Symmetrical Fourier Transform Lens Design for Signal Processing with Optics

W. Pijitrojana
Department of Electrical Engineering, Faculty of Engineering,
Thammasat University, Klong Luang, Pathum Thani 12121, Thailand
Phone 0-2564-3001-9 Ext. 3045, Fax 0-2564-3010, E-mail: pwanchai@engr.tu.ac.th

Abstract

Optical signal processors with the support of good coherence properties of light can perform one- or two-dimensional spatial functions using single linear operators, such as optical Fourier transformation. It is well known that a two-dimensional Fourier transformation can be obtained with an aberration-free positive lens. The lens has been designed in this paper by using a compound lens system in order to get rid of Seidel aberrations. The design of this lens system is governed by the imaging conditions and has been done with optical design program, CODE V. The predesign of this system is a symmetrical lens system.

Keywords: Fourier transform lens design, optical Fourier transformation.

1. Introduction

Optical processing can perform a myriad of processing operations. This is primarily due to its complex amplitude processing capability. Optical Fourier transform processing is frequently used in practice. The Fourier domain (filter) processor and the joint transform (spatial domain filter) processor are two types of Fourier transform processors. The concept of optical filtering was introduced originally by Abbe through his studies on image formation in the microscope with coherent illumination. These optical filter processors make use of the Fourier transform property of lenses that are free from Seidel aberrations and are located in a telecentric ray system. There is a restriction of small field angles of these lenses because the chief rays are kept close enough to the optical axis so that nonlinear terms of the field angle may be neglected. The restriction of small field angles is dropped completely if Abbe's sine condition is fulfilled for the chief rays. The Abbe's sine condition also outlines the resulting Fourier transform lens design. Early lens design (or optical system as it is called by optical designers) is a laborious work for the designers. The lens designer starts with drawing light rays (in itself a mathematical abstraction) originating from an object. These pass through a glass surface, and are bent or refracted through to the image plane. For example, in a 6 element design we need to calculate (by a mechanical and electrical calculator) 200 rays for every lens surface. Three thousand rays, in all, for the entire lens. That took a full three months of work [5]. Today the introduction of the computer and affordable optical design software revolutionized the way lenses are designed and evaluated. The process of designing lenses is more direct, much faster, and infinitely easier [6]. However, designing a lens is foremost a creative act, based on experience and insight into the real character of optical aberrations.

2. Theory of Optical Fourier Transforms

Fourier transforms have played an important role in wave propagation in homogeneous media and in the treatment of wave propagation through lenses. The amplitude distribution in the front and back focal plane of a lens forms, to a good approximation, a Fourier pair of which the point spread function is the Fourier transform of the complex amplitude distribution in the exit pupil. Considering a lens system in Fig. 1.

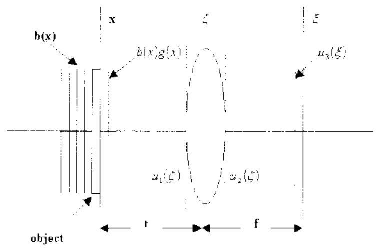


Fig. 1 Fourier Transform Lens.

Under certain conditions the field in the back focal plane of the lens can be shown as proportional to $G(v)$, the Fourier transform of $g(x)$, if the latter is illuminated by a plane wave. The field after the object $g(x)$ is $u_0(x)=b(x)g(x)$. To make the equation simple, a suitable choice $b(x)=1$ is made. By diffraction theory in the Fresnel region, the field u_1 at the first surface of lens is:

$$u_1(\varsigma)=\frac{e^{jkt/2}}{\sqrt{j\lambda t}}\int_{-\infty}^{\infty} g(x)e^{\left[j\frac{k}{2t}(\varsigma-x)^2\right]}dx \quad (1)$$

The overall transmittance is $p(\varsigma)e^{\frac{-jk}{2f}\varsigma^2}$ where $p(\varsigma)$ is the pupil function associated with the lens aperture and can be generalized to include aberrations. Assume that the lens is nonabsorbent lens of diameter D,

$$p(\varsigma)=rect\left(\varsigma/D\right) \quad (2)$$

where

$$rect\left(\varsigma/D\right)\equiv\begin{cases}1, & |\varsigma|<\frac{D}{2}\\ 0, & \text{otherwise}\end{cases} \quad (3)$$

After the lens, the field $u_2(\varsigma)$ is given by:

$$u_2(\varsigma)=u_1(\varsigma)p(\varsigma)e^{-j\frac{k}{2f}\varsigma^2} \quad (4)$$

Then, the field at the back focal plane is:

$$u_3(\xi)=\frac{e^{jkf/2}}{\sqrt{j\lambda f}}\int_{-\infty}^{\infty} u_2(\varsigma)e^{\left[j\frac{k}{2f}(\xi-\varsigma)^2\right]}d\varsigma \quad (5)$$

Substituting $u_1(\varsigma)$, we get:

$$u_3(\xi)=\frac{e^{jkt/2}e^{jkf/2}}{j\lambda\sqrt{ft}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x)e^{\left[j\frac{k}{2t}(\varsigma-x)^2\right]}p(\varsigma)e^{-j\frac{k}{2f}\varsigma^2}e^{\left[j\frac{k}{2f}(\xi-\varsigma)^2\right]}d\varsigma dx \quad (6)$$

Rearranging eq. (6), the equation below can be obtained as:

$$u_3(\xi)=\frac{e^{jk\left(\frac{t}{2}+\frac{f}{2}\right)}}{j\lambda\sqrt{ft}}e^{\left[j\frac{k}{2f}\left(1-\frac{t}{f}\right)\xi^2\right]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x)e^{-j\frac{k}{f}x\xi}p(\varsigma)e^{\left\{j\frac{k}{2t}\left[\varsigma-\left(x+\frac{t}{f}\xi\right)\right]^2\right\}}d\varsigma dx \quad (7)$$

Since $\int_{-\infty}^{\infty} p(\varsigma)e^{\left\{j\frac{k}{2t}\left[\varsigma-\left(x+\frac{t}{f}\xi\right)\right]^2\right\}}d\varsigma$ is a slowly varying function for value λ in an optical regime. To approximate this term, the method of stationary phase [1] is applied so that it becomes $\sqrt{j\lambda t}\,p\left[x+\left(\frac{t}{f}\right)\xi\right]$. The equation (7) becomes

$$u_3(\xi)=\frac{e^{jk\left(\frac{t}{2}+\frac{f}{2}\right)}}{\sqrt{j\lambda f}}e^{\left[j\frac{k}{2f}\left(1-\frac{t}{f}\right)\xi^2\right]}\int_{-\infty}^{\infty} g(x)p\left(x+\frac{t}{f}\xi\right)e^{-j\frac{k}{f}x\xi}dx \quad (8)$$

Let ξ be written in terms of the spatial frequency, $\xi=\lambda f v$, so

$$u_3'(v)=\frac{e^{jk\left(\frac{t}{2}+\frac{f}{2}\right)}}{\sqrt{j\lambda f}}e^{\left[j\pi\left(1-\frac{t}{f}\right)\lambda f v^2\right]}\int_{-\infty}^{\infty} g(x)p(x+\lambda t v)e^{-j2\pi v x}dx \quad (9)$$

The two dimensional form of equation (9), becomes:

$$u_3'(v,w)=\frac{e^{jk\left(\frac{t}{2}+\frac{f}{2}\right)}}{j\lambda f}e^{\left[j\pi\lambda f\left(1-\frac{t}{f}\right)\left(v^2+w^2\right)\right]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x,y)p(x+\lambda tv,y+\lambda tw)e^{-j2\pi(vx+wy)}dxdy \quad (10)$$

Let a coordinate system in phase space be written as (q,p), where q is the height of a light ray above the optical axis of a lens system and p is the angle that the light ray makes with the optical axis. The lens system transforms the coordinate system by rotating the axes 90° as in Fig 2:

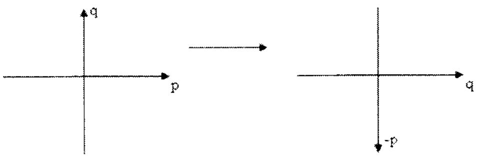


Fig. 2 Fourier Transformation of the lens system transforms the coordinates by rotating the axes 90°.

$$(q,p)\rightarrow(-p,q) \quad (11)$$

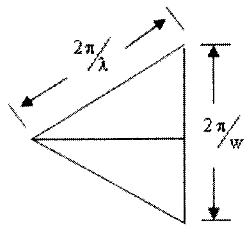
The Fourier transformation in Fig. 2 happen when an object sits at the position of the front focal plane and the image sits at the position of the back focal plane. Therefore, if substituting $t=f$ in eq. (10), then the phase factor outside the integral vanishes so that an exact Fourier transform relationship between the front and back focal plane of the lens system can be obtained:

$$G(v,w)\equiv u_3(v,w)=\frac{1}{j\lambda f}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x,y)e^{-j2\pi(vx+wy)}dxdy \quad (12)$$

3. Theory of Fourier Transform Lens

As mentioned above, the lens used to perform a Fourier transform is an aberrationless lens. Thus, a single lens cannot be used to perform a Fourier transform when the field angles are not small or the chief rays are not close enough to the optical axis. In practice it is feasible to achieve an accurate Fourier transform with a lens system of more than one component. A perfect Fourier transform lens should be capable of transforming plane waves into perfect diffraction limited point images and point objects into plane waves, i.e. plane waves in the space to the left of the lens should form diffraction limited point images in the focal plane to the right of the lens and likewise plane waves entering the lens from the right side should form point images in the first focal plane to the left of the lens.

