Dufour and Soret Effects on Steady MHD Combined Free-Forced Convective and Mass Transfer Flow Past a Semi-Infinite Vertical Plate

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Abstract

In this paper the Dufour (diffusion-thermo) and Soret (thermal-diffusion) effects on combined free-forced convective and mass transfer flow past a semi-infinite vertical flat plate, under the influence of transversely applied magnetic field, have been studied numerically. The non-linear partial differential equations, governing the problem under consideration, have been transformed by a similarity transformation into a system of ordinary differential equations which is solved numerically by applying the Nachtsheim-Swigert shooting iteration technique together with sixth order Runge-Kutta integration scheme. For hydrogen-air mixture as a non-chemical reacting fluid pair, profiles of the dimensionless velocity, temperature and concentration distributions are shown graphically for various values of the parameters entering into the problem. The present solutions are compared with Kafoussias [20] and found to be in excellent agreement. Finally, the corresponding local skin-friction coefficient, local Nusselt number and local Sherwood number are also shown in tabular form.

Keywords: MHD, Combined convection, Mass transfer flow, Dufour effect, Soret effect

1. Introduction

The application of free convection flows, which occur in nature and in engineering practice, are very wide and have been extensively, considered by Jaluria [1]. Flows that are subjected to a combination of free and forced convection are known as combined free-forced convective flows. The simplest physical model of such a flow is the two-dimensional laminar flow along a vertical flat plate. Extensive studies have been conducted on this type of flow by several authors [2-5]. Application of this model can be found in the area of reactor safety, combustion flames and solar collectors, as well as building energy conservation [6]. This model has also been used by many investigators to analyze the combined free-forced convective boundary layer flow, for micropolar fluids, or for the flow through porous media [7-9].

The previous studies, dealing with the transport phenomena of momentum and heat transfer, have dealt with one component phases which possess a natural tendency to reach equilibrium conditions. However, there are activities, especially in industrial and chemical engineering processes, where a system contains two or more components whose concentrations vary from point to point. In such a system there is a natural tendency for mass to be transferred; minimizing the concentration differences within the system and the transport of one constituent, from a region of higher concentration to that of a lower concentration.This is called mass transfer. For heat and mass transfer over plates by either natural, forced or combined convection, many studies involving theoretical or experimental investigations have been published in the literature and most of these studies are based upon the laminar boundary-layer approach [10-14]. The combined free-forced convective and mass transfer flow is a comparatively recent development in the field of fluid mechanics and the different mathematical models and correlations which have been developed can be applied to many industrial applications, such as chemical or drying processes.

In the above studies, the diffusion-thermo (Dufour) and thermal-diffusion (Soret) term were neglected from the energy and concentration equations respectively. But when heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradients and this is the Soret or thermal-diffusion effect. In general, the thermal-diffusion (Dufour) and the diffusion thermo (Soret) effects are of a smaller order of magnitude than the effects described by Fourier's or Fick's laws and are often neglected in heat and mass-transfer processes. There are, however, exceptions. The Soret effect, for instance, has been utilized for isotope separation, and in mixtures between gases with very light molecular weight (H_2, He). For medium molecular weight (N_2, air), the Dufour effect was found to be of a considerable magnitude such that it cannot be neglected (Eckert and Drake[15]). In view of the importance of these above effects, Kafoussias and Williams [16] studied thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Recently, Anghel et al.[17] investigated the Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium. Very recently, Postelnicu[18] used an implicit finite difference method to study the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in a porous media considering Soret and Dufour effects.

Therefore, the objective of this paper is to investigate the Dufour and Soret effects on steady combined free-forced convective and mass transfer flow past a semi-infinite vertical flat plate in the presence of a uniform transverse magnetic field.

2. Mathematical formulation

A two-dimensional steady combined free-forced convective and mass transfer flow of a viscous, incompressible and electrically conducting fluid over an isothermal semi-infinite vertical flat plate under the influence of a transversely applied magnetic field is considered. The flow is assumed to be in the x-direction, which is taken along the vertical plate in the upward direction and the y-axis is taken to be normal to the plate. The surface of the plate is maintained at a uniform constant temperature T_w and a uniform constant concentration C_w, of a foreign fluid, which are higher than the corresponding values T_∞ and C_∞, respectively, sufficiently far away from the flat surface. It is also assumed that the free stream velocity U_∞, parallel to the vertical plate, is constant. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is assumed to be negligible in comparison with the applied magnetic field so that $\boldsymbol{B}=(0, B_0, 0)$ where B_0 is the uniform magnetic field action normal to the plate. The equation of conservation of electric charge $\nabla J=0$ gives $J_y=$ constant, where $\mathbf{J}=(J_x, J_y, J_z)$ Since the plate is electrically non-conduction, this constant is zero and hence $j_y=0$ everywhere in the flow (for detailed discussion see Nanda and Mohanty [19], Sattar and Hossain [20]. Then the problem is governed by the following boundary layer equations under the usual Boussinesq's approximations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)$$
$$+g\beta^*(C-C_\infty)-\frac{\sigma B_0^2 u}{\rho}, \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \qquad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \qquad (4)$$

where u, v are the velocity components in the x and y directions respectively, ν the kinematic viscosity, g the acceleration due to gravity, ρ the density, β the coefficient of volume expansion, β^* the volumetric coefficient of expansion with concentration, T, T_w and T_∞ are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, while C, C_w and C_∞ are the corresponding concentrations. Also, σ is the electrical conductivity of the fluid, B_0 the magnetic induction, α the thermal diffusivity, D_m the coefficient of mass diffusivity, c_p the specific heat at constant pressure, T_m the mean fluid temperature, k_T the thermal diffusion ratio and c_s the concentration susceptibility.

The appropriate boundary conditions for the above problem are as follows:

$$u=0, v=0, T=T_w, C=C_w \text{ at } y=0, \qquad (5a)$$

$$u=U_\infty, T=T_\infty, C=C_\infty \text{ as } y\to\infty. \qquad (5b)$$

The last term on the right-hand side of the energy equation (3) and concentration equation (4) signifies the Dufour or diffusion-thermo effect and the Soret or thermal-diffusion effect, respectively.

