Multi-Product Process Mean with Customer's Loss Consideration

Jirarat Teeravaraprug
Industrial Engineering Department
Faculty of Engineering, Thammasat University
Khlong Luang, Pathum Thani 12121 Thailand

Abstract

This paper considers the problem of determining the optimum value of the process mean for a production process where multiple products are produced. Every outgoing item is inspected and each item failing to meet the specification limits is scrapped. The process is assumed to be normally distributed with a known variance. A profit model is presented which involves revenue, manufacturing cost, scrap cost, and customer loss. A multi-product process mean is determined so that the expected profit is maximized. A numerical example is given to illustrate the use of the profit model and to observe the behavior of the optimal value of the multi-product process mean when the variance and price structure are changed.

Keywords: process target, canning problem, multi-product process, Taguchi loss function

1. Introduction

The selection of the appropriate process mean is of major interest in a wide variety of industrial processes. This problem is often referred to as the "filling problem" or "canning problem" because it is concerned with placing a specific amount of filling into a container subject to a set of specifications. Selecting the optimal process mean is critically important since it has a large impact on both the manufacturer and customers. Although the quality engineering literature related to this issue contains a vast collection of work, some questions still remain unanswered. This research gives an attempt to determine the optimal process mean when several products are produced by the same process assuming that the process setting is not changed when altering the product types. An example of this process situation is the plating process of electronic devices. The plating process should guarantee an acceptable plating thickness of each product type. Altering the process for each product type is costly. Hence, the process setting should be rigid. In this research, the process mean setting is then called a "multi-product process mean".

Normally, two objective functions, maximizing profit functions and minimizing cost functions are used in the literature. Minimizing cost functions may be used in many other cases, but it cannot be used in this case. The reason is that several product types are considered and those may have different prices. Hence, the revenue should also be considered. Maximizing the profit function is then more appropriate. The optimization model provided in this research includes revenue generated by several product types, manufacturing cost, quality loss, and scrap cost. Manufacturing cost includes both fixed and variable costs. Detailed discussion on fixed and variable costs are shown later in this research. Quality loss is the loss due to a product quality less than the customers' desired values. Bhuyan [1] suggested that customers have an ideal value for a quality characteristic and the customer satisfaction is inversely proportional to the deviation between the ideal value and the quality perceived by the customer. Taguchi [2] indicated that the quality should be measured in monetary units and that quality cost, which is the cost incurred by imperfect quality, can be approximated by a quadratic function of the quality deviation from the ideal value. The concept of quality loss provided by Taguchi is then considered in the model. Lastly, scrap cost is the loss due to scrapping an item. Detailed descriptions of these items are presented later in this research.

This research is structured as follows. The next section presents a review of related literature. Then a need to maximize the profit is presented in the model development section. Profit determination is described and also the elements of profit determination, which are revenue, manufacturing cost, quality cost, and scrap cost are discussed in detail. Then an optimization model is developed and a numerical example is given. Finally, conclusions and discussions are given.

2. Related Literature

Techniques to determine the optimal process target have been discussed and developed for more than forty years. The initial work probably began with Springer [3] who considered the problem of determining the optimal process target with specified upper and lower specification limits under the assumption of constant net income functions. There are some situations in which the minimum content is often dictated by legislation. In such a case, underfilled cans need to be reprocessed. Along this line, Bettes [4] modeled the process target setting with a fixed lower specification limit and arbitrary upper specification limit when underfilled and overfilled cans are reprocessed at a fixed cost. In some situations, however, the cans that do not meet the minimum content requirement may be sold at a reduced price. Hunter and Kartha [5] presented a model to determine the optimal process target under the assumption that the cans meeting the minimum content requirement are sold in a regular market at a fixed price, while the underfilled cans are sold at a reduced price in a secondary market. Nelson [6,7] determined approximate solutions to the Hunter and Kartha model [5] and developed a nomograph for the Springer model [3]. The Hunter and Kartha model [5] was later modified by Bisgaard *et al.* [8] who assumed that underfilled cans are sold at a price proportional to their content, and by Carlsson [9] who included a more general income function. In addition, Arcelus and Banerjee [10] extended the work of Bisgaard *et al.* [8], assuming a linear shift in the process mean. Golhar [11] developed a model for the optimal process target under the assumptions that overfilled cans can be sold in a regular market, while underfilled ones can be reprocessed. Golhar and Pollock [12] modified this model by treating both the upper specification limit and The process mean as control variables, and Golhar [13] developed a computer program to solve the Golhar and Pollock model [12]. Arcelus and Rahim [14] presented a model for the most profitable process target where both variable and attribute quality characteristics of a product are considered simultaneously, while Boucher and Jafari [15] addressed the same problem by extending the line of research under the context of a sampling plan. Schmidt and Pfeifer [16] extended the models of Golhar [11] and Golhar and Pollock [12] by considering a limited process capacity. Al-Sultan [17] developed an algorithm to find the optimal machine setting when two machines are connected in series, and Das [18] presented a non-iterative numerical method for solving the Hunter and Kartha model [5]. Usher *et al.* [19] considered the process target problem in a situation where demand for a product does not exactly meet the capacity of a filling operation. Liu and Taghavachari [20] considered the general problem of determining both optimal process target and upper specification limit when a filling amount follows an arbitrary continuous distribution, and showed that the optimal upper specification limit can be presented by a very simple formula regardless of the shape of the distribution. Pulak and Al-Sultan [21] developed a set of FORTRAN-based computer codes, and Pollock and Golhar [22] reconsidered the process target problem under the environment of capacitated production and fixed demand. Hong and Elsayed [23] studied the effects of measurement errors on process target, and Pfeifer [24] showed the use of an electronic spreadsheet program as a solution method.