Two coherent plane waves at an angle of 2θ with respect to each other will form a set of interference fringes in the first focal plane. The fringe width w is given by:



$$\frac{\pi}{w}=\frac{2\pi}{\lambda}\sin\theta \Rightarrow w=\frac{\lambda}{2\sin\theta} \quad (13)$$

Any periodic structure of spacing w in the first focal plane which is illuminated with a coherent plane wave will then form plus and minus diffracted plane waves which propagate at the angle $\theta=\sin^{-1}(\lambda/w)$. The first order image point focused in the second focal plane will be spaced from the zero order by the amount:

$$d=f\cdot\sin\theta \quad (14)$$

where f is the focal length of the lens. When a single thin lens performs as a perfect Fourier transform lens, it will form the two images at the heights given by $f\cdot\tan\theta$. This difference means that the lens must have a slight amount of distortion. Thus, a single lens with the aperture in contact with it can not provide this distortion. This is because a Fourier transform lens has an effective aperture stop at the first focal plane of the system, and the rays through the point $x=y=z=0$ represent the chief rays. With help of Abbe's sine condition, the formation of a perfect image of a point object can be made so that the rays penetrate through the system at the coordinate $x=y=z=0$. To perform Fourier transforms by optical means, the simplest way is to place a diffracting aperture in front of the lens so that the point of observation is moved from infinity into the back focal plane of the lens. The diffracting aperture at the front focal plane can also be replaced by a number of coherent light sources at infinity, where the phase and amplitude of the light oscillations as well as the lateral locations of the light sources are chosen such that each light source corresponds to a particular diffracted plane parallel wave. To have an equivalent ray path, all rays from the light sources at infinity must penetrate the front focal plane, and the resolution in the image plane is solely determined by the aperture limits in the front focal plane. If the numerical aperture for this imaging process is to be changed, it can be accomplished only by changing the aperture in the front focal plane. Therefore, the effective aperture stop of the system is located in the front focal plane, and the rays through the point $x = y = z = 0$ represent the chief rays [2].

With this interpretation of the diffraction process in the front focal plane, the imaging conditions for a Fourier transform lens is such that:

1. Images in the front focal plane must be aplanatic, i.e. free from spherical aberration and coma.
2. Images in the back focal plane must be aplanatic, i.e. free from spherical aberration and coma; and anastigmatic, i.e. having a flat field free from astigmatism and it is necessary to reduce the Petzval sum drastically.

Due to the Stop-Shift Theorem [3], Petzval curvature is independent of entrance pupil coordinates as well as object distance; a flat field in the back focal plane will automatically provide a flat field in the front focal plane.

4. Symmetrical Fourier Transform Lens Design

A lens system in this paper is a symmetrical optical Fourier transform system with symmetrical location of entrance pupil at the front focal plane and image plane at the back focal plane relative to the lens system. One of the advantages of a symmetrical system is that the correction for only the first four Seidel aberrations is necessary. By the way of symmetry through the correction of spherical aberration and coma in the plane at the back focal plane, distortion from the fifth Seidel aberration, receives its proper value (other than zero) on account of the sine condition for the axial image point in the plane at the front focal plane. In another words, it is virtually impossible to nullify distortion while the sine condition is fulfilled because there is a symmetrical relationship between distortion and aplanatism. Therefore, to achieve a simultaneous correction for the first four Seidel aberrations, distortion may not be specified. The lens system, such as photographic objectives, which have all five Seidel aberrations minimized (especially the freedom from distortion) are restricted only to paraxial Fourier transform applications.

The symmetric optical Fourier transform system has been designed by using lens elements with refracting surfaces that are spherical and rotationally symmetric. The design consists of one pair of identical triplets that are arranged symmetrically on a common optical axis. The reduction of the overall length from the front focal plane to the back focal plane of the lens system has been accomplished by placing all lens elements relatively close to the input and output planes, thereby providing a large overlap of the principal planes. Furthermore, one of important criteria of Fourier transform lens design is to fix the stop plane at the front focal point of the lens system (see Fig. 4, Lens Data Manager Table) according to eq. (10). At this point the Fourier transform lens can replace a single lens exactly.

Since the first four Seidel aberrations need to be corrected, six lens elements are the minimum number required in this Fourier transform unit. The six thin lens elements provide five degrees of freedom. These are two axial separations between the lens elements and three bendings of the lens elements of one triplet. Thus, the number of variables is larger than the number of aberrations. Four lens elements, two doublets, provide only three degrees of freedom, thereby making a stable correction impossible.

Fig. 3 shows the diagram of the Fourier transform lens system displayed by the VIEW function in CODE V [4].The effective focal length (EFL) is 147.86 mm. The field of view (FOV) is 10°. The image height of the lens system can be solved according to the infinite-object-distance relationship:

$$\begin{aligned}\text{Image height} &= f \cdot \sin(\text{semi} - \text{FOV}) \\ &= 25.8\ \text{mm}.\end{aligned} \qquad (15)$$

The f-number of the lens system is fixed aperture and $f/1.5$. The entrance pupil is circular with diameter 100 mm.

The lens system in fig. 3 is found by using the optimization function in CODE V. The purpose of optimization is to generate the best possible optical system that can be achieved within a given set of physical and other constraints. The term "best " is measured by an error function which combines image error data into a single number that is made as small as possible, i.e. the perfect lens system has a zero error function.

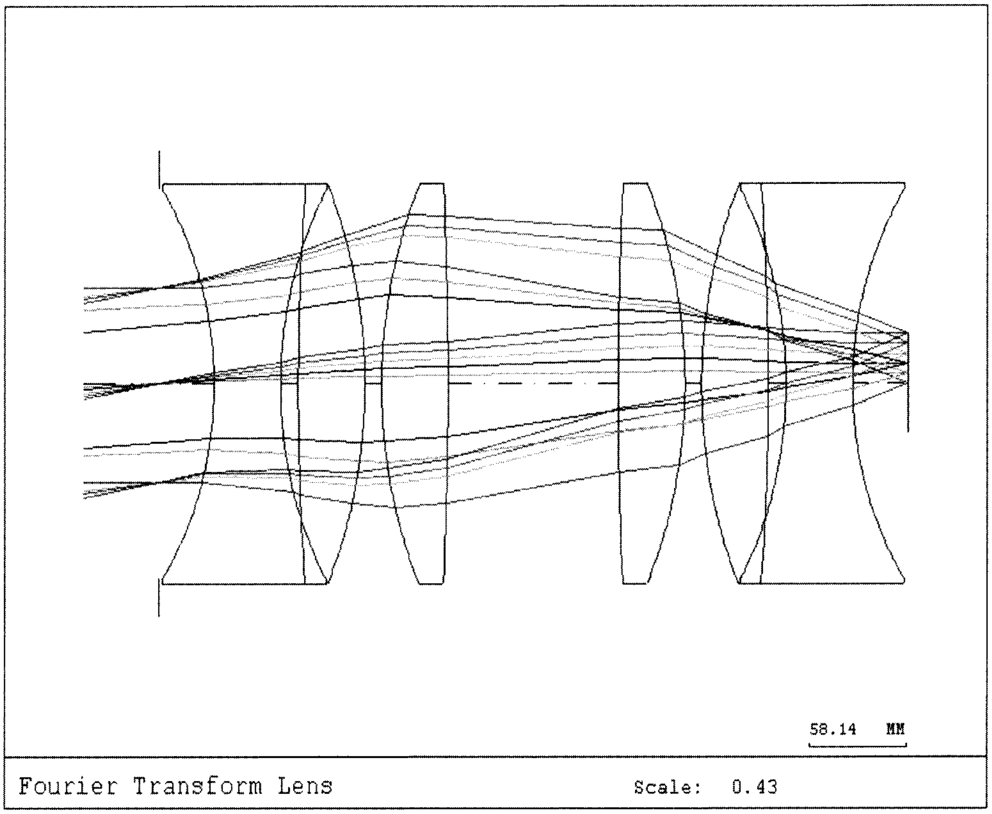


Fig. 3 Fourier transform lens system diagram.