Now we introduce the following dimensionless similarity transformation:

$$\left.\begin{aligned}&\eta=y\sqrt{\frac{U_\infty}{\upsilon x}},\\&u=U_\infty f'(\eta),\\&\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\\&\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}.\end{aligned}\right\} \qquad (6)$$

From the continuity equation (1), we have

$$\frac{\partial v}{\partial y}=-\frac{\partial u}{\partial x} \qquad (7)$$

Integrating both sides of (7) with respect to y, we get

$$v=-\frac{1}{2}\sqrt{\frac{U_\infty \upsilon}{x}}\left[f(\eta)-\eta f'(\eta)\right] \qquad (8)$$

Then substituting the relations (6) and (8) into equations (2)-(4), we get the following dimensionless equations which are locally similar:

$$f'''+\frac{1}{2}ff''+g_s\theta+g_c\phi-Mf'=0, \qquad (9)$$

$$\theta''+\frac{1}{2}\Pr f\theta'+\Pr Df\phi''=0, \qquad (10)$$

$$\phi''+\frac{1}{2}Scf\phi'+SrSc\theta''=0. \qquad (11)$$

The relevant boundary conditions in dimensionless form are:

$$f=0, f'=0, \theta=1, \phi=1 \text{ at } \eta=0, \qquad (12a)$$

$$f'=1, \theta=0, \phi=0 \quad \text{as } \eta\to\infty, \qquad (12b)$$

where primes denote differentiation with respect to the variable η and the dimensionless parameters introduced in the above equations are defined as follows:

$\Pr=\frac{\upsilon}{\alpha}$ is the Prandtl number, $Sc=\frac{\upsilon}{D_m}$ is the Schmidt number, $\mathrm{Re}_x=\frac{U_\infty x}{\upsilon}$ is the local Reynolds number, $M=\frac{\sigma B_0^2 x}{\rho U_\infty}$ is the Magnetic field parameter,

$Sr = \frac{D_m k_T (T_w - T_\infty)}{T_m \upsilon (C_w - C_\infty)}$ is the Soret number, $Df = \frac{D_m k_T (C_w - C_\infty)}{c_s c_p \upsilon (T_w - T_\infty)}$ is the Dufour number, $Gr_t = \frac{g\beta (T_w - T_\infty) x^3}{\upsilon^2}$ is the local temperature Grashof number, $Gr_m = \frac{g\beta^* (C_w - C_\infty) x^3}{\upsilon^2}$ is the local mass Grashof number, $g_s = \frac{Gr_t}{\text{Re}_x^2}$ is the temperature buoyancy parameter and $g_c = \frac{Gr_m}{\text{Re}_x^2}$ is the mass buoyancy parameter.

3. Numerical solution:

The system of non-linear ordinary differential equation (9)-(11) together with the boundary conditions (12) are solved numerically using the Nuchtsheim-Swigert shooting iteration technique together with sixth order Runge-Kutta integration scheme. In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial value problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy. For this type of iterative approach, one naturally inquires whether or not there is a systematic way of finding each succeeding (assumed) value of the missing initial condition.

The Nachtsheim-Swigert iteration technique thus needs to be discussed elaborately. The boundary condition (12) associated with the non-linear ordinary differential equations (9)-(11) are the two-point asymptotic class. Two-point boundary conditions have values of the dependent variable specified at two different values of independent variable. Specification of an asymptotic boundary condition implies that the first derivative (and higher derivatives of the boundary layer equations, if they exist) of the dependent variable approaches zero as the outer specified value of the independent variable is approached.

The method of numerically integrating a two-point asymptotic boundary-value problem of the boundary-layer type, the initial-value method is similar to an initial-value problem. Thus it is necessary to estimate as many boundary conditions at the surface as were (previously) given at infinity. The governing differential equations are then integrated with these assumed surface boundary conditions. If the required outer boundary condition is satisfied, a solution has been achieved. However, this is not generally the case. Hence, a method must be devised to estimate logically the new surface boundary conditions for the next trial integration. Asymptotic boundary value problems such as those governing the boundary-layer equations are further complicated by the fact that the outer boundary condition is specified at infinity. In the trial integration,infinity is numerically approximated by some large value of the independent variable. There is no a priori general method of estimating these values. Selecting too small a maximum value for the independent variable may not allow the solution to asymptotically converge to the required accuracy. Selecting too large a value may result in divergence of the trial integration or in slow convergence of surface boundary conditions. Selecting too large a value of the independent variable is expensive in terms of computer time.

Nachtsheim-Swigert [19] developed an iteration method to overcome these difficulties. Extension of the Nachtsheim-Swigert iteration scheme to the system of equation (9)-(11) and the boundary conditions (12) is straightforward. In equation (12) there are three asymptotic boundary conditions and hence three unknown surface conditions $f''(0)$, $\theta'(0)$ and $\phi'(0)$.

Within the context of the initial-value method and Nachtsheim-Swigert iteration technique, the outer boundary conditions may be functionally represented as:

$$f'(\eta_{\max}) = f'(f''(0), \theta'(0), \phi'(0)) = \delta_1, \quad (13)$$

$$\theta(\eta_{\max}) = \theta(f''(0), \theta'(0), \phi'(0)) = \delta_2, \quad (14)$$

$$\phi(\eta_{\max}) = \phi(f''(0), \theta'(0), \phi'(0)) = \delta_3, \quad (15)$$

with the asymptotic convergence criteria given by

$$f''(\eta_{\max}) = f''(f''(0), \theta'(0), \phi'(0)) = \delta_4, \quad (16)$$

$$\theta'(\eta_{\max}) = \theta'(f''(0), \theta'(0), \phi'(0)) = \delta_5, \quad (17)$$

$$\phi'(\eta_{\max}) = \phi'(f''(0), \theta'(0), \phi'(0)) = \delta_6. \quad (18)$$

