MHD Free Convective Heat and Mass Transfer Flow Past an Inclined Surface with Heat Generation

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Abstract

A steady two-dimensional MHD free convection and mass transfer flow past an inclined semi-infinite surface in the presence of heat generation has been studied numerically. The governing partial differential equations are reduced to a system of ordinary differential equations by introducing similarity transformations. The non-linear similarity equations are solved numerically by applying the Nachtsheim-Swigert shooting iteration technique together with a sixth order Runge-Kutta integration scheme. The numerical results are presented graphically for different values of the parameters entering into the problem. Finally, the numerical values of the local skin-friction coefficients, local Nusselt number and local Sherwood number are also shown in a tabular form.

Keywords: MHD, Free convection, Mass transfer flow, Inclined surface, Heat generation.

1. Introduction

The problem of free convection and mass transfer flow of an electrically conducting fluid past an inclined heated surface under the influence of a magnetic field has attracted interest in view of its application to geophysics, astrophysics and many engineering problems, such as cooling of nuclear reactors, the boundary layer control in aerodynamics and cooling towers. In light of these applications, Umemura and Law [1] developed a generalized formulation for the natural convection boundary layer flow over a flat plate with arbitrary inclination. They found that the flow characteristics depend not only on the extent of inclination but also on the distance from the leading edge. Hossain et al. [2] studied the free convection flow from an isothermal plate inclined at a small angle to the horizontal. Recently, Anghel et al. [3] presented a numerical solution of free convection flow past an inclined surface. Very recently, Chen [4] performed an analysis to study the natural convection flow over a permeable inclined surface with variable wall temperature and concentration. He observed that increasing the angle of inclination decreases the effect of buoyancy force.

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Vajravelu and Hadjinicolaou [5] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. In that study they considered that the volumetric rate of heat generation, q''' [w.m^3], should be $q''' = Q_0(T - T_\infty)$, for $T \geq T_\infty$ and equal to zero for $T < T_\infty$, where Q_0 is the heat generation/absorption constant. The above relation is valid as an approximation of the state of some exothermic process and having T_∞ as the onset temperature. When the inlet temperature is not less than T_∞ they used $q''' = Q_0(T - T_\infty)$. The effect of conjugate conduction-natural convection heat transfer

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along a thin vertical plate with non-uniform heat generation was studied by Mendez and Trevino [6]. Hossain et al. [7] studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption. Therefore the aim of the present work is to study the free convective heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation.

2. Mathematical formulation

We consider a steady two-dimensional hydromagnetic flow of a viscous incompressible, electrically conducting fluid past a semi-infinite inclined plate with an acute angle α to the vertical. The flow is assumed to be in the x-direction, which is taken along the semi-infinite inclined plate and y-axis normal to it. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. In the analysis, we assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. It is also assumed that all fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term. The surface is maintained at a constant temperature T_w, which is higher than the constant temperature T_∞ of the surrounding fluid and the concentration C_w is greater than the constant concentration C_∞. Then under the usual Boussinesq's and boundary layer approximations, the governing equations (see Ramadan and Chamkha [8] and Sivasankaran et al. [9]) are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)\cos\alpha$$

$$+g\beta^*(C-C_\infty)\cos\alpha-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty), \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

with boundary conditions:

$$u=0, v=0, T=T_w, C=C_w \text{ at } y=0, \tag{5a}$$

$$u=0, \quad T=T_\infty, C=C_\infty \quad \text{at } y\to\infty. \tag{5b}$$

where the variables and related quantities are defined in the nomenclature.

In order to obtain a similarity solution of the problem we introduce the following non-dimensional variables:

$$\left.\begin{aligned} &\eta=y\sqrt{\frac{U_\infty}{\upsilon x}},\\ &u=U_\infty f'(\eta),\\ &v=\frac{1}{2}\sqrt{\frac{U_\infty \upsilon}{x}}(\eta f'-f),\\ &\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\\ &\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}.\end{aligned}\right\} \tag{6}$$