Lens Data Manager

Surface #	Surface Name	Surface Type	Y Radius	Thickness	Glass	Refract Mode	Y Semi-Aperture
Object		Sphere	Infinity	Infinity		Refract	0
Stop		Sphere	Infinity	32.9966 P		Refract	99.9996 O
2		Sphere	-176.6451 V	40.0000 P	FK5_SCHOT	Refract	99.9996 O
3		Sphere	196.8981 V	10.7158 P		Refract	99.9996 O
4		Sphere	1267.5096 V	40.0000 P	SF4_SCHOT	Refract	99.9996 O
5		Sphere	-246.4429 V	10.0000 P		Refract	99.9996 O
6		Sphere	243.5921 V	40.0000 P	SF4_SCHOT	Refract	99.9996 O
7		Sphere	-1965.4260 V	102.2088 V		Refract	99.9996 O
8		Sphere	1965.4260 P	40.0000 V	SF4_SCHOT	Refract	99.9996 O
9		Sphere	-243.5921 P	10.0000 V		Refract	99.9996 O
10		Sphere	246.4429 P	40.0000 V	SF4_SCHOT	Refract	99.9996 O
11		Sphere	-1267.5096 P	10.7158 V		Refract	99.9996 O
12		Sphere	-196.8981 P	40.0000 V	FK5_SCHOT	Refract	99.9996 O
13		Sphere	176.6451 P	32.9966 V		Refract	99.9996 O
Image		Sphere	Infinity	0.0000		Refract	26.0717 O
End Of Data							

Fig. 4 Lens Data Manager show variables of the lens used for Lens design. 'V' stands for vary indicating that it is a variable and 'P' pickup.

The error function of the lens system in Fig. 3 is 155.42016084. The initial value of the error function of the lens system before optimization is 0.1303629×10^{6}. The LDM (Lens Data Manager) spreadsheet is shown in Fig. 4. First order data is obtained after scaling the lens system so that the scaled lens system has the correct first order focal length and f-number. The list of the first order data is shown in Fig. 5.

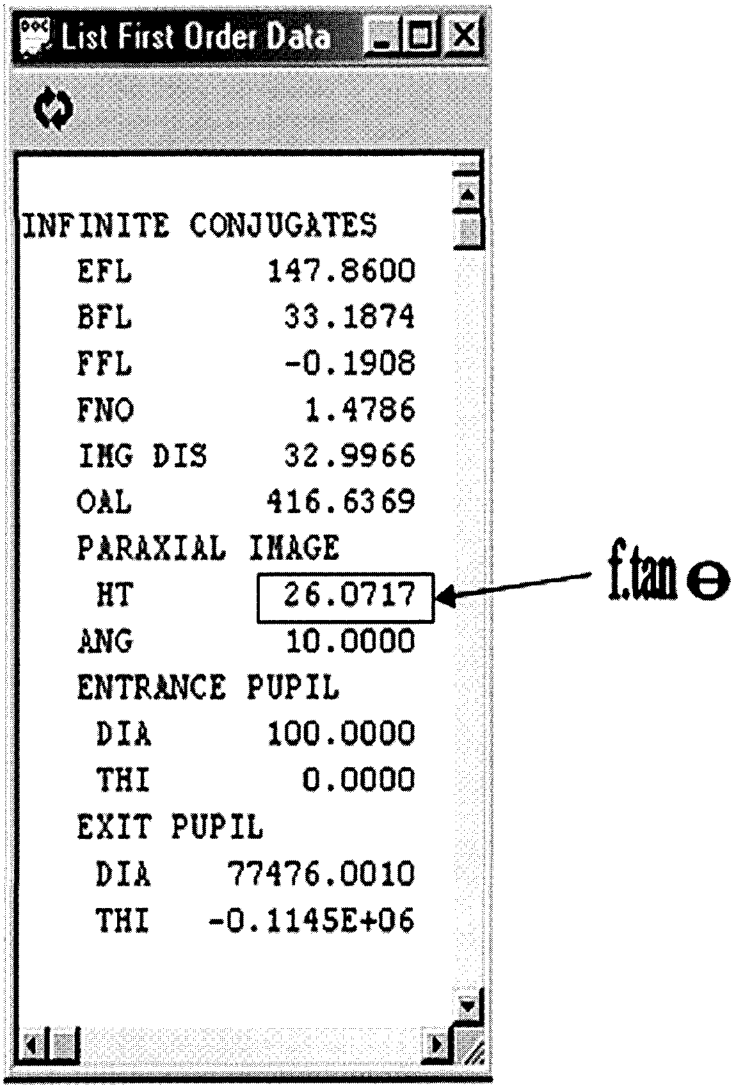


Fig. 5 The First Order Data Sheets Show paraxial image height of rays.

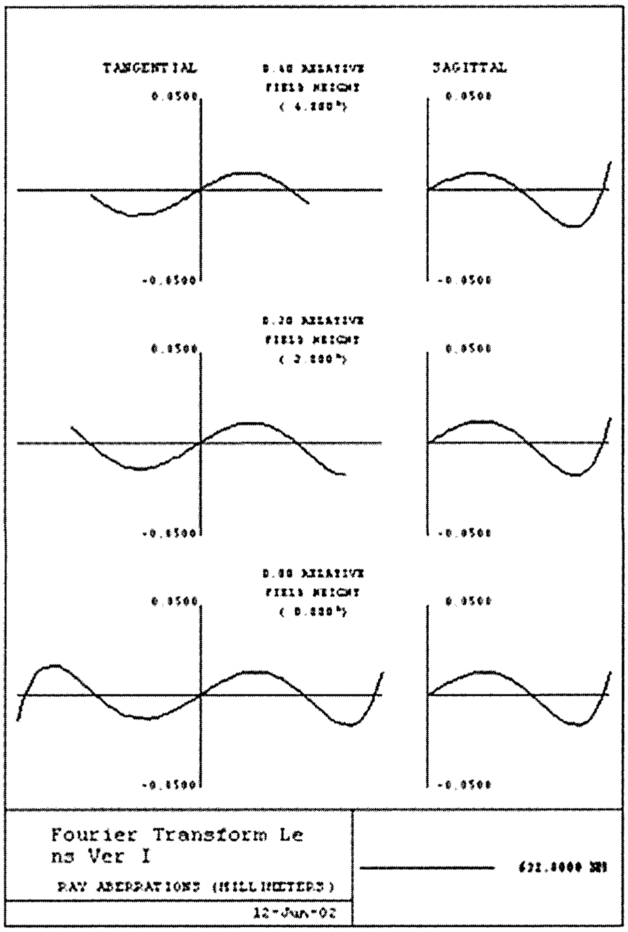
5. Analysis of the Lens System

There are many types of analysis in CODE V. The results of these analyses will also guide the setup for optimization.