Choosing $f''(0) = g_1$, $\theta'(0) = g_2$ and $\phi'(0) = g_3$ and expanding in a first-order Taylor's series after using equations (13)-(18) yields:

$$f'(\eta_{\max}) = f'_C(\eta_{\max}) + \frac{\partial f'}{\partial g_1}\Delta g_1 + \frac{\partial f'}{\partial g_2}\Delta g_2 + \frac{\partial f'}{\partial g_3}\Delta g_3 = \delta_1, \quad (19)$$

$$\theta(\eta_{\max}) = \theta_C(\eta_{\max}) + \frac{\partial \theta}{\partial g_1}\Delta g_1 + \frac{\partial \theta}{\partial g_2}\Delta g_2 + \frac{\partial \theta}{\partial g_3}\Delta g_3 = \delta_2, \quad (20)$$

$$\phi(\eta_{\max}) = \phi_C(\eta_{\max}) + \frac{\partial \phi}{\partial g_1}\Delta g_1 + \frac{\partial \phi}{\partial g_2}\Delta g_2 + \frac{\partial \phi}{\partial g_3}\Delta g_3 = \delta_3, \quad (21)$$

$$f''(\eta_{\max}) = f''_C(\eta_{\max}) + \frac{\partial f''}{\partial g_1}\Delta g_1 + \frac{\partial f''}{\partial g_2}\Delta g_2 + \frac{\partial f''}{\partial g_3}\Delta g_3 = \delta_4, \quad (22)$$

$$\theta'(\eta_{\max}) = \theta'_C(\eta_{\max}) + \frac{\partial \theta'}{\partial g_1}\Delta g_1 + \frac{\partial \theta'}{\partial g_2}\Delta g_2 + \frac{\partial \theta'}{\partial g_3}\Delta g_3 = \delta_5, \quad (23)$$

$$\phi'(\eta_{\max}) = \phi'_C(\eta_{\max}) + \frac{\partial \phi'}{\partial g_1}\Delta g_1 + \frac{\partial \phi'}{\partial g_2}\Delta g_2 + \frac{\partial \phi'}{\partial g_3}\Delta g_3 = \delta_6, \quad (24)$$

where subscript 'C' indicates the value of the function at $\eta_{\max}$ determined from the trial integration. Solution of these equations in a least-squares sense requires determining the minimum value of :

$$E = \delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2 + \delta_5^2 + \delta_6^2 \quad (25)$$

Differentiating E with respect to g_1, g_2 and g_3 respectively, we obtain:

$$\left[\left(\frac{\partial f'}{\partial g_1}\right)^2 + \left(\frac{\partial \theta}{\partial g_1}\right)^2 + \left(\frac{\partial \phi}{\partial g_1}\right)^2\right]\Delta g_1 + \left[\left(\frac{\partial f''}{\partial g_1}\right)^2 + \left(\frac{\partial \theta'}{\partial g_1}\right)^2 + \left(\frac{\partial \phi'}{\partial g_1}\right)^2\right]\Delta g_1 +$$
$$\left[\frac{\partial f'}{\partial g_1}\frac{\partial f'}{\partial g_2} + \frac{\partial \theta}{\partial g_1}\frac{\partial \theta}{\partial g_2} + \frac{\partial \phi}{\partial g_1}\frac{\partial \phi}{\partial g_2}\right]\Delta g_2 + \left[\frac{\partial f''}{\partial g_1}\frac{\partial f''}{\partial g_2} + \frac{\partial \theta'}{\partial g_1}\frac{\partial \theta'}{\partial g_2} + \frac{\partial \phi'}{\partial g_1}\frac{\partial \phi'}{\partial g_2}\right]\Delta g_2 +$$
$$\left[\frac{\partial f'}{\partial g_1}\frac{\partial f'}{\partial g_3} + \frac{\partial \theta}{\partial g_1}\frac{\partial \theta}{\partial g_3} + \frac{\partial \phi}{\partial g_1}\frac{\partial \phi}{\partial g_3}\right]\Delta g_3 + \left[\frac{\partial f''}{\partial g_1}\frac{\partial f''}{\partial g_3} + \frac{\partial \theta'}{\partial g_1}\frac{\partial \theta'}{\partial g_3} + \frac{\partial \phi'}{\partial g_1}\frac{\partial \phi'}{\partial g_3}\right]\Delta g_3$$
$$= -(f'_C\frac{\partial f'}{\partial g_1} + \theta_C\frac{\partial \theta}{\partial g_1} + \phi_C\frac{\partial \phi}{\partial g_1} + f''_C\frac{\partial f''}{\partial g_1} + \theta'_C\frac{\partial \theta}{\partial g_1} + \phi'_C\frac{\partial \phi'}{\partial g_1}), \quad (26)$$

$$\left[\left(\frac{\partial f'}{\partial g_2}\right)^2 + \left(\frac{\partial \theta}{\partial g_2}\right)^2 + \left(\frac{\partial \phi}{\partial g_2}\right)^2\right]\Delta g_2 + \left[\left(\frac{\partial f''}{\partial g_2}\right)^2 + \left(\frac{\partial \theta'}{\partial g_2}\right)^2 + \left(\frac{\partial \phi'}{\partial g_2}\right)^2\right]\Delta g_2 +$$
$$\left[\frac{\partial f'}{\partial g_1}\frac{\partial f'}{\partial g_2} + \frac{\partial \theta}{\partial g_1}\frac{\partial \theta}{\partial g_2} + \frac{\partial \phi}{\partial g_1}\frac{\partial \phi}{\partial g_2}\right]\Delta g_1 + \left[\frac{\partial f''}{\partial g_1}\frac{\partial f''}{\partial g_2} + \frac{\partial \theta'}{\partial g_1}\frac{\partial \theta'}{\partial g_2} + \frac{\partial \phi'}{\partial g_1}\frac{\partial \phi'}{\partial g_2}\right]\Delta g_1 +$$
$$\left[\frac{\partial f'}{\partial g_2}\frac{\partial f'}{\partial g_3} + \frac{\partial \theta}{\partial g_2}\frac{\partial \theta}{\partial g_3} + \frac{\partial \phi}{\partial g_2}\frac{\partial \phi}{\partial g_3}\right]\Delta g_3 + \left[\frac{\partial f''}{\partial g_2}\frac{\partial f''}{\partial g_3} + \frac{\partial \theta'}{\partial g_2}\frac{\partial \theta'}{\partial g_3} + \frac{\partial \phi'}{\partial g_2}\frac{\partial \phi'}{\partial g_3}\right]\Delta g_3$$
$$= -(f'_C\frac{\partial f'}{\partial g_2} + \theta_C\frac{\partial \theta}{\partial g_2} + \phi_C\frac{\partial \phi}{\partial g_2} + f''_C\frac{\partial f''}{\partial g_2}$$