Rahim and Shaibu [25] and Rahim and Al-Sultan [26] applied the Taguchi loss function to determine the optimal process target and variance. Shao *et al.* [27] examined several methods for process target optimization when several grades of customer specifications are sold within the same market. Kim *et al.* [28] proposed a model for determining the optimal process target with the consideration of variance reduction and process capability. There are situations in which empirical data concerning the costs associated with product performance are available. Under this situation, Teeravaraprug *et al.* [29] developed a model for the most cost-effective process target using regression analysis and Terravaraprug, and Cho [30] developed a model for multiple quality characteristics. Recently, Teeravaraprug [31] developed a model to determine the optimal process mean when a product is classified into two grades with respect to market specifications and Bowling et, al. [32] developed a model for process target levels within the framework of a multi-stage serial production process.

3. Model Development

Frequently, the problem of setting the process mean is solved by minimizing the product cost. When using the product cost as an objective function of the optimization model, it is inherently assumed that the revenue of the product is constant. In this research, several types of products are considered. The revenue then depends on the price and quantity of each product type. Therefore, the expected profit is preferred as the objective function. Generally, the expected profit comprises revenue, manufacturing cost, quality cost, and scrap cost. Detailed discussion is given in the following subsection.

3.1 Revenue

Since several product types are considered, the revenue depends on the product of price and quantity of each product type assuming that each product can be sold independently. Let P_i and q_i be the price and manufacturing quantity of product *i* when $i = 1, 2, ..., n$. A saleable product is needed to pass the inspection process. Assume that a 100% inspection process is applied and only one quality characteristic is employed. For a single quality characteristic, specifications are generally defined as two discrete values, such as a lower specification limit (L_i) and an upper specification limit(U_i) for product *i*. Therefore, the probability of a product falling between those specification limits is $\Pr(L_i \le x_i \le U_i)$ for product *i* when x_i is a performance setting variable of product *i*. The expected revenue, $E[R]$, can be shown in Eq. 1 or Eq. 2.

$$E(R) = \sum_{i=1}^{n} P_i q_i \Pr(L_i \le x_i \le U_i). \quad (1)$$

$$E(R) = \sum_{i=1}^{n} P_i q_i \left[\Phi\left(\frac{U_i - \mu}{\sigma}\right) - \Phi\left(\frac{\mu - L_i}{\sigma}\right) \right], \quad (2)$$

where μ and σ are the mean and standard deviation of the multi-product process, and Φ and ϕ are the cumulative and probability density functions of a normal distribution respectively.

3.2 Manufacturing cost

Manufacturing cost normally comprises fixed and variable costs. Fixed cost is the cost that in total will not change as a function of the proposed change in activity level, while variable cost is the cost that in total will change proportionately as levels of activity are changed. Hence, different product types may give different variable costs per unit and the variable costs per unit do not depend on the product quality. Therefore, even if the product fails in the inspection process, the variable cost per unit is still paid. The expected manufacturing cost, $E[MC]$, can be shown in Eq. (3).

$$E[MC] = FC + \sum_{i=1}^{n} v_i q_i , \quad (3)$$

where FC is a fixed cost and v_i is a variable cost per unit of product *i*.