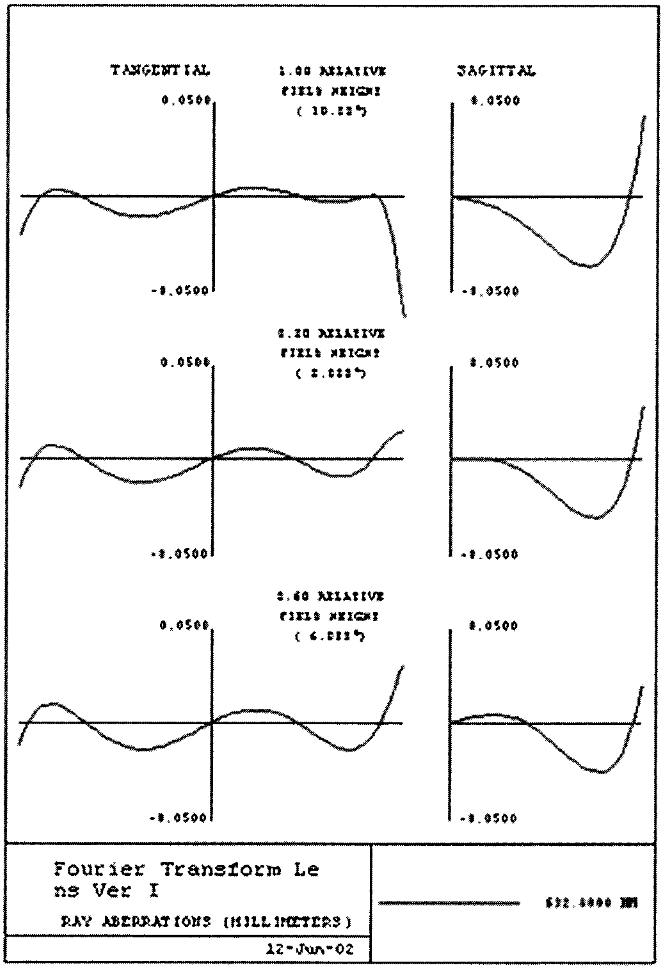
Diagnostic Analysis

- Ray Aberration Curve (RIM)

A ray aberration curve is a useful way of looking at ray trace data to see patterns that may cause problems. Transverse ray aberrations are measured on the image surface as the distance from a particular ray to the chief ray for the same field point (for a perfect lens, this should be zero for every ray traced from the same field point). Fig. 6 shows Ray Aberration Curve plots in different fields of view. In Fig.6, the cubic curves show mainly spherical aberration. The automatic scale value in Fig. 7 is 0.05 mm., 50 μm.



(a) Ray aberration curves (1)



(b) Ray aberration curves (2)

Fig. 6 Ray Aberration Curves at Aperture Stop with the vertical coordinate; measured in lens units on the image surface and the horizontal coordinate; the normalized aperture stop or pupil, the end point = 100% of the normalized stop.

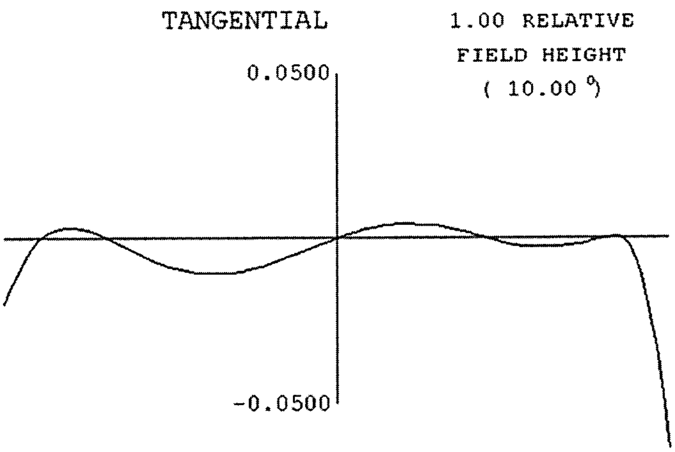


Fig. 7 Automatic scale value of one of the ray aberration curves.

- Third Order Aberrations

THO SO..I
Fourier Transform Lens Ver I
Position 1, Wavelength = 632.8 NM

	SA	TCO	TAS	SAS	PTB	DST	AX	LAT	PTZ
STO	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2	0.132109	-0.324151	0.416215	0.239469	0.151095	-0.195859	0.000000	0.000000	0.001315
3	4.357567	6.466562	3.402359	1.269853	0.203600	0.628148	0.000000	0.000000	0.001772
4	-1.276026	-2.823565	-2.135946	-0.747517	-0.053303	-0.551364	0.000000	0.000000	-0.000464
5	-0.084134	0.110863	-0.209266	-0.176803	-0.160572	0.077657	0.000000	0.000000	-0.001397
6	-0.715383	-1.698704	-1.539173	-0.642809	-0.194627	-0.508792	0.000000	0.000000	-0.001693
7	-0.265263	0.403486	-0.206727	-0.070342	-0.002149	0.035665	0.000000	0.000000	-0.000019
8	0.167886	-0.270596	0.143232	0.046311	-0.002149	-0.024881	0.000000	0.000000	-0.000019
9	-2.753462	-2.831718	-1.165360	-0.518205	-0.194627	-0.177644	0.000000	0.000000	-0.001693
10	0.210645	-0.391930	0.082505	-0.079546	-0.160572	0.049335	0.000000	0.000000	-0.001397
11	-1.549839	-1.710493	-0.682570	-0.263059	-0.053303	-0.096776	0.000000	0.000000	-0.000464
12	1.735852	2.856966	1.770987	0.726063	0.203600	0.398332	0.000000	0.000000	0.001772
13	-0.151709	0.151614	0.100589	0.134260	0.151095	-0.044725	0.000000	0.000000	0.001315
SUM	-0.191757	-0.061667	-0.023154	-0.082325	-0.111910	-0.410904	0.000000	0.000000	-0.000974

Surface number of the lens system

Fig. 8 Third Order Aberrations Table.

Note: The abbreviations of the third order aberrations stand for:

Transverse:
AX Axial colour **LAT** Lateral colour
SA 3rd order spherical aberration
TCO 3rd order tangential coma
SAS 3rd order sagittal astigmatic blur
TAS 3rd order tangential astigmatic blur
PTB 3rd order Petzval blur
DST 3rd order distortion
Curvature:
PTZ Petzval surface curvature.

Geometrical Analysis

- Spot Diagram

A spot diagram gives a quick and easy analysis of the image quality. The various field positions are plotted vertically, while multiple focal positions (if defined by command in CODE V) are plotted horizontally. Fig. 9 shows a spot diagram with one focal position and Fig. 10 shows a spot diagram with seven focal positions.

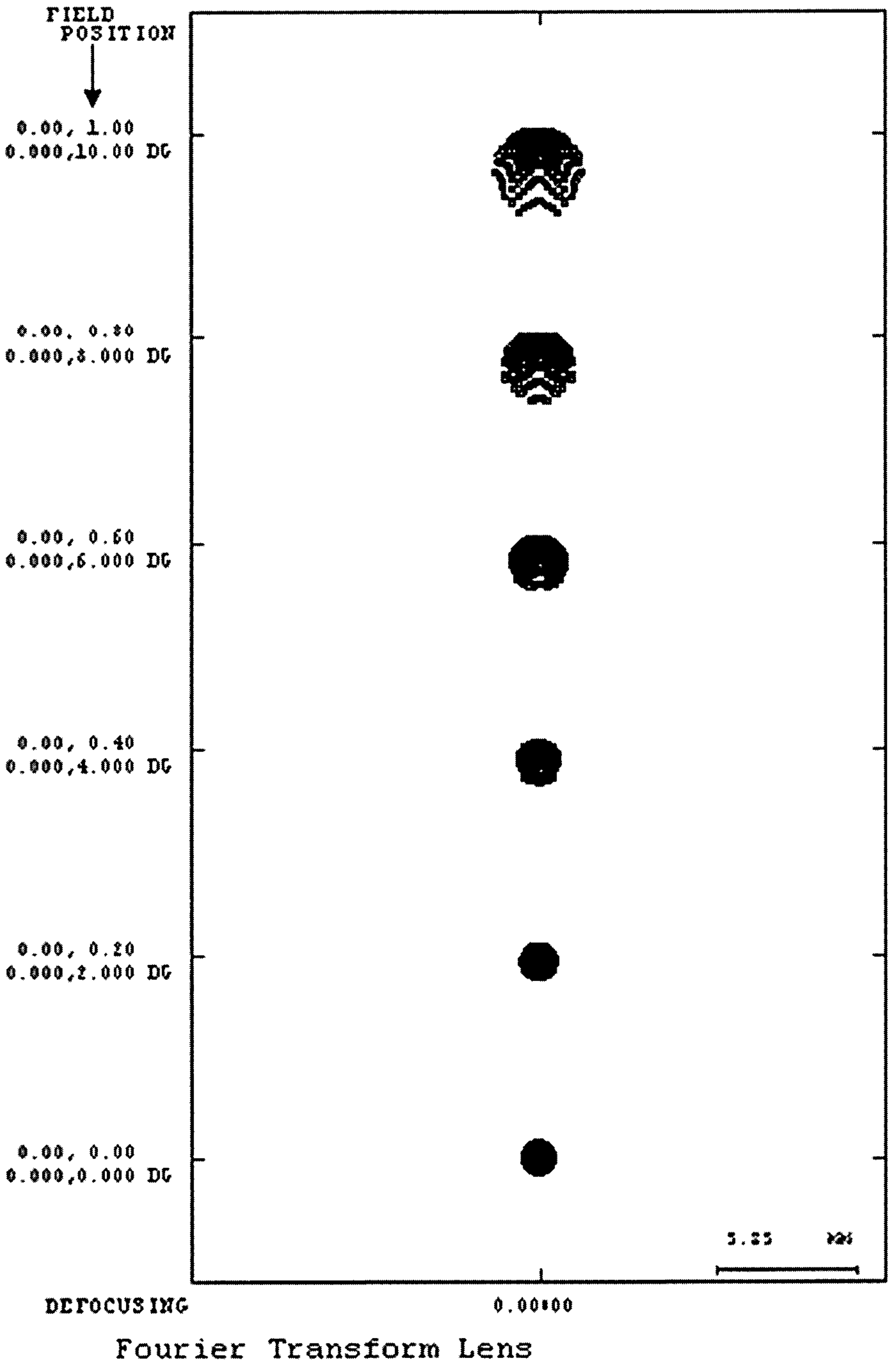


Fig. 9 Spot Diagram of a Fourier transform compound lens with single focal point. DG stands for degree.