Now substituting equation (6) in equations (2)-(4) we obtain:

$$f'''+\frac{1}{2}ff''+Gr\theta\cos\alpha+Gm\phi\cos\alpha-Mf'=0, \tag{7}$$

$$\theta''+\frac{1}{2}\Pr f\theta'+\Pr Q\theta=0, \tag{8}$$

$$\phi''+\frac{1}{2}Scf\phi'=0. \tag{9}$$

The boundary conditions (5) then turn into:

$$f=0, f'=0, \theta=1, \phi=1 \text{ at } \eta=0, \tag{10a}$$

$$f'=0, \quad \theta=0, \quad \phi=0 \text{ as } \eta\to\infty. \tag{10b}$$

The dimensionless parameters introduced in the above equations are defined as follows:

$M = \frac{\sigma B_0^2 x}{\rho U_\infty}$ is the local magnetic field parameter, $Gr = \frac{g\beta(T_w - T_\infty)x}{U_\infty^2}$ is the local temperature Grashof number, $Gm = \frac{g\beta^*(C_w - C_\infty)x}{U_\infty^2}$ is the local mass Grashof number, $\Pr = \frac{\upsilon\rho c_P}{k}$ is the Prandtl number, $Q = \frac{Q_0 x}{\rho c_P U_\infty}$ is the local heat generation parameter and $Sc = \frac{\upsilon}{D}$ is the Schmidt number.

3. Method of Numerical solution

Numerical solutions to the transformed set of nonlinear ordinary differential equations (7)-(9) together with boundary condition (10) were obtained, utilizing a modification of the program suggested by Nachtsheim and Swigert [10]. Within the context of the initial value method and the Nachtsheim-Swigert shooting iteration technique, the outer boundary conditions may be functionally represented by the first order Taylor's series as:

$$f'(\eta_{\max}) = f'(X,Y,Z)$$
$$= f_0'(\eta_{\max}) + \Delta X f_X' + \Delta Y f_Y' + \Delta Z f_Z' = \delta_1,$$
$$\theta(\eta_{\max}) = \theta(X,Y,Z)$$
$$= \theta_0(\eta_{\max}) + \Delta X\theta_X + \Delta Y\theta_Y + \Delta Z\theta_Z = \delta_2,$$
$$\phi(\eta_{\max}) = \phi(X,Y,Z)$$
$$= \phi_0(\eta_{\max}) + \Delta X\phi_X + \Delta Y\phi_Y + \Delta Z\phi_Z = \delta_3,$$

with the asymptotic convergence criteria given by:

$$f''(\eta_{\max}) = f''(X,Y,Z)$$
$$= f_0''(\eta_{\max}) + \Delta X f_X'' + \Delta Y f_Y'' + \Delta Z f_Z'' = \delta_4,$$
$$\theta'(\eta_{\max}) = \theta'(X,Y,Z)$$
$$= \theta_0'(\eta_{\max}) + \Delta X\theta_X' + \Delta Y\theta_Y' + \Delta Z\theta_Z' = \delta_5,$$
$$\phi'(\eta_{\max}) = \phi'(X,Y,Z)$$
$$= \phi_0'(\eta_{\max}) + \Delta X\phi_X' + \Delta Y\phi_Y' + \Delta Z\phi_Z' = \delta_6,$$

where, $X = f''(0)$, $Y = \theta'(0)$, $Z = \phi'(0)$ and X, Y, Z subscripts indicate partial differentiation, e. g., $f_X' = \frac{\partial f'}{\partial f''(0)}$. The subscript 0 indicates the value of the function at $\eta_{\max}$ to be determined from the trial integration. Solution of these equations in a least-squares sense requires determining the minimum value of $E = \delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2 + \delta_5^2 + \delta_6^2$ with respect to X, Y and Z. To solve ΔX, ΔY and ΔZ we differentiate E with respect to X, Y and Z, respectively. Thus adopting this numerical technique, a computer program was set up for the solutions of the governing non-linear partial differential equations of our problem where the integration technique was adopted as a sixth-order Runge-Kutta method of integration. Various groups of the parameters *M*, *Q*, *Pr*, *Sc*, *Gr*, *Gm* and α were considered in different phases. In all the computations, the step size $\Delta\eta = 0.01$ was selected that satisfied a convergence criterion of 10^{-6} in almost all of the different phases mentioned above.

The value of η_∞ was found for each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞, for each group of parameters, has been obtained when the value of the unknown boundary conditions at $\eta = 0$ do not change for a successful loop with error less than 10^{-6}.

