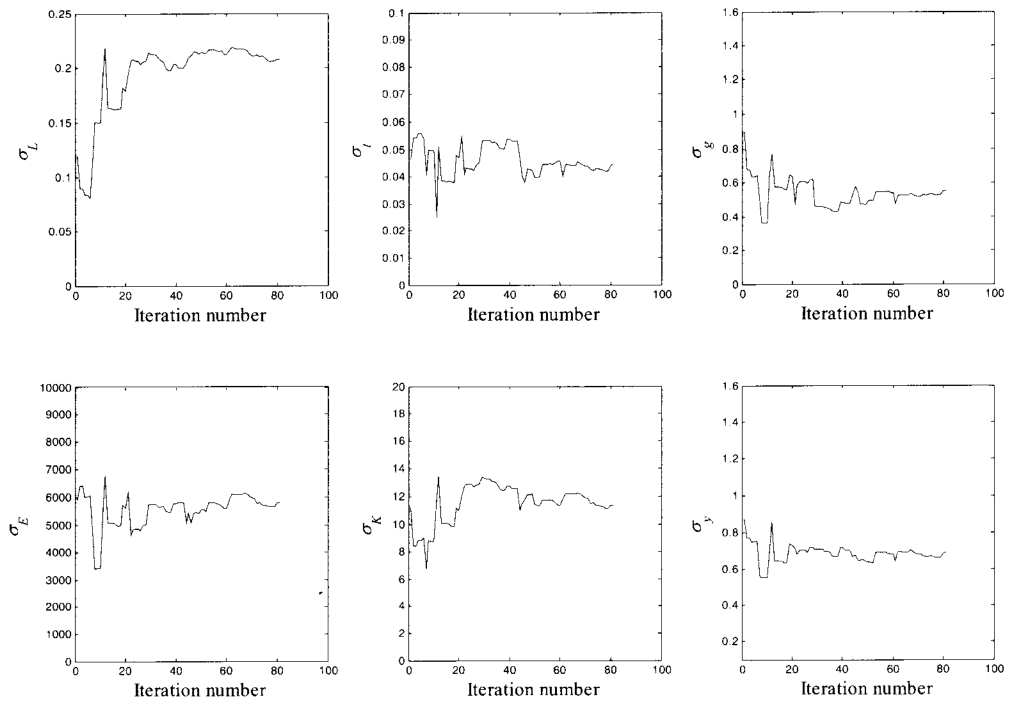

Figure 11. An optimal combination of σ_x from the second set of initial values.

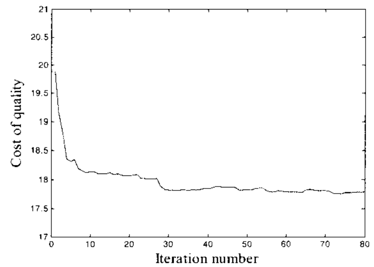

Figure 12. The cost of quality from the first set of initial values.

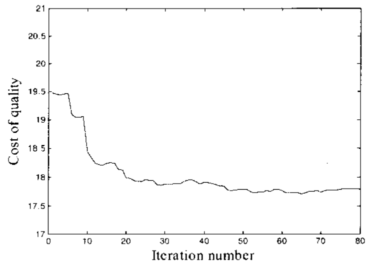

Figure 13. The cost of quality from the second set of initial values.