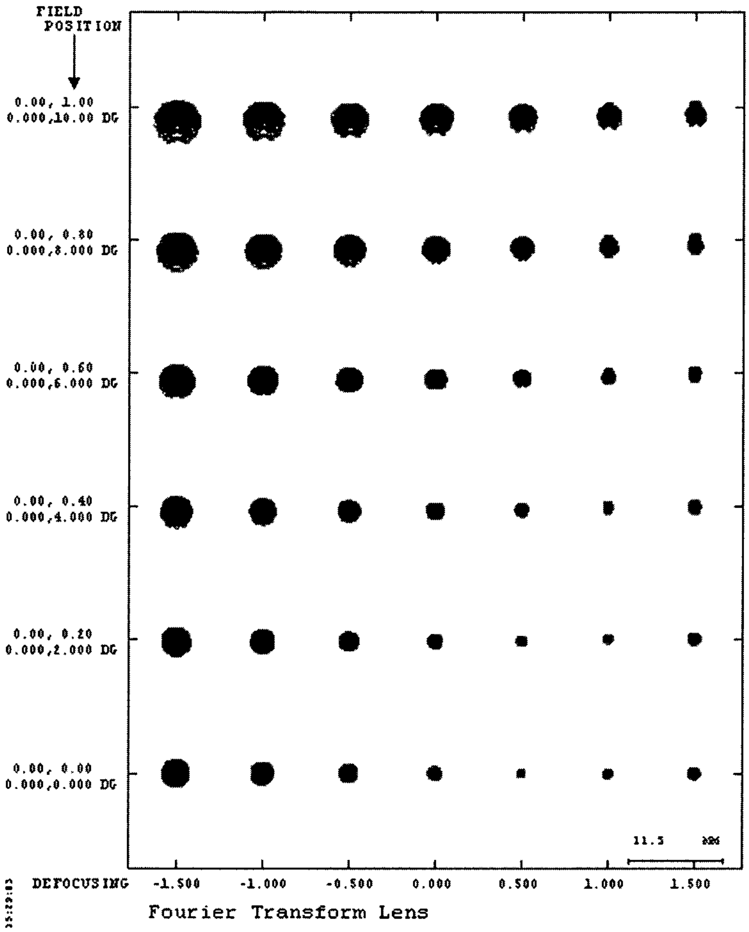


Fig. 10 Through-Focus Spot Diagram Shows spot diagrams at different seven focal points. DG stands for degree.

- Field Aberrations (Astigmatism and Distortion Analysis)

The image height is related to field angle by the relationship $h = f \cdot \sin\theta$. If the real image height differs from this prediction, the image has distortion. Fig. 11 shows the resulting distortion curve.

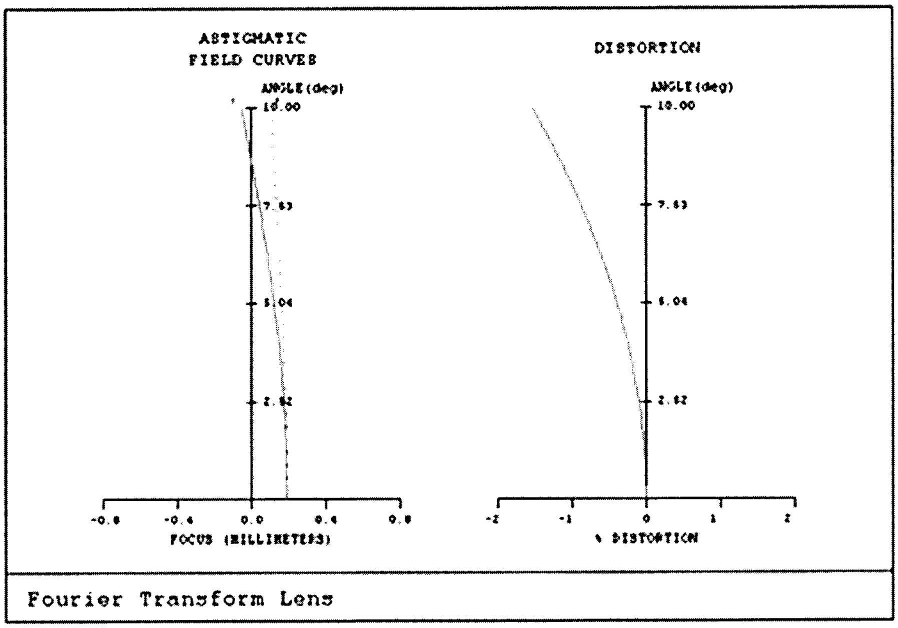


Fig. 11 Field Curves and Distortion of a Fourier transform compound lens.

Units of focus are MILLIMETERS

ROTATIONALLY SYMMETRIC FIELD ABERRATIONS

POSITION 1 Fourier Transform Lens Ver I

S C A N L I N E A R I T Y

RELATIVE FIELD HEIGHT (A)	ANGLE (DEGREES) (B)	IMAGE HEIGHT (C)	REFERENCE IMAGE HEIGHT (D)	ERROR (PERCENT) (E)	LOCAL ERROR (PERCENT) (F)
0.00	0.0000	0.000000	0.000000	0.0000	0.3357
0.05	0.5000	1.290308	1.286009	0.3343	0.3343
0.10	1.0000	2.580512	2.572017	0.3303	0.3262
0.15	1.5000	3.870508	3.858026	0.3235	0.3101
0.20	2.0000	5.160193	5.144035	0.3141	0.2859
0.25	2.5000	6.449463	6.430043	0.3020	0.2536
0.30	3.0000	7.738215	7.716052	0.2872	0.2133
0.35	3.5000	9.026346	9.002061	0.2698	0.1650
0.40	4.0000	10.313754	10.288070	0.2497	0.1088
0.45	4.5000	11.600337	11.574078	0.2269	0.0447
0.50	5.0000	12.885993	12.860087	0.2014	-0.0274
0.55	5.5000	14.170622	14.146096	0.1734	-0.1073
0.60	6.0000	15.454123	15.432104	0.1427	-0.1949
0.65	6.5000	16.736399	16.718113	0.1094	-0.2903
0.70	7.0000	18.017349	18.004122	0.0735	-0.3933
0.75	7.5000	19.296877	19.290130	0.0350	-0.5040
0.80	8.0000	20.574884	20.576139	-0.0061	-0.6222
0.85	8.5000	21.851278	21.862148	-0.0497	-0.7476
0.90	9.0000	23.125965	23.148156	-0.0959	-0.8804
0.95	9.5000	24.398848	24.434165	-0.1445	-1.0207
1.00	10.0000	25.669836	25.720174	-0.1957	-1.1680

Calibrated Focal Length = 147.3657

(A) Fraction of angle in object space

(B) Chief ray angle in object space in degrees

(C) Ray trace image height

(D) (Calibrated focal length-f) $*(B * \pi/180)$

(E) $100 * (C/D - 1)$

(F) $F(i) = 100 * \{[C(i) - C(i-1)]/[D(i) - D(i-1)] - 1\}$

(a) Field Aberration-Linearity table. Show the actual image height in column C and $f * \theta$ image height in column D.