3.3 Quality cost

Due to product performance variation, a quality evaluation is needed. One of the quality evaluation systems is based on the concept of quality cost. Quality cost is the loss to the customer incurred when the product performance deviates from the customer-desired point. The loss may be estimated by the quality loss function. The quality loss function is a way to quantify the quality cost of a product on a monetary scale when a product or its production process deviates from the customer-desired value for one or more key characteristics. The quality cost includes long-term losses related to poor reliability and the cost of warranty, excess inventory, customer dissatisfaction, and eventually loss of market share.

Even though researchers attempt to construct many types of quality loss functions, there is a general consensus that Taguchi's loss function may be a better approximation for the measurement of customer dissatisfaction of product quality. Assuming that t is the customer-desired point, the Taguchi's loss function (L) is defined as Eq. (4).

$$L = k(x-t)^2 . \quad (4)$$

The well-known expected quality cost based on Taguchi's loss function is

$$E[L] = \sum_{i=1}^{n} k_i \left[(\mu - t_i)^2 + \sigma^2 \right], \quad (5)$$

where k_i is a positive loss coefficient based on estimated loses at a given specification limit of product i, and t_i is a customer-desired value of product i.

3.4 Scrap cost

Scrap cost is the loss due to scrapping an item. Therefore, scrap cost incurs only when product performance is out of its specification limit ($x_i < L_i$ and $x_i > U_i$). Let S_i be the scrap cost per unit of product i and $E[S]$ be the expected value of scrap costs. The expected scrap costs are then:

$$E[S] = \sum_{i=1}^{n} S_i q_i \left[1 - \left[\Phi\left(\frac{U_i - \mu}{\sigma}\right) - \Phi\left(\frac{L_i - \mu}{\sigma}\right) \right] \right]. \quad (6)$$

3.5 The model

As previously discussed, the objective of the optimization model in this research is to maximize the expected profit. The expected profit is composed of four components, which are revenue, manufacturing cost, quality cost, and scrap cost. Hence, the expected profit can be determined as:

$$E[\text{Profit}] = E[R] - E[MC] - E[L] - E[S] . \quad (7)$$

To seek an optimal setting of the process mean, the expected profit shown in Eq. (7) is maximized and the optimization model is shown in Eq. (8).

Maximize

$$E[\text{Profit}] = E[R] - E[MC] - E[L] - E[S] \quad (8)$$

Integrating all the equations, the expected profit turns to be as follows.

Maximize

$$E[\text{Profit}] = \sum_{i=1}^{n} P_i q_i \left[\Phi\left(\frac{U_i - \mu}{\sigma}\right) - \Phi\left(\frac{\mu - L_i}{\sigma}\right) \right] - \left[FC + \sum_{i=1}^{n} v_i q_i \right] - \sum_{i=1}^{n} k_i \left[(\mu - t_i)^2 + \sigma^2 \right] - \sum_{i=1}^{n} S_i q_i \left[1 - \left[\Phi\left(\frac{U_i - \mu}{\sigma}\right) - \Phi\left(\frac{L_i - \mu}{\sigma}\right) \right] \right] \quad (9)$$

Constraints for this objective function includes $U_i \le \mu \le L_i$ for all i.

4. Numerical Example

A printed circuit board firm encounters a problem of excessive warranty costs and customer dissatisfaction associated with component failures in their electronic equipment. The company wants to find the most economical process mean for producing three different product types, A, B and C. For confidentiality reasons, the data have been coded and reported in Table 1. Each type carries a quality characteristic of interest, x_i (i=1,2,3), where the customer identified target value is 40 for all product types. Assume that the process follows a normal distribution with a variance of 0.25. Using the optimization model shown in

Table. 1 Numerical Example Problem

	Product Type		
	A	B	C
Price, P_i	10	20	30
Demand Quantity, q_i	50,000	20,000	10,000
Upper Specification Limit, U_i	40	50	60
Lower Specification Limit, L_i	20	30	40
Variable Cost, v_i	3	5	7
Scrap Cost, S_i	2	3	5
Loss function factor, k_i	1	2	3