From the process of numerical computation, the local skin-friction coefficients, the local Nusselt number and the local Sherwood number, which are respectively proportional to $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$, are also sorted out and their numerical values are presented in Table-1.

4. Results and discussion

As a result of the numerical calculations, the dimensionless velocity, temperature and concentration distributions for the flow are obtained from equations (7)-(9) and are displayed in Figs. 1-5 for different values of *M*, *Q*, *Pr*, *Sc* and α and for fixed values of *Gr* and *Gm*. To be realistic, the values of Prandtl number Pr are chosen for air $(\Pr = 0.71)$, electrolyte solution such as salt water $(\Pr = 1.0)$ and water $(\Pr = 7.0)$.

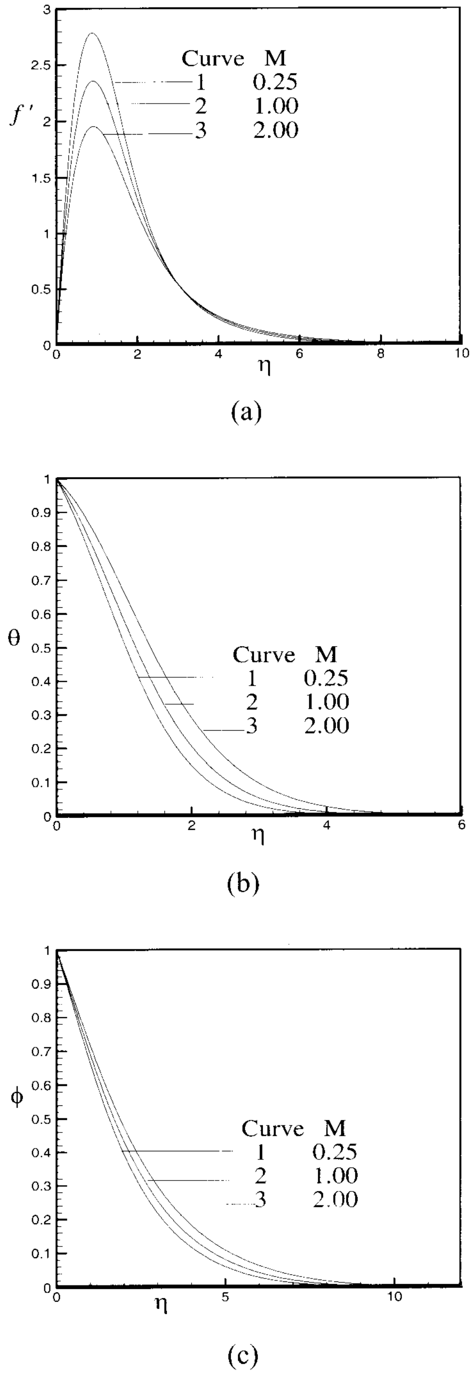


Fig. 1: Variation of dimensionless (a) velocity, (b) temperature and (c) concentration profiles across the boundary layer for different values of M while $Gr = 12$, $Gm = 6$, $Pr = 0.71$, $Sc = 0.22$, $Q = 0.5$ and $\alpha = 30^0$.