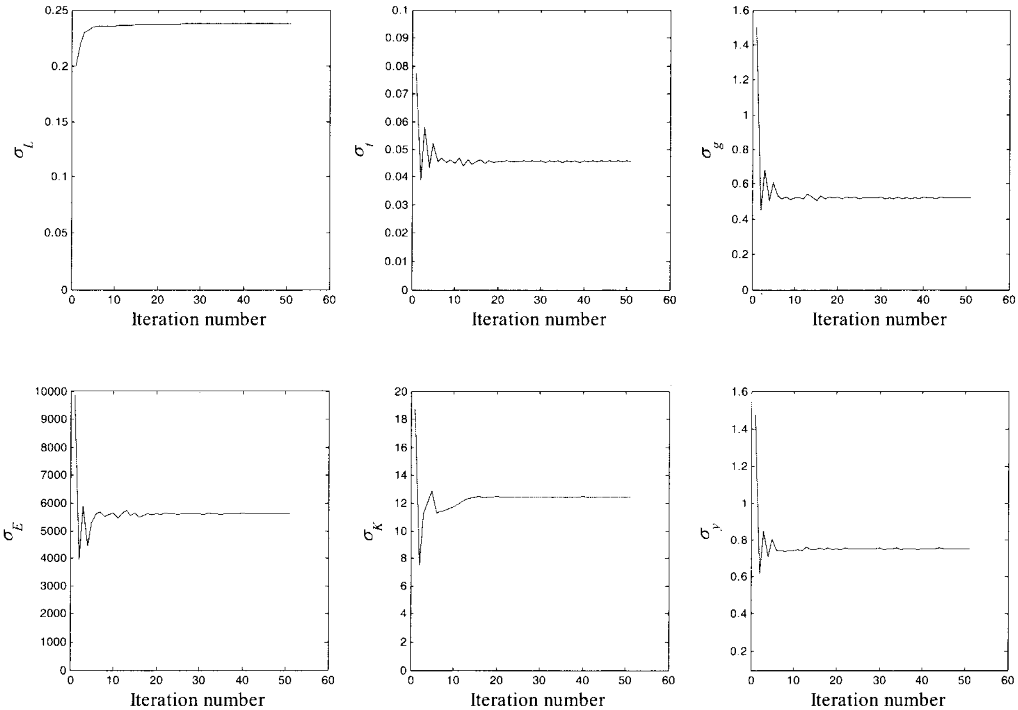

Figure 14. An optimal combination of σ_x from the first set of initial values.

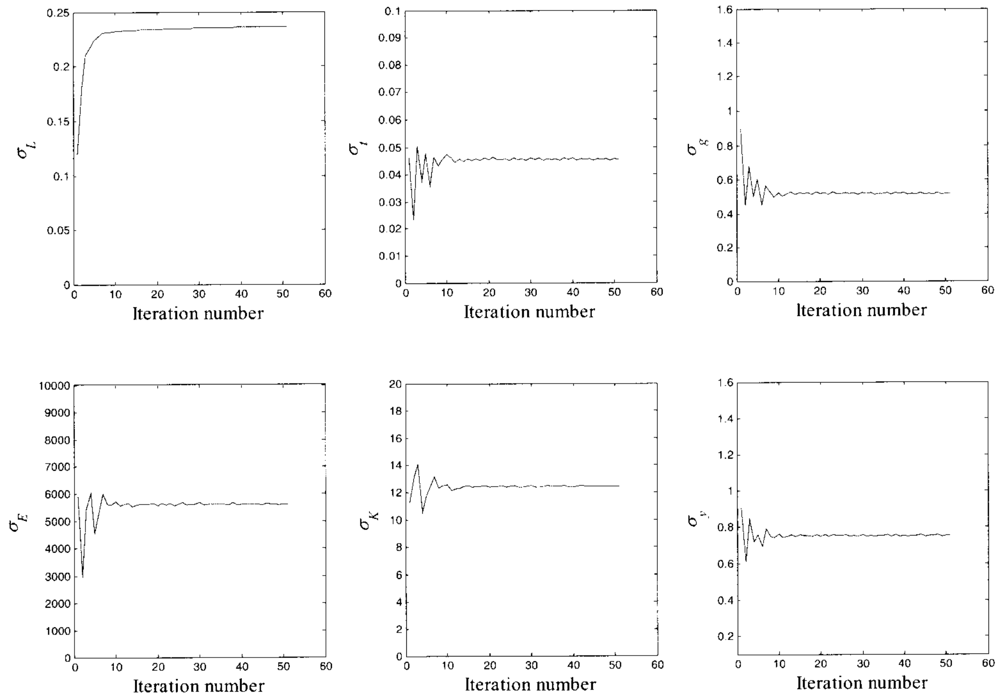

Figure 15. An optimal combination of σ_x from the second set of initial values.

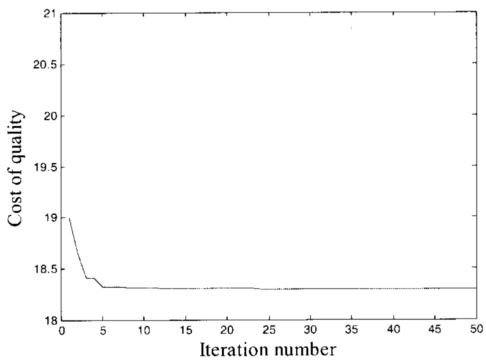

Figure 16. The cost of quality from the first set of initial values.