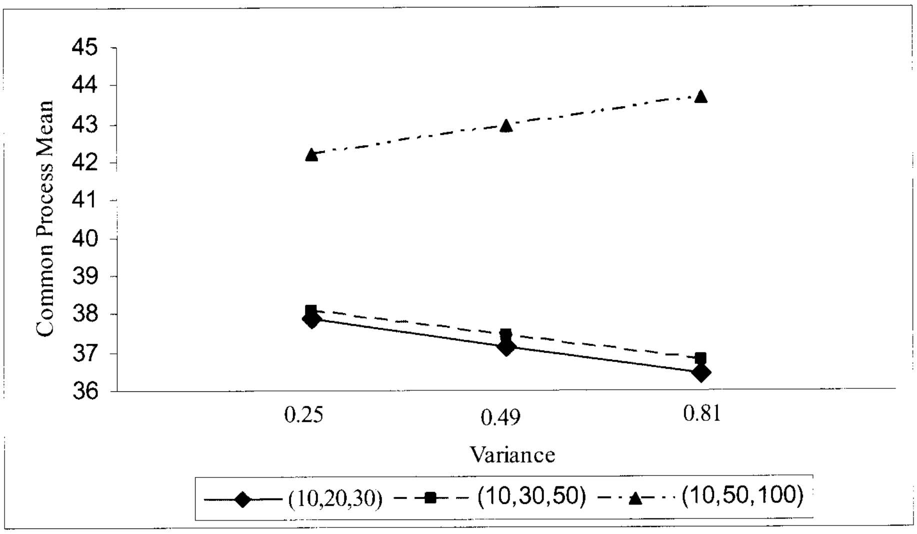

Figure. 1 Optimal Value of Multi-product Process Mean When Varying Variance and Price

Eq. (9) and Excel Solver®, the result shows that the optimal value of multi-product process mean is 37.88 providing an expected profit of 479,969. Note that if the process mean is set at the customer identified target value, the expected profit is 354,999. For this particular example, an extra profit of 35.20 % (=((479,969-354,999)/354,999)x100) would be realized by implementing the optimal value of multi-product process mean, which indicates that the customer-identified target values may not be the most cost-effective levels for the process. In the case of varying the variances and the prices, the result is shown in Figure 1.

It is seen that when the prices are deviated, the optimal values are changed. If the difference of price among product type is not high, the optimal value is less than 40. The reason of that is, the demand quantity of type A is higher than that of the other two types and the demand quantity of type B is higher than that of type C. Note that the lower and upper specification limits of type A are 20 and 40, respectively. Hence, the optimal value tends to be in the specification limits of type A. While the prices of type B and C are higher, the optimal values tend to move up. That is because when the revenues of type B and C are increased and then the optimal values tend to move to obtain those revenues.

Considering the variance, when the variance increases, the optimal values tend to deviate in different ways. When the price differences among product types are not high, the optimal values move down. That is because of the high probability in obtaining type A. It should be reminded that the demand quantity of type A is the highest one. Similarly, while the prices of type B and C are higher, the optimal values tend to move up. That is because when the revenues of type B and C are increased, the optimal values tend to move to obtain those revenues.

5. Conclusion

Quality engineers are often faced with the problem of determining the most cost-effective process target level. In this paper, an attempt is made to determine the optimal process mean where there are several products using the process by incorporating the customer's overall perception of product quality into design. An optimization model is presented for the most economical process target value by considering revenue, manufacturing cost and the loss due to variability to the customer. The numerical example reveals that a savings of 35.20% would be realized by implementing the optimal value of multi-product process mean, indicating that the customer-identified target value may not be the most cost effective setting level.

6. References

[1] Bhuyan, S.K., Cost of Quality as a Customer Perception, ASQC Quality Congress Transactions, Detroit, pp. 459-484, 1982.

[2] Taguchi, G., Quality Evaluation for Quality Assurance, Romulus, MI: American Supplier Institute, 1984.

[3] Springer, C.H., A Method for Determining the Most Economic Position of a Process Mean, Ind Qual Control, Vol. 8, pp. 36-39, 1951.

[4] Bettes, D.C., Finding an Optimum Target Value in Relation to a Fixed Lower Limit and an Arbitrary Upper Limit, Appl Stat, Vol. 11, pp. 202-210, 1962.

[5] Hunter, W.G. and Kartha, C.P., Determining the Most Profitable Target Value for a Production Process, J Qual Technol, Vol. 9, pp. 176-181, 1977.

[6] Nelson, L.S., Best Target Value for Production Process, J. Qual Technol, Vol. 10, pp. 88-89, 1978.

[7] Nelson, L.S., Nomograph for Setting Process to Minimize Scrap Cost, J. Qual Technol, Vol. 11, pp. 48-49, 1979.

[8] Bisgaard S., Hunter, W.G., and Pallesen, L., Economic Selection of Quality of Manufactured Product, Technometrics, Vol. 26, pp. 9-18, 1984.

[9] Carlsson, O., Determining the Most Profitable Process Level for a Production Process under Different Sales Conditions, J Qual Technol, Vol. 16, pp. 44-49, 1984.