$$\theta'_C \frac{\partial \theta}{\partial g_2} + \phi'_C \frac{\partial \phi'}{\partial g_2}), \tag{27}$$

$$\left[\left(\frac{\partial f'}{\partial g_3}\right)^2 + \left(\frac{\partial \theta}{\partial g_3}\right)^2 + \left(\frac{\partial \phi}{\partial g_3}\right)^2\right]\Delta g_3$$
$$+\left[\left(\frac{\partial f''}{\partial g_3}\right)^2 + \left(\frac{\partial \theta'}{\partial g_3}\right)^2 + \left(\frac{\partial \phi'}{\partial g_3}\right)^2\right]\Delta g_3 +$$
$$\left[\frac{\partial f'}{\partial g_1}\frac{\partial f'}{\partial g_3} + \frac{\partial \theta}{\partial g_1}\frac{\partial \theta}{\partial g_3} + \frac{\partial \phi}{\partial g_1}\frac{\partial \phi}{\partial g_3}\right]\Delta g_1 +$$
$$\left[\frac{\partial f''}{\partial g_1}\frac{\partial f''}{\partial g_3} + \frac{\partial \theta'}{\partial g_1}\frac{\partial \theta'}{\partial g_3} + \frac{\partial \phi'}{\partial g_1}\frac{\partial \phi'}{\partial g_3}\right]\Delta g_1 +$$
$$\left[\frac{\partial f'}{\partial g_2}\frac{\partial f'}{\partial g_3} + \frac{\partial \theta}{\partial g_2}\frac{\partial \theta}{\partial g_3} + \frac{\partial \phi}{\partial g_2}\frac{\partial \phi}{\partial g_3}\right]\Delta g_2 +$$
$$\left[\frac{\partial f''}{\partial g_2}\frac{\partial f''}{\partial g_3} + \frac{\partial \theta'}{\partial g_2}\frac{\partial \theta'}{\partial g_3} + \frac{\partial \phi'}{\partial g_2}\frac{\partial \phi'}{\partial g_3}\right]\Delta g_2$$
$$= -(f'_C \frac{\partial f'}{\partial g_3} + \theta_C \frac{\partial \theta}{\partial g_3} + \phi_C \frac{\partial \phi}{\partial g_3} + f''_C \frac{\partial f''}{\partial g_3}$$
$$\theta'_C \frac{\partial \theta}{\partial g_3} + \phi'_C \frac{\partial \phi'}{\partial g_3}), \tag{28}$$

We can write equations (26)-(28) in a system of linear equations as follows:

$$a_{11}\Delta g_1 + a_{12}\Delta g_2 + a_{13}\Delta g_3 = b_1, \tag{29}$$

$$a_{21}\Delta g_1 + a_{22}\Delta g_2 + a_{23}\Delta g_3 = b_2, \tag{30}$$

$$a_{31}\Delta g_1 + a_{32}\Delta g_2 + a_{33}\Delta g_3 = b_3. \tag{31}$$

where

$$a_{11} = \left[\left(\frac{\partial f'}{\partial g_1}\right)^2 + \left(\frac{\partial \theta}{\partial g_1}\right)^2 + \left(\frac{\partial \phi}{\partial g_1}\right)^2\right]$$
$$+\left[\left(\frac{\partial f''}{\partial g_1}\right)^2 + \left(\frac{\partial \theta'}{\partial g_1}\right)^2 + \left(\frac{\partial \phi'}{\partial g_1}\right)^2\right],$$

$$a_{12} = \left[\frac{\partial f'}{\partial g_1}\frac{\partial f'}{\partial g_2} + \frac{\partial \theta}{\partial g_1}\frac{\partial \theta}{\partial g_2} + \frac{\partial \phi}{\partial g_1}\frac{\partial \phi}{\partial g_2}\right]$$
$$+\left[\frac{\partial f''}{\partial g_1}\frac{\partial f''}{\partial g_2} + \frac{\partial \theta'}{\partial g_1}\frac{\partial \theta'}{\partial g_2} + \frac{\partial \phi'}{\partial g_1}\frac{\partial \phi'}{\partial g_2}\right] = a_{21},$$

$$a_{13} = \left[\frac{\partial f'}{\partial g_1}\frac{\partial f'}{\partial g_3} + \frac{\partial \theta}{\partial g_1}\frac{\partial \theta}{\partial g_3} + \frac{\partial \phi}{\partial g_1}\frac{\partial \phi}{\partial g_3}\right]$$
$$+\left[\frac{\partial f''}{\partial g_1}\frac{\partial f''}{\partial g_3} + \frac{\partial \theta'}{\partial g_1}\frac{\partial \theta'}{\partial g_3} + \frac{\partial \phi'}{\partial g_1}\frac{\partial \phi'}{\partial g_3}\right] = a_{31},$$

$$a_{22} = \left[\left(\frac{\partial f'}{\partial g_2}\right)^2 + \left(\frac{\partial \theta}{\partial g_2}\right)^2 + \left(\frac{\partial \phi}{\partial g_2}\right)^2\right]$$
$$+\left[\left(\frac{\partial f''}{\partial g_2}\right)^2 + \left(\frac{\partial \theta'}{\partial g_2}\right)^2 + \left(\frac{\partial \phi'}{\partial g_2}\right)^2\right],$$

$$a_{23} = \left[\frac{\partial f'}{\partial g_2}\frac{\partial f'}{\partial g_3} + \frac{\partial \theta}{\partial g_2}\frac{\partial \theta}{\partial g_3} + \frac{\partial \phi}{\partial g_2}\frac{\partial \phi}{\partial g_3}\right]$$
$$+\left[\frac{\partial f''}{\partial g_2}\frac{\partial f''}{\partial g_3} + \frac{\partial \theta'}{\partial g_2}\frac{\partial \theta'}{\partial g_3} + \frac{\partial \phi'}{\partial g_2}\frac{\partial \phi'}{\partial g_3}\right] = a_{32},$$