ROTATIONALLY SYMMETRIC FIELD ABERRATIONS

POSITION 1 Fourier Transform Lens Ver I

RELATIVE FIELD HEIGHT	ANGLE (DEG)	X-FOCUS AT THE IMAGE SURFACE	Y-FOCUS AT THE IMAGE SURFACE	X-FOCUS (DISPLACED BY 0.455010)	Y-FOCUS (DISPLACED BY 0.455010)	DISTORTION (PER CENT)
0.00	0.00	-0.264103	-0.264103	0.190907	0.190907	0.00000
0.10	1.01	-0.266974	-0.265380	0.188037	0.189630	-0.01584
0.20	2.02	-0.275536	-0.269130	0.179474	0.185880	-0.06329
0.30	3.03	-0.289642	-0.275191	0.165369	0.179819	-0.14221
0.40	4.03	-0.309042	-0.283290	0.145968	0.171720	-0.25233
0.50	5.04	-0.333392	-0.293053	0.121619	0.161957	-0.39329
0.60	6.04	-0.362248	-0.304009	0.092762	0.151001	-0.56463
0.70	7.04	-0.395075	-0.315594	0.059935	0.139416	-0.76580
0.80	8.03	-0.431242	-0.327166	0.023768	0.127844	-0.99616
0.90	9.02	-0.470029	-0.338013	-0.015019	0.116998	-1.25498
1.00	10.00	-0.510626	-0.347367	-0.055616	0.107643	-1.54146

Units of focus are MILLIMETERS

(b) Field Aberration-Astigmatism and Distortion Analysis Shows the distortion of the real rays displaced from the chief ray.

Fig. 12 The Table (a) replaces the astigmatism and distortion Table (b) with a linearity table to show actual image height and $f * \theta$ image height for 20 equally spaced points across the field; % difference of these is also listed.

In Fig. 12, the Table (a) replaces astigmatism and distortion table with a linearity table to show actual image and $f \cdot \theta$ image height for 20 equally spaced points across the field; % difference of these is also listed. The distortion table shows the distortion (percent) which is about 1.54%. These calculations are based on tracing real rays that are slightly displaced from the chief ray. This distortion is calculated from Fig. 13 and Fig. 14. In Fig. 13, the image height is calculated from $f \cdot \tan\theta = 26.071774$ and, in Fig. 14, the image height is calculated from $f \cdot \sin\theta = 25.66982$.

FIO SO..I
Fourier Transform Lens Ver I
Position 1, Wavelength = 632.8 NM

	HMY	UMY	N * IMY	HCY	UCY	N * ICY
EP	50.000013	0.000000		0.000000	0.176327	
STO	50.000013	0.000000	0.000000	0.000000	0.176327	0.176327
2	50.000013	0.092575	-0.283054	5.818186	0.129430	0.143390
3	53.703004	0.270124	0.542870	10.995396	0.219475	0.275318
4	56.597592	0.135252	0.314777	13.347239	0.120925	0.230005
5	62.007676	0.048020	-0.203591	18.184235	0.156264	0.082477
6	62.487871	-0.082469	0.304546	19.746872	0.054576	0.237329
7	59.189105	-0.166872	-0.196987	21.929892	0.087125	0.075967
8	42.133300	-0.104558	-0.145435	30.834855	0.043073	0.102814
9	37.950989	-0.299743	-0.455540	32.557760	-0.024837	-0.158494
10	34.953563	-0.232083	-0.157910	32.309387	-0.070369	0.106266
11	25.670247	-0.421256	-0.441508	29.494635	-0.140568	-0.163838
12	21.156174	-0.248339	-0.528703	27.988337	-0.048104	-0.282715
13	11.222602	-0.338158	-0.274626	26.064158	0.000228	0.147779
IMG	-0.089308	-0.338158		26.071774	0.000228	

Fig. 13 Paraxial Ray Tracing Table.

RSI SO..I W1 F6 0 0
Lens is assumed to have been changed--TOR coefficients reinitialized.
Tolerances/Compensators are unchanged.

Fourier Transform Lens Ver I
Position 1, Wavelength = 632.8 NM

	X	Y	Z	TANX	TANY	LENGTH
OBJ	0.00000	-0.175E+12	0.00000	0.00000	0.17633	
STO	0.00000	0.00000	0.00000	0.00000	0.17633	0.00000
2	0.00000	5.80138	-0.09529	0.00000	0.12872	33.40884
3	0.00000	11.00216	0.30763	0.00000	0.22292	40.73627
4	0.00000	13.33796	0.07018	0.00000	0.12064	10.73550
5	0.00000	18.07516	-0.66375	0.00000	0.15636	39.55080
6	0.00000	19.86941	0.81171	0.00000	0.05285	11.61488
7	0.00000	21.93401	-0.12239	0.00000	0.08426	39.12041
8	0.00000	30.57637	0.23785	0.00000	0.04134	102.93251
9	0.00000	32.13203	-2.12855	0.00000	-0.02736	37.66573
10	0.00000	31.74405	2.05301	0.00000	-0.07116	14.18687
11	0.00000	29.06730	-0.33334	0.00000	-0.14297	37.70878
12	0.00000	27.76901	-1.96800	0.00000	-0.04790	9.17343
13	0.00000	25.66886	1.87496	0.00000	0.00003	43.89323
IMG	0.00000	25.66982	0.00000	0.00000	0.00003	31.12161
		OPD =	0.000 Waves			

Fig. 14 Real Ray Tracing Table.

- Through-Focus Spot Diagrams

A through-focus spot diagram is shown in Fig. 10 with seven focal positions starts at −1.5 mm. in steps of 0.5 mm.

Diffraction Analysis

Diffraction analysis calculations take into account the wave nature of light, so that even a perfectly corrected system will have a finite image spot size and frequency response.

- Point Spread Function

The Point Spread Function option gives detailed image structure information including all diffraction effects. Fig. 15 shows the spread function diagram which is 3-D projection plots and colour displays. The Strehl ratio is 0.0188 and the total energy through the pupil, contained within the central maximum, is about 99%. The point spread function in Fig. 15 is plotted with some amount of spherical aberration and coma.

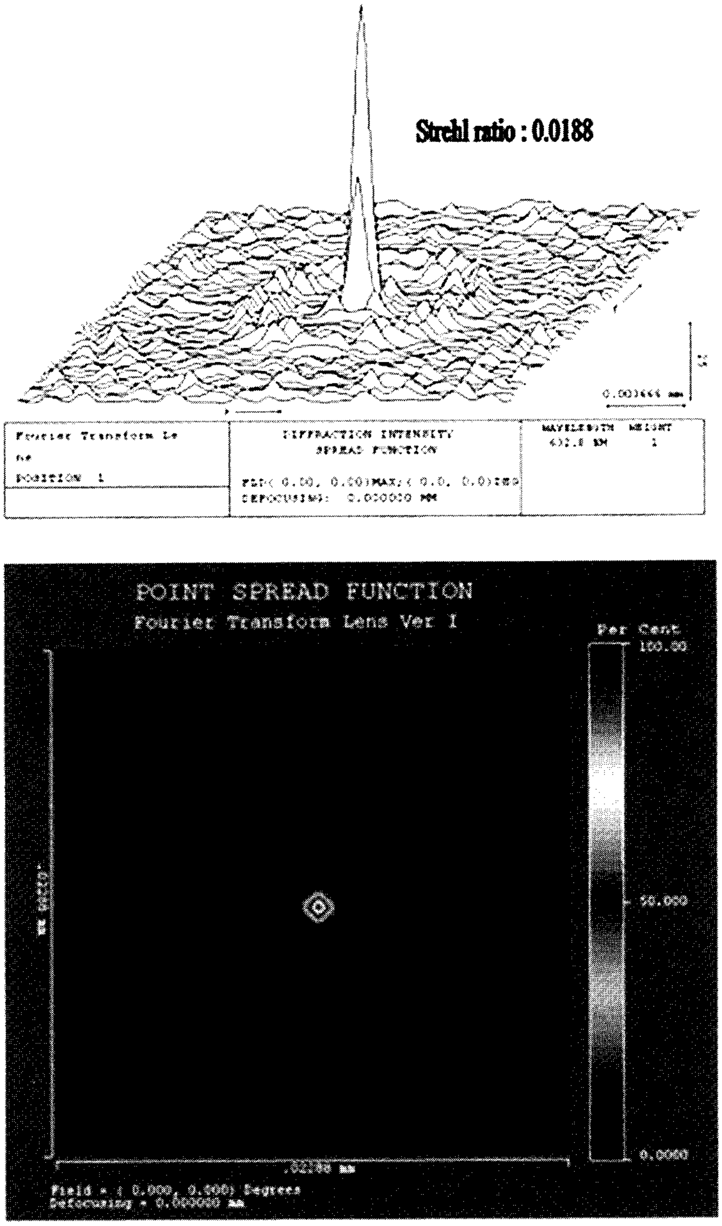
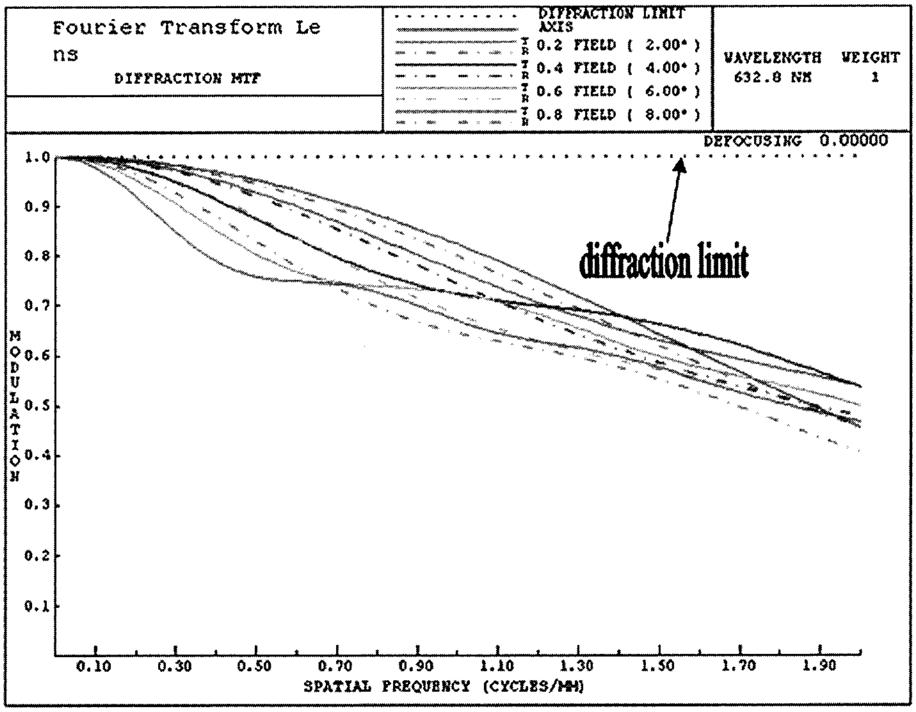