[10] Arcelus, F. J. and Rahim, M.A., Optimal Process Levels for the Joint Control of Variables and Attribute, Eur J Oper Res, Vol. 45, pp. 224-230, 1990.

[11] Golhar, D.Y., Determination of the Best Mean Contents for a Canning Problem, J Qual Technol, Vol. 19, pp. 82-84, 1987.

[12] Golhar, D.Y. and Pollock, S.M., Determination of the Optimal Process Mean and the Upper Limit for a Canning Problem, J Qual Technol, Vol. 20, pp. 188-195, 1988.

[13] Golhar, D.Y., Computation of the Optimal Process Mean and the Upper Limit for a Canning Problem, J Qual Technol, Vol. 20, pp. 193-195, 1988.

[14] Arcelus, F.J. and Banerjee, P.K., Selection of the Most Economical Production Plan in a Tool-wear Process, Technometrics, Vol. 27, pp.433-437, 1985.

[15] Boucher, T.O. and Jafari, M.A., The Optimal Target Value for Single Filling Operations with Quality Sampling Plans, J Qual Technol, Vol. 23, pp. 44-47, 1991.

[16] Schmidt, R.L., and Pfeifer, P.E., Economic Selection of the Mean and Upper Limit for a Canning Problem, K Qual Technol, Vol. 23, pp. 312-317, 1991.

[17] Al-Sultan, K.S., An Algorithm the Determination of the Optimum Target Values of Two Machines in Series with Quality Sampling Plan, Int J Prod Res, Vol. 32, pp. 37-45, 1994.

[18] Das, C., Selection and Evaluation of Most Profitable Process Targets for Control of Canning Quality, Comput Ind Eng, Vol. 28, 259-266, 1995.

[19] Usher, J.S., Alexander, S.M., and Duggines, D.C., The Filling Problem Revisited, Qual Eng, Vol. 9, pp. 35-44, 1996.

[20] Liu, W. and Taghavachari, M., The Target Mean Problem for an Arbitrary Quality Characteristic Distribution, Int J Prod Res, Vol. 35, pp. 1713-1727, 1997.

[21] Pulak, M.F. and Al-Sultan, K.S., A Computer Program for Process Mean Targeting, J Qual Technol, Vol. 29, pp. 477-484, 1997.

[22] Pollock, S.M. and Golhar, D., The Canning Problem Revisited: The Case of Capacitated Production and Fixed Demand, Eur J Oper Res, Vol. 105, pp. 475-482, 1988.

[23] Hong, S.H. and Elsayed, E.A., The Optimum Mean for Processes with Normally Distributed Measurement Error, J Qual Technol, Vol. 31, pp. 338-344, 1999.

[24] Pfeifer, P.E., A General Piecewise Linear Canning Problem Model, J Qual Technol, Vol. 31, pp. 326-337, 1999.

[25] Rahim, M.A. and Shaibu, A.B., Economic Selection of Optimal Target Values, Proc Cont Qual, Vol. 11, pp. 369-381, 2000.

[26] Rahim, M.A. and Al-Sultan, K.S., Joint Detemination of the Target Mean and Variance of a Process, J Qual Maint Eng, Vol. 6, pp. 192-199, 2000.

[27] Shao, Y.E., Fowler, J.W., and Runger, G.C., Determining the Optimal Target for a Process with Multiple Markets and

Variable Holding Cost, Int J Prod Econ, Vol. 65, pp. 229-242, 2000.

[28] Kim, Y.J., Cho, B.R., and Phillips, M.D., Determination of the Optimal Process Mean with the Consideration of Variance Reduction and Process Capability, Qual Eng, Vol. 13, pp. 251-260, 2000.

[29] Teeravaraprug, J., Cho, B. R., and Kennedy, W. J., Designing the Most Cost Effective Process Target under Regression Analysis: A Case Study, Proc Cont Qual, Vol. 11, pp. 469-477, 2000.

[30] Teeravaraprug, J., and Cho, B.R., Designing the Optimal Process Target Levels for Multiple Quality Characteristics, Int J Prod Res, Vol. 40, pp. 37-54, 2002.

[31] Teeravaraprug, J., Determining Optimal Process Mean of Two-market Products, Int J of Adv Manu Technol, Vol. 25, pp. 1248-1253, 2004.

[32] Bowling, S.R., Khasawneh, M.T., Kaewkuekool, S., and Cho, B.R., A Markovian Approach to Determining Optimum Process Target Levels for a Multi-stage Serial Production System, Eur J Oper Res, Vol. 159, pp. 636-650, 2004.