$$a_{33} = \left[\left(\frac{\partial f'}{\partial g_3}\right)^2 + \left(\frac{\partial \theta}{\partial g_3}\right)^2 + \left(\frac{\partial \phi}{\partial g_3}\right)^2\right]$$
$$+\left[\left(\frac{\partial f''}{\partial g_3}\right)^2 + \left(\frac{\partial \theta'}{\partial g_3}\right)^2 + \left(\frac{\partial \phi'}{\partial g_3}\right)^2\right],$$

$$b_1 = -(f'_C \frac{\partial f'}{\partial g_1} + \theta_C \frac{\partial \theta}{\partial g_1} + \phi_C \frac{\partial \phi}{\partial g_1} + f''_C \frac{\partial f''}{\partial g_1}$$
$$\theta'_C \frac{\partial \theta}{\partial g_1} + \phi'_C \frac{\partial \phi'}{\partial g_1}),$$

$$b_2 = -(f'_C \frac{\partial f'}{\partial g_2} + \theta_C \frac{\partial \theta}{\partial g_2} + \phi_C \frac{\partial \phi}{\partial g_2} + f''_C \frac{\partial f''}{\partial g_2}$$
$$\theta'_C \frac{\partial \theta}{\partial g_2} + \phi'_C \frac{\partial \phi'}{\partial g_2}),$$

$$b_3 = -(f'_C \frac{\partial f'}{\partial g_3} + \theta_C \frac{\partial \theta}{\partial g_3} + \phi_C \frac{\partial \phi}{\partial g_3} + f''_C \frac{\partial f''}{\partial g_3}$$
$$\theta'_C \frac{\partial \theta}{\partial g_3} + \phi'_C \frac{\partial \phi'}{\partial g_3}).$$

Now solving the equations (29)-(31) by using Cramer's rule, we have:

$$\Delta g_1 = \frac{\det A_1}{\det A}, \Delta g_2 = \frac{\det A_2}{\det A} \text{ and}$$
$$\Delta g_3 = \frac{\det A_3}{\det A}$$

where

$$\det A_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \det A_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix},$$

$$\det A_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}, \det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

Then we obtain the missing (unspecified) values g_1, g_2 and g_3 as:

$$g_1 = g_1 + \Delta g_1,$$
$$g_2 = g_2 + \Delta g_2,$$
$$g_3 = g_3 + \Delta g_3.$$

Thus adopting the numerical technique aforementioned, the solution of the nonlinear ordinary differential equations (9)-(11) with boundary conditions (12) are obtained together with the sixth-order implicit Runge-Kutta initial value solver. A step size of $\Delta\eta = 0.01$ was selected to be satisfactory for a convergence criterion of 10^{-6} in all cases. The value of η_∞ was found for each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞, for each group of parameters *M*, *Df*, *Sr*, *Pr*, *Sc*, g_s and g_c is determined when the value of the unknown boundary conditions at $\eta = 0$ does not change (successful loop with error less than 10^{-6}.)

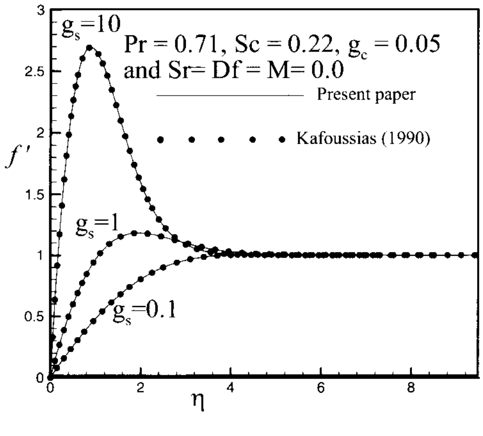

Fig. 1: Comparison of velocity profiles with Kafoussias (1990).

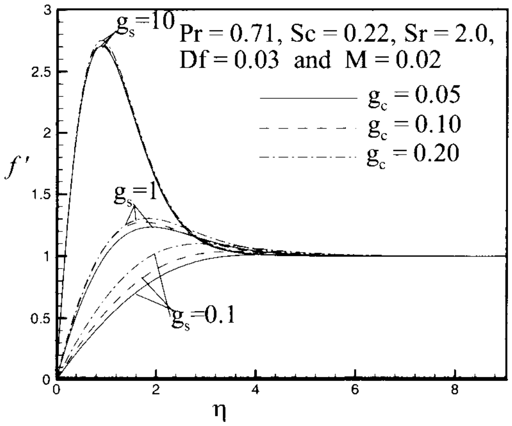

Fig. 2: Velocity profiles for different values of g_s and g_c.

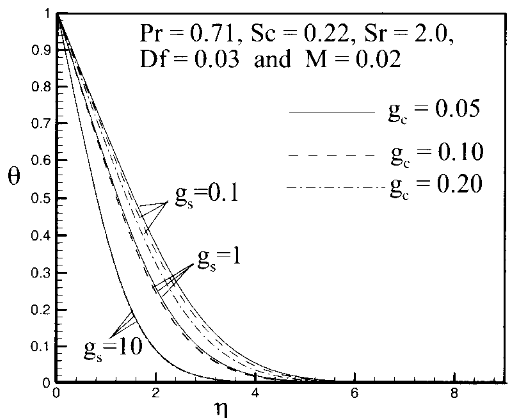

Fig. 3: Temperature profiles for different values of g_s and g_c.

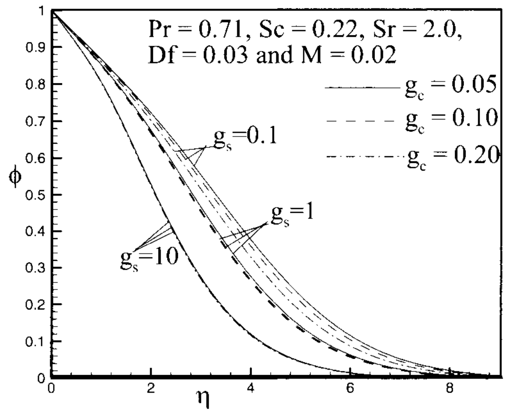

Fig. 4: Concentration profiles for different values of g_s and g_c.