Fig. 15 Point Spread Function of a field angle of 0 degree.

- Modulated Transfer Function

MTF essentially analyzes the spatial frequency response of an optical system. The maximum frequency is the axial diffraction limit, and the increment in frequency is 1/60 of the axial diffraction limit. Modulation is relative contrast, with 1.0 representing ideal contrast (perfect black and white, no degradation to intermediate grays). For large features (low spatial frequency), even a poor lens will have a good contrast, while for higher frequencies (fine details), aberrations and diffraction blend the dark and light areas. Fig. 16 shows the modulation transfer function at different field angles of Fourier transform lens. Fig. 17 shows the modulation transfer function for the lens with spherical aberration.



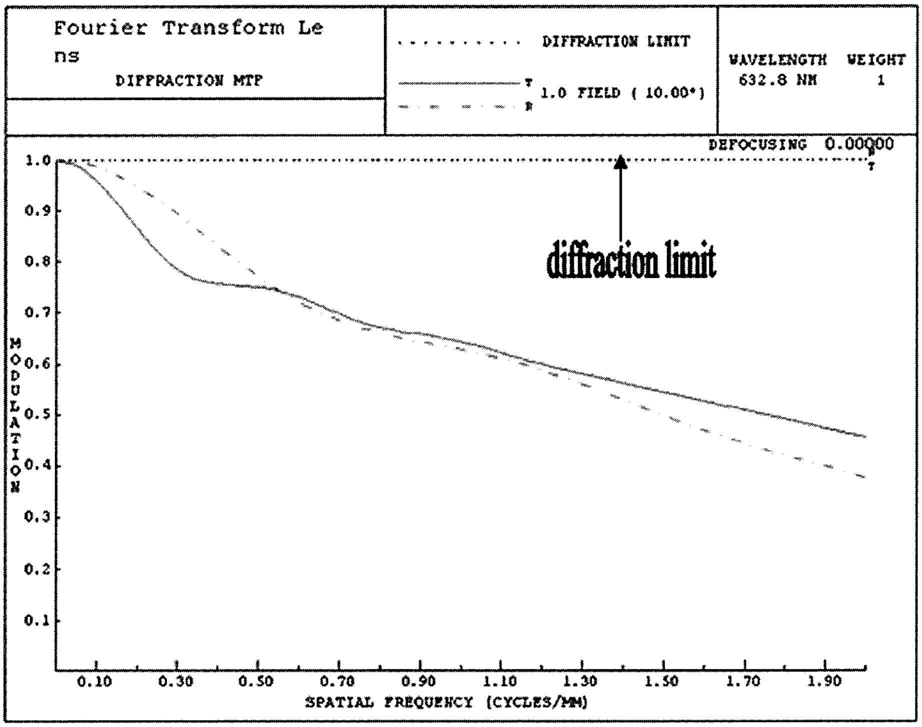
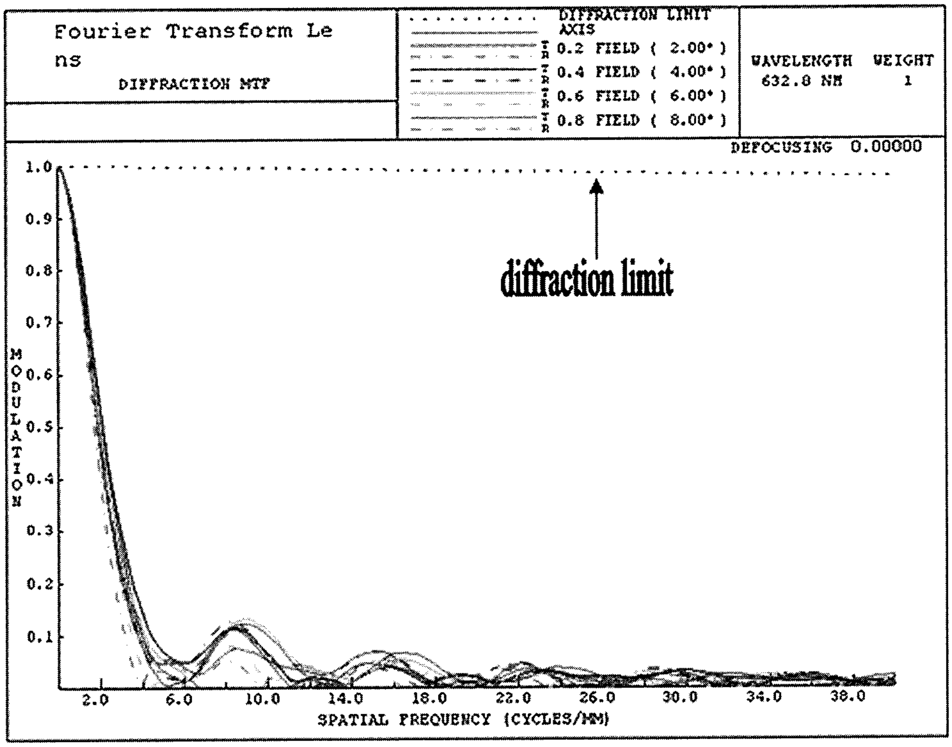


Fig. 16 Modulation Transfer Function.



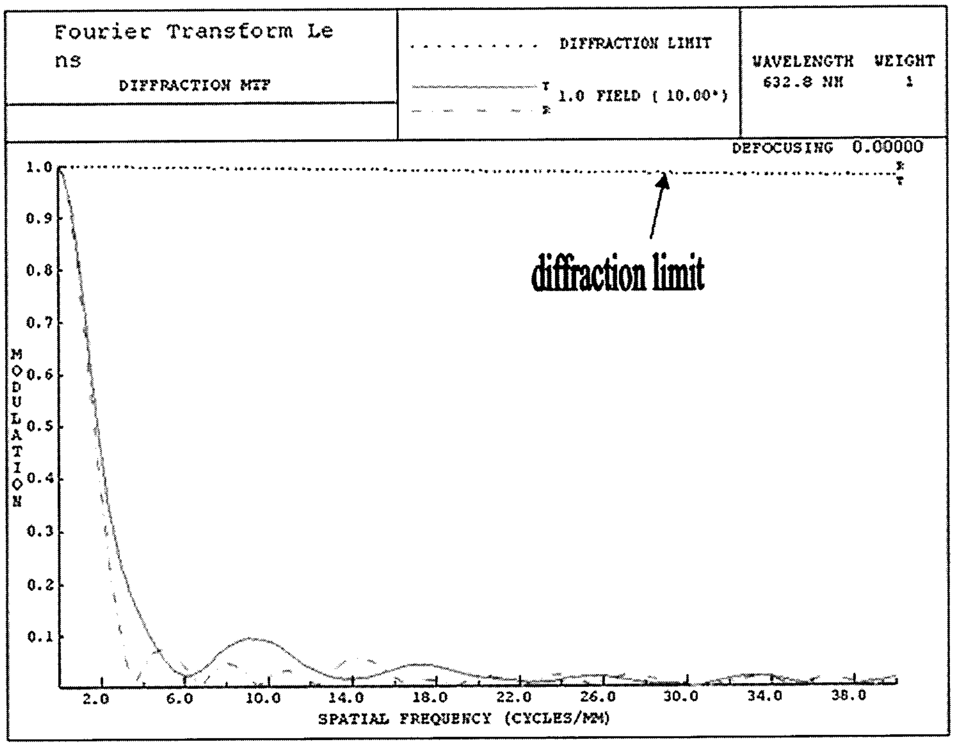


Fig. 17 Modulation Transfer Function with a spherical aberration.