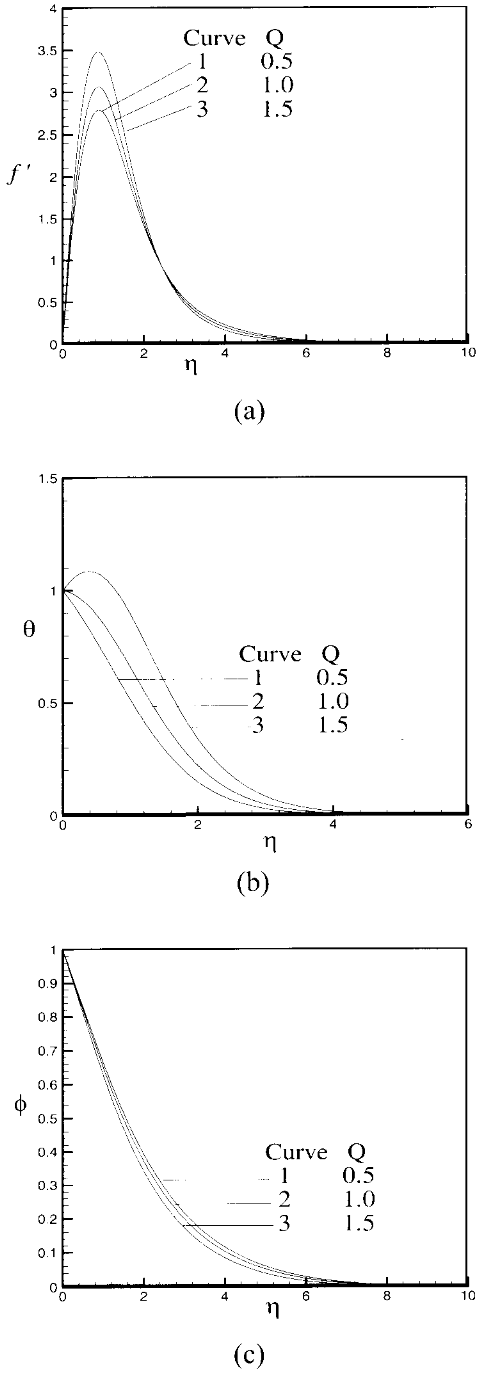
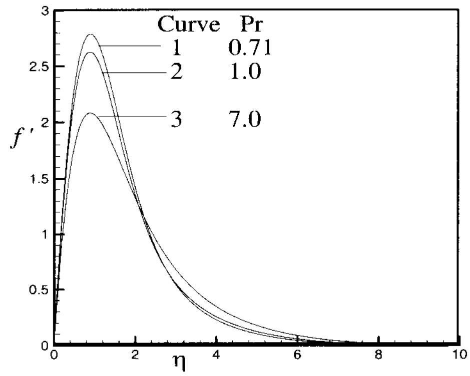
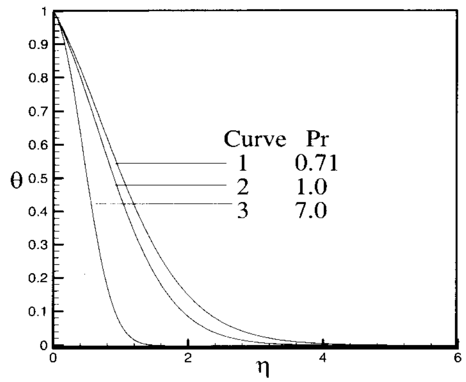


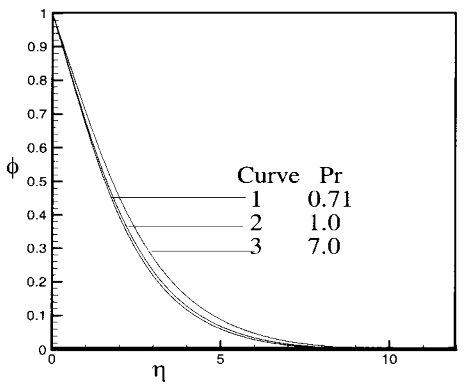
Fig. 2: Variation of dimensionless (a) velocity, (b) temperature and (c) concentration profiles across the boundary layer for different values of Q while $Gr = 12$, $Gm = 6$, $Pr = 0.71$, $Sc = 0.22$, $M = 0.25$ and $\alpha = 30^0$.



(a)

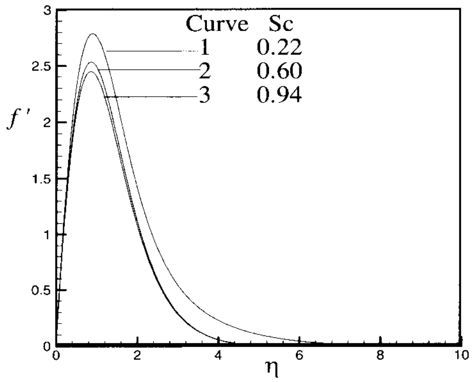


(b)

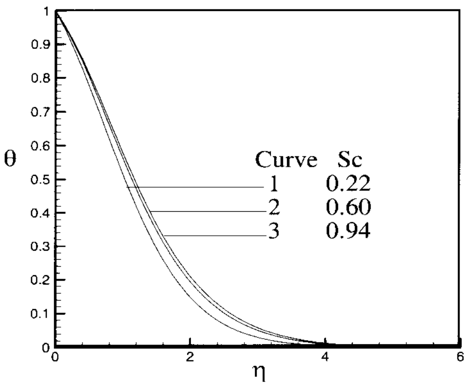


(c)