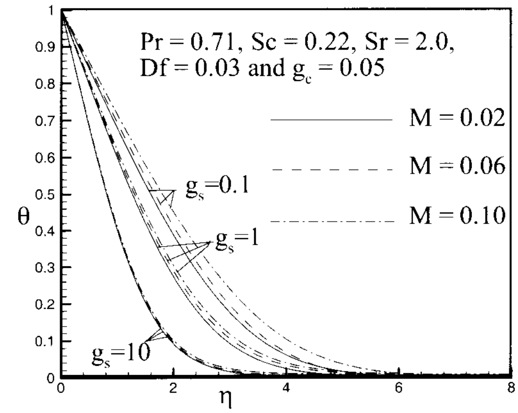

Fig. 7: Concentration profiles for different values of M.

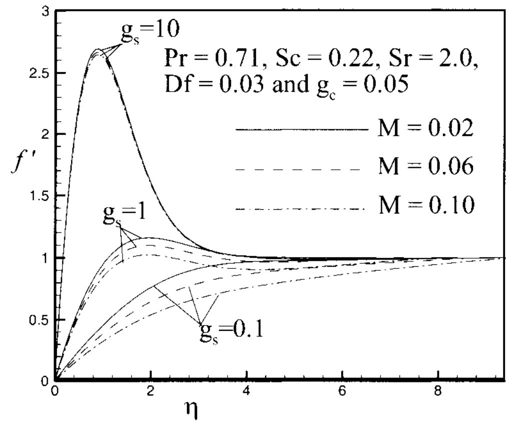

Fig. 5: Velocity profiles for different values of M.

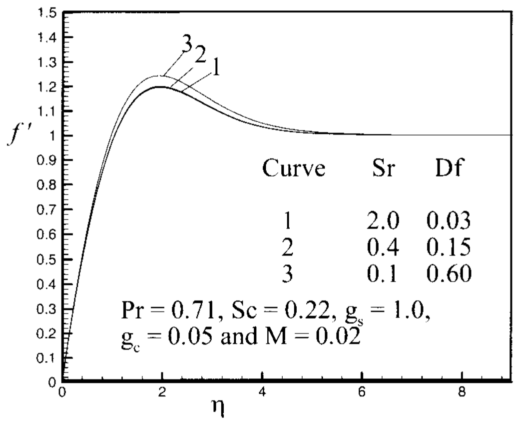

Fig. 8: Velocity profiles for different values of Sr and Df.

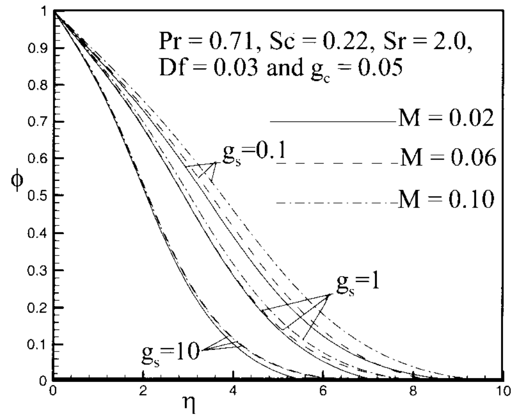

Fig. 6: Temperature profiles for different values of M.

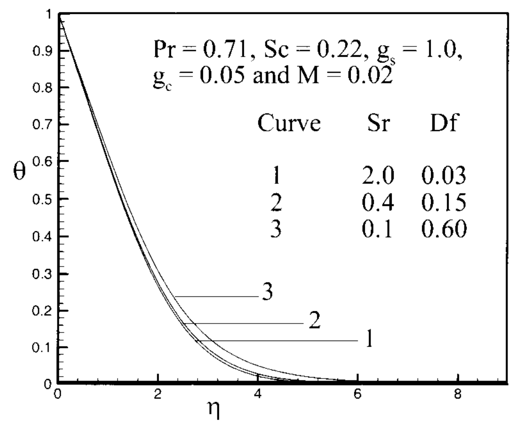

Fig. 9: Temperature profiles for different values of Sr and Df.

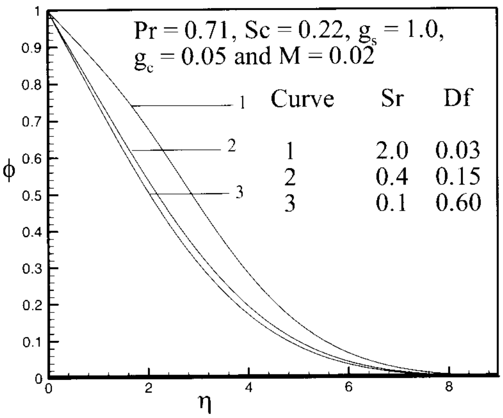

Fig. 10: Concentration profiles for different values of Sr and Df.

To assess the accuracy of our code, the present result has been compared with Kafoussias [20], when $M = Df = Sr = 0$ (see Fig. 1 and Table-1). From this figure we see an excellent agreement between them.

4. Skin-friction coefficient, rate of heat and mass transfer

The physical quantities of most interest in such problems are the skin-friction coefficient (C_f), the Nusselt number (Nu_x) and the Sherwood number (Sh_x) which are defined by the following relations:

$$C_f = \frac{2\tau_w}{\rho U_\infty^2},\ Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \quad \text{and}$$

$$Sh_x = \frac{xM_w}{D_m(C_w - C_\infty)} \tag{32}$$

respectively, where k is the thermal conductivity of the fluid, and the skin-friction on the flat plate

τ_w, rate of heat transfer q_w and rate of mass transfer M_w are given by:

$$\tau_w = \left[\mu \frac{\partial u}{\partial y}\right]_{y=0},\ q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0} \quad \text{and}$$

$$M_w = -D_m\left[\frac{\partial C}{\partial y}\right]_{y=0} \tag{33}$$

Using (6) and (33) we can write the quantities of (32) in the following form:

$$\frac{1}{2}C_f(\text{Re}_x)^{\frac{1}{2}} = f''(0),\ Nu_x(\text{Re}_x)^{-\frac{1}{2}} = -\theta'(0)$$

$$\text{and}\quad Sh_x(\text{Re}_x)^{-\frac{1}{2}} = -\phi'(0). \tag{34}$$

From the process of numerical computation, the above coefficients sorted out in Table-1 and Table-2.