To show that the compound lens obtained in this paper is well-designed, the lens system is compared with the bi-convex single lens. The bi-convex is modeled and simulated using CODE V. The results of simulations with a comparison table are as the following (Figs. 18-22):

Lens Data Manager

Surface #	Surface Name	Surface Type	Y Radius	Thickness	Glass	Refract Mode	Y Semi-Aperture
Object		Sphere	Infinity	Infinity		Refract	0
Stop		Sphere	Infinity	148.7253		Refract	100.0000 O
2		Sphere	156.6000	19.5000	BK7_SCHOT	Refract	100.0000 O
3		Sphere	-156.6000	128.3600		Refract	100.0000 O
Image		Sphere	Infinity	0.0000		Refract	13.5353 O
End Of Data							

Fig. 18 Lens Data Manager of a single lens.

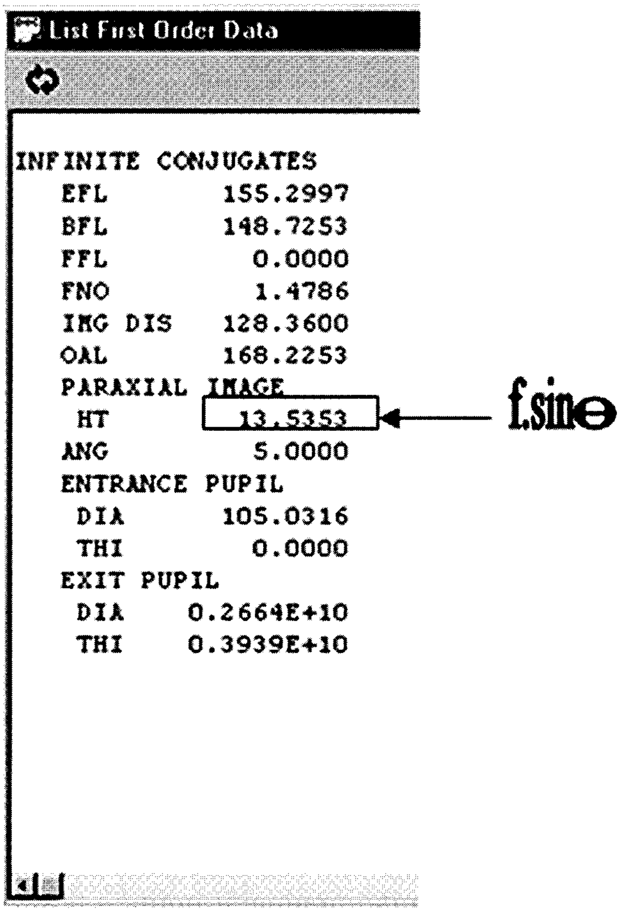


Fig. 19 List of First Order Data of a single lens.

THO SO..I
Single Lens
Position 1, Wavelength = 632.8 NM

	SA	TCO	TAS	SAS	PTB	DST	AX	LAT	PTZ
STO	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2	-0.657115	-1.002737	-0.577811	-0.237778	-0.067762	-0.120948	0.000000	0.000000	-0.002171
3	-8.498445	-3.355066	-0.509273	-0.214932	-0.067762	-0.028284	0.000000	0.000000	-0.002171
SUM	-9.155560	-4.357804	-1.087084	-0.452711	-0.135524	-0.149232	0.000000	0.000000	-0.004342

Fig. 20 Third Order Aberrations of a single lens.

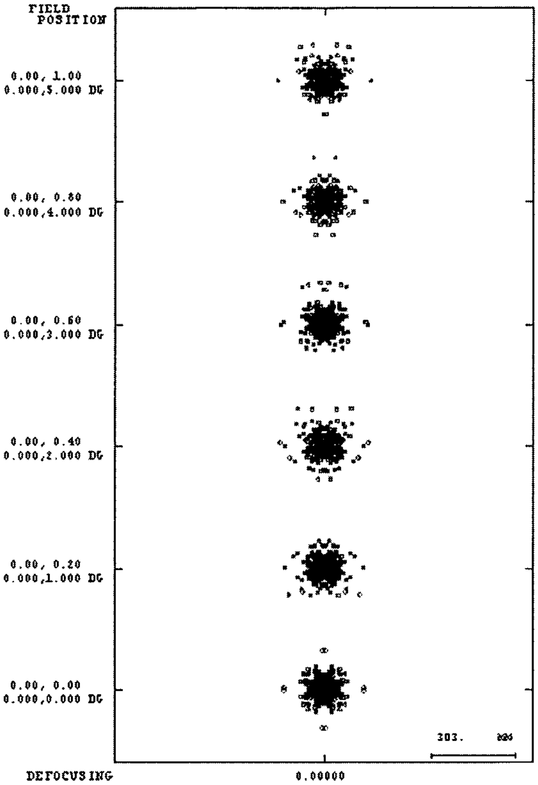


Fig. 21 Spot Diagram of a single lens.

Aberration coefficients	A single lens	A compound lens	Correction
SA (spherical aberration)	-9.155560	-0.191757	97.9 %
TCO (tangential coma)	-4.357804	-0.061667	99.3 %
TAS (tangential astigmatic blur)	-1.087084	-0.023154	99.5 %
SAS (sagittal astigmatic blur)	-0.452711	-0.082325	95.5 %
PTB (Petzval blur)	-0.135524	-0.111910	79.7 %
DST (distortion)	-0.149232	-0.410904	66.4 %
PTZ (Petzval surface curvature)	-0.004342	-0.000974	77.6 %

Fig. 22 Shows the degree to which the design change of the Fourier transform lens has corrected for the aberrations.

5. Conclusion

In this paper, the Fourier transform lens has been designed using a compound lens system which is comprised of six elements of lenses. The Fourier transform lens is well-designed because most of the aberrations are corrected as shown in Fig. 8 (Third Order Aberrations Table.)

6. References

[1] A Walther, *The Ray and Wave Theory of Lenses*, Cambridge Univeristy Press (1995).

[2] K. von Bieren, *Lens Design for Optical Fourirer Transform Systems*, Applied Optics, vol. 10, 2739-2742 (1971).

[3] Pantazis Mouroulis and John Macdonald, *Geometrical Optics and Optical Design*", Oxford University Press, 1997.

[4] CODE V is a Tool for Designing and Analyzing Optical System. CODE V is the Proprietary Product of Optical Research Associates.

[5] Erwin Puts, *Lens Design: Criteria and Considerations*, http://www.imx.nl/photosite/leica/technics/optics01/lensdesign01.html.

[6] Gregory Hallock Smith, *Practical Computer-Aided Lens Design*, Willmann-Bell Press, ISBN 0943396573.

Wanchai Pijitrojana received the B.Eng. degree in Telecommunication Engineering from King's Mongkut Institute of Technology, Ladkrabang, Bangkok, Thailand. He also received the M.S. degree in both, Computer Technology and Nonlinear Optics (Electrophysics) from the Asian Institute of Technology, Bangkok, Thailand and the University of Southern California, California, U.S.A., respectively, and the Ph.D. degree in Optoelectronics from King's College, University of London, London, England. He is currently a lecturer in the EE Department, Thammasat University, Bangkok, Thailand. His current research interests are Optical Interconnection systems, Optical Design, Optics, Nonlinear Optics, Physical Optics, PBG, and Mathematics applied to Optics.