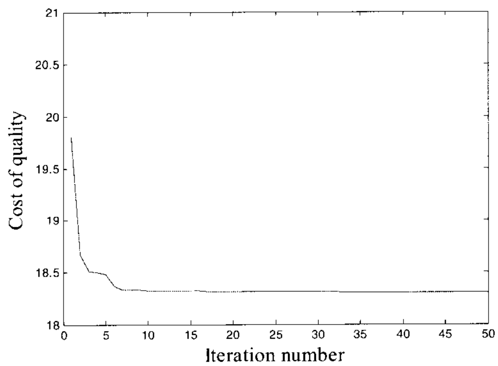

Figure 17. The cost of quality from the second set of initial values.

5.2.2. Uncertainty analysis with the second response surface approximation

By applying the first set of initial values, an optimal combination of σ_x and the cost of quality are shown in Figs. 14 and 16, respectively. Similarly, by applying the second set of initial values, an optimal combination of σ_x and the cost of quality are shown in Figs. 15 and 17, respectively.

5.3. Results

The computational results for the tolerance design from each case for the sample flanging process are presented to observe the big picture in Table 4.

Table 4. The computational results for the tolerance design (same units as Table 1).

Case	Initial	σ_L	σ_t	σ_g	σ_E	σ_K	σ_y
5.1.1.	#1	0.1228	0.0755	0.0214	15.27	14.8465	0.69980
	#2	0.0786	0.0710	0.4254	3853.07	8.2503	0.69817
5.1.2.	#1	0.2123	0.0459	0.5226	4746.68	9.1160	0.70089
	#2	0.1224	0.0436	0.5527	4711.63	8.8892	0.69999
5.2.1.	#1	0.2212	0.0428	0.5005	4982.84	12.5391	0.65794
	#2	0.2204	0.0397	0.5501	5608.04	12.4253	0.68935
5.2.2.	#1	0.2380	0.0454	0.5168	5617.01	12.4470	0.75146
	#2	0.2365	0.0453	0.5165	5614.44	12.4430	0.75115

6. Discussions and concluding remarks

In this paper, the tolerance (uncertainty) management in the straight flanging process was conducted. Two systematic approaches are presented for the tolerance design in an engineering problem requiring expensive numerical methods. The final configuration (including springback) of sheet metals is considered to be the output variable for the deterministic system and is originally computed by using finite element methods. The material properties and process variables (L, t, g, E, and K) are the design variables in this paper. The statistical descriptions of the design variables are determined by experiments in Buranathiti *et al.* (2006) and assumed that all design variables are normally distributed. In the tolerance design, the system was incorporated with uncertainty of the design variables. The computational effort for the uncertainty analysis was reduced by using the response surface techniques to substitute the prediction from the finite element methods. Monte Carlo simulation was conducted to compute the statistical descriptions of the system. In addition, the response surface techniques were further utilized to substitute the standard deviation of the final configuration from Monte Carlo simulation. The effectiveness of the second approximation was shown by evaluating the mean of sum of squared error in Section 5.

The two tolerance design schemes were incorporated with optimization techniques. The first tolerance design scheme shows that the specified tolerance can be achieved by using a number of combinations of input tolerances in Figs. (6) – (9) due to the non-uniqueness of the problem. In addition, some solutions may not be feasible due to the high cost associated with a very tight tolerance. This problem formulation is sensitive to the initial values, i.e., a good set of initial values can reach the optimal combination. Therefore, this class of problem has potential difficulties with non-uniqueness, and the model does not take the difficulty of tight tolerance control into account.

The second tolerance design scheme shows that the problem of non-uniqueness of solutions can be reduced by applying the cost-of-quality function to the model in Figs (10)-(17). The cost-of-quality formulation is crucial to the validity of the tolerance design.

For both tolerance design schemes, it can be seen that by directly applying Monte Carlo simulation to the system, numerical noise is a major cause reducing the efficiency and effectiveness of the optimization algorithms. The second approximations by the response surface techniques show a better convergent rate. However, the nature of the function of statistical descriptions of the output is more complicated than the deterministic case; as a result, a more complicated function is needed to substitute this characteristic. In addition, without normalizing the variables, there are severe problems. Finally, the control of the expected value E[y] can be further included into the models with the same frameworks without a significant change of the schemes by adding the expected value part into the models in Eqs. (3) and (7). Therefore, the tolerance design schemes presented are general and applicable to other manufacturing processes.

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8. References

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