Table-1: Comparison of skin-friction Coefficient (C_f) and local Nusselt number (Nu_x) for $M = Df = Sr = 0$.

g_s	g_c	Kafoussias[22] (C_f)	Present (C_f)	Kafoussias[22] (Nu_x)	Present (Nu_x)
0.10	0.05	0.5538	0.5538	0.3296	0.3296
0.10	0.10	0.6317	0.6315	0.3404	0.3404
0.10	0.20	0.7776	0.7772	0.3589	0.3589
1.00	0.05	1.4452	1.4451	0.4129	0.4129
1.00	0.10	1.5007	1.5001	0.4179	0.4178
1.00	0.20	1.6096	1.6081	0.4274	0.4272
10.0	0.05	6.8389	6.8385	0.6449	0.6450
10.0	0.10	6.8715	6.8712	0.6461	0.6463
10,0	0.20	6.9366	6.9356	0.6487	0.6488

Table-2: Numerical values of skin-friction coefficient (C_f), Nusselt number (Nu_x) and Sherwood number (Sh_x) for $Pr = 0.71$, $Sc = 0.22$, $g_s = 1.0$ and $g_c = 0.05$.

M	Sr	Df	C_f	Nu_x	Sh_x
0.02	2.00	0.03	1.4222	0.4086	0.1465
0.06	2.00	0.03	1.3671	0.3978	0.1440
0.10	2.00	0.03	1.3069	0.3869	0.1388
0.02	2.00	0.03	1.4222	0.4086	0.1465
0.02	0.40	0.15	1.4260	0.4012	0.2538
0.02	0.10	0.60	1.4650	0.3830	0.2545

5. Results and discussion

Numerical computations have been carried out for different values of magnetic field parameter M and for fixed values of Prandtl number Pr, Schmidt number Sc. The value of Prandtl number Pr is taken equal to 0.71 which corresponds physically to air. The value of Schmidt number $Sc = 0.22$ has been chosen to represent hydrogen at approx. $T_m = 25^0$C and 1 atm. The values of Soret number Sr and Dufour number Df are chosen in such a way that their product is constant according to their definition provided that the mean temperature T_m is kept constant as well. The dimensionless parameter $g_s = \frac{Gr_t}{\mathrm{Re}_x^2}$ is used to represent the free, forced and combined (free-forced) convection regimes. The case $g_s \ll 1$ corresponds to pure forced convection, $g_s = 1$ corresponds to combined free-forced convection and $g_s \gg 1$ corresponds to pure free convection. As the local mass Grashof number Gr_m is a measure of the buoyancy forces (due not to temperature but to concentration differences) to the viscous forces, the dimensionless parameter g_c has the same meaning as the parameter g_s. The dimensionless parameter g_s takes the values 0.1, 1 and 10 which correspond to three different flow regimes as already mentioned above. The corresponding parameter g_c takes the values 0.05, 0.10 and 0.20.

With the above-mentioned flow parameters, the results are displayed in Figs. 2-10, for the velocity, temperature and concentration profiles. In Fig.2, velocity profiles are shown for different values of g_s and g_c. We observe that velocity increases with the increase of g_s. This increment is greater for higher values of g_c and in the case of pure forced convection ($g_s \ll 1$). The velocity reaches maximum inside the boundary layer for pure free convection (g_s=10, g_c=0.20). The variations of temperature and concentration fields for different values of g_s and g_c are displayed in Figs. 3 and 4, respectively. As would be expected, both fields exhibit the same behavior. The influence of g_c on the temperature and concentration field is not so much evident for higher values of g_s.

In Fig.5, the effects of magnetic field parameter M for different values of g_s are shown. From this figure we see that the increase of magnetic field leads to the decrease the velocity field indicating that the magnetic field retards the flow field. On the other hand, in Figs.6 and 7 we see that an increase in the magnetic field leads to rises in both the temperature and concentration distributions.

The influence of Soret number Sr and Dufour number Df on the velocity, temperature and concentration profiles are shown in Figs. 8, 9 and and 10 respectively. From Fig. 8, we see that quantitatively, when $\eta = 2$ and Sr decreases from 2 to 0.4 (or Df increases from 0.03 to 0.15) there is a 0.33% increase in the velocity field, whereas the corresponding increase is 3.64% when Sr decreases from 0.4 to 0.1. From Fig. 9, when $\eta = 3$ and Sr decreases from 2 to 0.4 (or Df increases from 0.03 to 0.15) there is a 10.53% increase in the temperature field, whereas the corresponding increase is 32.19% when Sr decreases from 0.4 to 0.1. From Fig. 10, when $\eta = 3$ and Sr decreases from 2 to 0.4 (or Df increases from 0.03 to 0.15) there is a 38.22% decrease in the concentration field, whereas the corresponding decrease is 10.14% when Sr decreases from 0.4 to 0.1.

Finally, Table-2 represents the numerical values of local skin-friction coefficient, local Nusselt number and Sherwood number for some values of the parameters M, Sr and Df when g_s = 1. From this table it is evident that for fixed g_s, Sr and Df : skin-friction coefficient, Nusselt number and Sherwood number decrease as M increases. Finally we see that the local Nusselt number increases, while the local Sherwood number decreases as Df decreases and Sr increases.

6. Conclusions

In this paper, the Dufour and Soret effects on steady combined free-forced convective and mass transfer flow past a semi-infinite vertical flat plate under the influence of a transversely applied magnetic field has been studied theoretically for a hydrogen-air mixture as a non-chemical reacting fluid pair. Using usual similarity transformations, the governing equations have been transformed into non-linear ordinary differential equations. The similarity solutions are obtained numerically by applying Nachtsheim-Swigert shooting iteration technique together with sixth order Runge-Kutta method. Since no experimental results of the corresponding studies are available, comparison between the obtained results with existing results is numerically simulated.