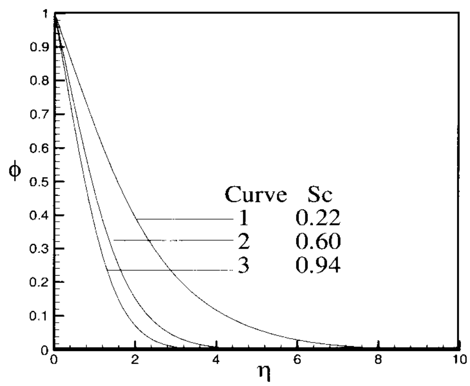
Fig. 3: Variation of dimensionless (a) velocity, (b) temperature and (c) concentration profiles across the boundary layer for different values of Pr while $Gr = 12$, $Gm = 6$, $Q = 0.50$, $Sc = 0.22$, $M = 0.25$ and $\alpha = 30^0$



(a)



(b)



(c)

Fig. 4: Variation of dimensionless (a) velocity, (b) temperature and (c) concentration profiles across the boundary layer for different values of Sc while $Gr = 12$, $Gm = 6$, $Q = 0.50$, $Pr = 0.71$, $M = 0.25$ and $\alpha = 30^0$.

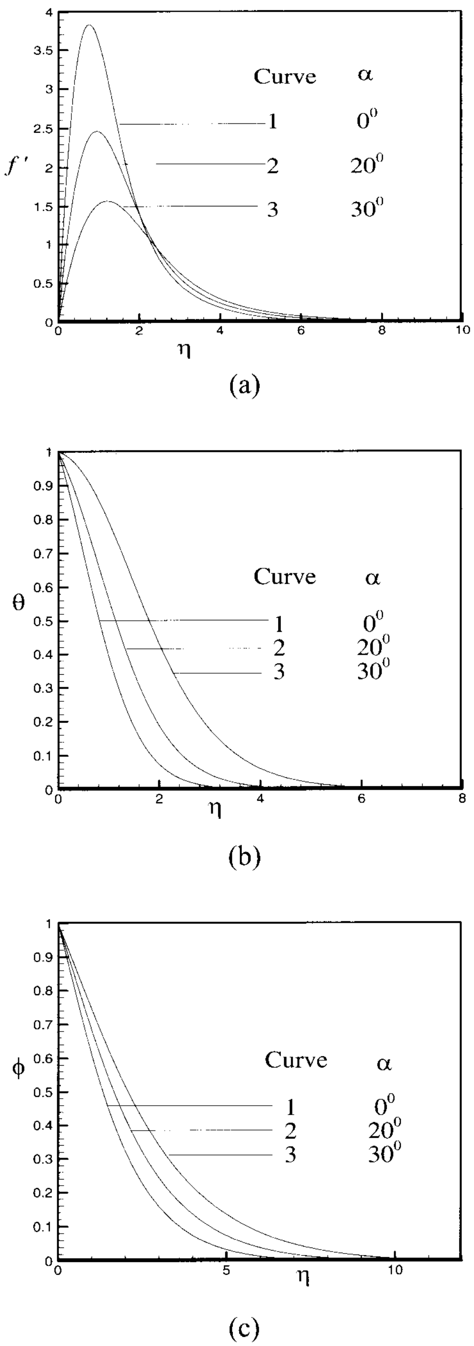


Fig. 5: Variation of dimensionless (a) velocity, (b) temperature and (c) concentration profiles across the boundary layer for different values of α while Gr =12, Gm = 6, Pr = 0.71, Sc = 0.22, Q = 0.5 and M = 0.25.

Table-1: Numerical values of C_f, Nu and Sh for Gr = 12, Gm = 6, Sc = 0.22, M=0.25 and α =30^0.