From the present study we see that the momentum boundary layer thickness decreases with an increase of magnetic field parameter (M), whereas both the thermal and species concentration boundary layer thickness increases with an increase of magnetic field parameter. The presented analysis has also shown that the flow field is appreciably influenced by the Dufour and Soret effects Therefore, we can conclude that for fluids of hydrogen-air mixtures, the Dufour and Soret effects should not be neglected.

Nomenclature:

B_0 = applied magnetic field
C = concentration
c_p = specific heat at constant pressure
c_s = concentration susceptibility
C_f = local skin-friction coefficient
Df = Dufour number
D_m = mass diffusivity
f = dimensionless stream function
g = acceleration due to gravity
g_s = temperature buoyancy parameter
g_c = mass buoyancy parameter
Gr_t = local temperature Grashof number
Gr_m = local mass Grashof number
k_T = thermal diffusion ratio
M = magnetic field parameter
M_w = mass flux
Nu_x = local Nusselt number
Pr = Prandtl number
q_w = heat flux
Sc = Schmidt number
Sh_x = local Sherwood number
Sr = Soret number
T = temperature
T_m = mean fluid temperature
U_∞ = free stream velocity
u, v = velocity components in the x- and y-direction respectively
x, y = Cartesian coordinates along the plate and normal to it

Greek Symbols:

η = similarity variable
α = thermal diffusivity
β = coefficient of thermal expansion
β^* = coefficient of concentration expansion
σ = electrical conductivity
ρ = density of the fluid
υ = kinematic viscosity
θ = dimensionless temperature
ϕ = dimensionless concentration
τ_w = wall shear stress

Subscripts:

w = condition at wall
∞ = condition at infinity

Superscript:

$'$ differentiation with respect to η

7. References

[1] Jaluria, Y., Natural Convection Heat and Mass Transfer, Pergamon Press, Oxford, 1980.

[2] Merkin, J. H., The Effect of Buoyancy Forces on the Boundary Layer Flow Over Semi-infinite Vertical Flat Plate in a Uniform Free Stream, J. Fluid Mech., Vol. 35, pp. 439-450, 1969.

[3] Lloyd, J. R. and Sparrow, E. M., Combined Forced and Free Convection Flow on

Vertical Surfaces, Int. J. Heat Mass Transfer, Vol. 13, pp. 434-438, 1970.
[4] Wilks, G., Combined Forced and Free Convection Flow on Vertical Surfaces, Int. J. Heat Mass Transfer, Vol. 16, pp. 1958-1964, 1973.
[5] Raju, M. S., Liu, X. R. and Law, C. K., A Formulation of Combined Forced and Free Convection Past Horizontal and Vertical Surfaces, Int. J. Heat Mass Transfer, Vol. 27, pp. 2215-2224, 1984.
[6] Hossian, M. A. and Ahmed, M., MHD Forced and Free Convection Boundary Layer Near the Leading Edge, Int. J. Heat Mass Transfer, Vol. 33(3), pp. 571-575, 1990.
[7] Lai, F. C. and Kulacki, F. A., The Effect of Variable Viscosity on Convective Hat Transfer Along a Vertical Surface in a Saturated Porous Medium, Int. J. Heat Mass Transfer, Vol. 33(5), pp. 1028-1031, 1990.
[8] Gorla, R. S. R., Lin, P. P. and Yang, A. J., Asymptotic Boundary Layer Solutions for Mixed Convection From a Vertical Surface in a Micropolar Fluid, Int. J. Eng. Sci., Vol. 28, pp. 525-533, 1990.
[9] Gorla, R. S. R., Mixed Convection in a Micropolar Fluid Along a Vertical Surface with Uniform Heat Flux, Int. J. Eng. Sci., Vol. 30, pp. 349-358, 1992.
[10] Schlichting, H., Boundary Layer Theory, 6^{th} Ed. McGraw-Hill, New York, 1968.
[11] Chen, T. S. and Yuh, C. F., Combined Heat and Mass Transfer in Natural Convection on Inclined Surfaces, Num. Heat Transfer, Vol. 2, pp. 233-250, 1979.
[12] Roshenow, W. M., Harnet, J. P. and Ganic, E. N., Handbook of Heat Transfer, Second Ed., McGraw-Hill , New York, 1985.
[13] Chen, T. S., Tien, H. C. and Armaly, B. F., Int. J. Heat Mass Transfer, Vol.29, pp. 1465-1478, 1986.
[14] Zeghmarti, B., Le Palec, G. and Daguenet, M., Study of Transient Laminar free Convection Over an Inclined Wet That Plate, Int. J. Heat Mass Transfer, Vol. 34, pp. 899-909, 1991.
[15] Eckert, E. R. G. and Drake, R. M., Analysis of Heat and Mass Transfer, McGraw-Hill, New York, 1972.
[16] Kafoussias, N. G. and Willaiams, E. W., Thermal-diffusion and Diffusion-thermo Effects on Mixed Free Forced Convective and Mass Transfer Boundary Layer Flow with Temperature Dependent Viscosity, Int. J. Eng. Sci. Vol.33, pp.1369-1384, 1995.
[17] Anghel, M., Takhar, H. S. and Pop, I., Dufour and Soret Effects on Free Convection Boundary Layer over a Vertical Surface Embedded in a Porous Medium, Studia Universitatis Babes-Bolyai, Mathematica, Vol.XL, V, pp.11-21, 2000.
[18] Postelnicu, A., Influence of a Magnetic Field on Heat and Mass Transfer by Natural Convection from Vertical Surfaces in Porous Media Considering Soret and Dufour Effects, Int. J. Heat Mass Transfer, Vol.47, pp.1467-1472, 2004.
[19]Nachtsheim, P. R. and Swigert, P., Satisfaction of the Asymptotic boundary Cnditions in Numerical Solution of the System of Non-linear Equations of Boundary Layer Type, NASA TND-3004, 1965.
[20]Kafoussias, N. G., Local similarity Solution for Cmbined Free-forced Convective and Mss Transfer Flow Past a Semi-infinite Vertical Plate, Int. J. Energy Res., Vol. 14, pp. 305-309, 1990.