Pr	M	Q	C_f	Nu	Sh
0.71	0.25	0.50	2.8114	0.0464	0.2580
0.71	1.00	0.50	2.3428	-0.1360	0.2334
0.71	1.50	0.50	2.1508	-0.2648	0.2237
0.71	0.25	0.50	2.8114	0.0464	0.2580
0.71	0.25	1.00	3.5287	-0.7431	0.2957
0.71	0.25	1.50	4.6943	-2.2242	0.3413
7.00	0.25	0.50	2.4523	-0.9334	0.2235
7.00	1.00	0.50	2.3672	-1.8918	0.2023
7.00	1.50	0.50	2.2569	-2.3219	0.1845
7.00	0.25	0.50	2.4523	-0.9334	0.2235
7.00	0.25	1.00	4.1744	-7.1718	0.2703
7.00	0.25	1.50	6.8052	-20.7015	0.3181

The values of Schmidt number Sc are taken for hydrogen $(Sc = 0.22)$, water-vapour $(Sc = 0.60)$ and carbon dioxide $(Sc = 0.94)$. Throughout the calculations, physical variables $Gr = 12$ and $Gm = 6$ are taken which correspond to a cooling problem that is generally encountered in nuclear engineering in connection with cooling of a reactor. Finally the values of M, Q and α are chosen arbitrarily.

In Fig. 1(a), the effects of magnetic field parameter M on the velocity profiles are shown. From this figure we see that the increase of magnetic field leads to a decrease in the velocity field, indicating that the magnetic field retards the flow field. On the other hand, in Figs.1 (b) and 1(c) we see that an increase in the magnetic field leads to rises both the temperature and concentration distributions.

The effects of heat generation parameter Q on the velocity profiles are shown on Fig. 2(a). It is seen from this figure that when heat is generated the buoyancy force increases, which induces the flow rate to increase, giving rise to the increase in the velocity profiles. From Fig. 2(b), we observed that when the value of heat generation parameter Q increases, the temperature distribution also increases significantly. On the other hand, from Fig. 2(c) we see that the concentration profiles decrease with the increase of heat generation parameter.

The effects of Prandtl number Pr on the velocity, temperature and concentration fields are shown in Figs. 3(a)-3(c) respectively. It is seen that the increase in the Prandtl number leads to a fall in the velocity and temperature of the fluid and a rise in the concentration of the fluid along the inclined surface as shown in Fig. 3.

Figs. 4(a)-(c) show the effect of Schmidt number Sc on velocity, temperature and concentration of the air (Pr = 0.71) boundary layer. It is seen that increasing the Schmidt number decreases both the velocity and concentration layer thickness. But the thermal boundary layer increases with the increase of Schmidt number. We also observe that the variation in the thermal boundary layer is very small corresponding to a moderate change in Schmidt number.

The effect of inclination of the surface on velocity, temperature and concentration fields are shown in Figs.5 (a)-(c) respectively. From Fig. 5(a) we observe that the fluid (air) velocity is decreased for increasing angle α. The fluid has higher velocity when the surface is vertical $(\alpha = 0)$ than when inclined because of the fact that the buoyancy effect decreases due to gravity components $(g\cos\alpha)$, as the plate is inclined. On the other hand, both the temperature and concentration of air boundary layer are increased with an increase of α.

Finally, the effects of the above-mentioned parameters on the local skin-friction coefficients (C_f), local Nusselt number (Nu) and local Sherwood number (Sh) for air (Pr = 0.71) and water (Pr = 7.0) are shown in Table 1. The behavior of these parameters is self evident from Table-1 and hence any further discussions about them seem to be redundant.

Nomenclature

B_0 = applied magnetic field
C = concentration
c_p = specific heat at constant pressure
D = mass diffusivity
f = dimensionless stream function
g = acceleration due to gravity
Gr = local temperature Grashof number
Gm = local mass Grashof number
k = thermal conductivity of fluid
M = magnetic field parameter
Nu = Nusselt number
Pr = Prandtl number
Q_0 = heat generation constant
Q = heat generation parameter
Sc = Schmidt number
Sh = Sherwood number
T = temperature
U_∞ = Uniform velocity
u, v = velocity components in the x- and y-direction respectively
x, y = Cartesian coordinates along the plate and normal to it

Greek Symbols:

η = similarity variable
α = angle of inclination
β = coefficient of thermal expansion
β^* = coefficient of concentration expansion
σ = electrical conductivity
ρ = density of the fluid
υ = kinematic viscosity
θ = dimensionless temperature
ϕ = dimensionless concentration

Subscripts:

w = condition at wall
∞ = condition at infinity

Superscript:

$'$ differentiation with respect to η

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