Normalized Graphs for Seepage Analyses along Sheet Pile in Double Soil Layers

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Abstract

Sheet pile systems are very popularly used in a variety of civil works. The objective of this paper is to create the normalized graphs of the seepage analyses along a sheet pile in double soil layers by using the finite element program called SEEP/W. The model is simulated by using the properties and characteristics of Bangkok soil layers. To verify and accept the simulation of the case studies, the effects of numbers of elements in finite element method is done and the proportion of model dimensions is studied. After the program is simulated, it can be seen that the proper number of elements is about 4000. There is a suitable proportion of distance to depth models, 4:1. Normalized graphs are presented, their utilization shown through some examples of flow rate calculation under sheet pile. Finally, some limitations of this study are proposed and further recommendation also presented.

Keywords: Normalized Graph / Seepage / Sheet pile / Finite element method.

1. Introduction

Traditionally, seepage problems are solved by flow nets. The quantity of flow can be computed directly from the flow net. The major disadvantage in the flow net method is that it requires substantial effort in drawing a good flow net. In the preliminary design stages, where several alternatives are tried, drawing a separate flow net every individual configuration becomes a tedious task. This method was known to be limited to complication of boundary conditions such as configuration of the problem, the number and different of soil layers, including their transient conditions. Under such circumstances, it is desirable to have methodology with sufficient accuracy.

Use of the finite element method for geotechnical engineering began in 1966, when Clough and Woodward [1] used it to determine stress and movements in embankments, and Reyes and Deene [2] described its application in the analysis of underground openning in rocks. Many research studies and practical applications have taken place in the intervening 36 years [3,4,5,6]. During this period, considerable advances have been made in theory and practice, and the cost of computers has diminished to a small fraction of the cost of 36 years ago.

The finite element method presented in this paper is a very simple and efficient technique. By this method, the quantity of flow can be estimated without drawing the flow nets and can be solved with all cases of situations mentioned above. The method provides quick and powerful solutions, solutions, sufficient for all practical purposes.

2. Theory

2.1 Flow of water in soil

The governing differential equation that expresses the flow water in two-dimensional case in Figure 1 is:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial h}{\partial z}\right) + Q = \frac{\partial \Theta}{\partial t} \quad (1)$$

where
h = head loss
k_x = hydraulic conductivity in the x-direction
k_z = hydraulic conductivity in the z-direction
Q = applied boundary flux or flow rate
Θ = volumetric water content
t = time

This equation states that the difference between the flow (flux) entering and leaving an elemental volume at a point in time is equal to the change in the volumetric water content. More fundamentally, it states that the sum of the rates of change of flows in the *x*- and z-directions plus the external applied flux is

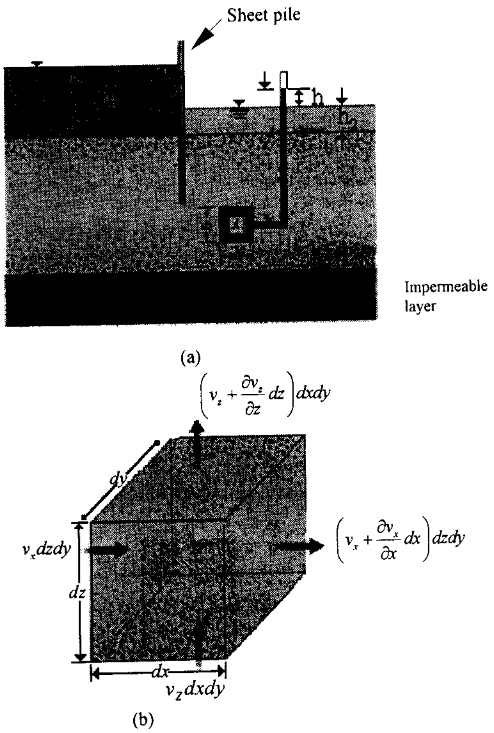

Figure 1 (a) Single sheet pile driven into permeable layer; (b) flow at point A

equal to the rate of change of the volumetric water content with respect to time. Under steady-state conditions, the flux entering and leaving an elemental volume is the same at all times. The right side of the equation consequently vanishes and the equation reduces to:

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial h}{\partial z}\right)+Q=0 \qquad (2)$$

2.2 Finite Element Equations

The finite element equation that follows from applying the Galerkin method of weighed residual to the governing differential equation (Eq.2) becomes:

$$\int_v \left([B]^T[C][B]\right)dv\,\{H\} + \int_v \left(\lambda <N>^T <N>\right)dv\,\{H\},t$$

$$= q\int_A \left(<N>^T\right)dA \qquad (3)$$

where: $[B]$ = gradient matrix
$[C]$ = element hydraulic conductivity matrix
$\{H\}$ = vector of nodal heads
λ = $m_w\,\gamma w$ where m_w is the slope of the storage curve, γw = unit
$<N>^T<N>$ = $[M]$ = mass matrix
$\{H\},t$ = $\frac{\partial h}{\partial t}$ = change in head with time
q = unit flux across the side of an element
$<N>$ = vector of interpolating function

For a two-dimensional analysis the thickness of the *dx* element is considered to be constant over the entire element. The finite element equation (3) can consequently be written as:

$$t\int_A \left([B]^T[C][B]\right)dA\,\{H\} + t\int_A \left(\lambda <N>^T<N>\right)dA\,\{H\},t$$

$$dy \;=\; qt\int_L \left(<N>^T\right)dL \qquad (4)$$

where *t* is the element thickness. When *t* is a constant the integral over the volume $\int_v$ becomes the integral over the area $\int_A$ and the integral over the area $\int_A$ becomes the integral over the length $\int_L$ from corner node to corner node. In abbreviated form, the finite element equation is

$$[K]\{H\}+[M]\{H\},t \;=\; \{Q\} \qquad (5)$$

where:

$[K]$ = element characteristic matrix

$= t \int_A \left([B]^T\ [C]\ [B]\right) dA$, or

$= \int_A \left([B]^T\ [C]\ [B]\ R\right) dA$

$[M]$ = mass matrix

$= t \int_A \left(\lambda < N >^T < N >\right) dA$, or

$= \int_A \left(\lambda < N >^T < N > R\right) dA$

$\{Q\}$ = applied flux vector

$= qt \int_L \left(< N >^T\right) dL$, or

$= q \int_L \left(< N >^T\ R\right) dL$

R is radial thickness in the axisymmetric case of three-dimensional analysis. Equation 5 is the general finite element equation including a transient seepage analysis. For a steady-state analysis, the head is not a function of time and consequently the term $[M]$ $\{H\},t$ vanishes, reducing the finite element equation to:

$$[K]\ \{H\} = \{Q\} \tag{6}$$

The Gauss numerical integration is used to form the element characteristic matrix $[K]$. The integrals are sampled at specifically defined points in the elements and then summed for all the points. The following integral (from Equation 5) $\int_A \left([B]^T\ [C]\ [B]\right) dA$ can be replaced by:

$$\sum_{j=1}^{n} [B_j]^T\ [C_j]\ [B_j]\ \det|J_j|\ W_{1j} W_{2j} \tag{7}$$

where: j = integration point

n = number of integration points

$\det|J_j|$ = determinant of the Jacobian matrix

W_{1j}, W_{2j} = weighting factors

The number of integration points required in an element depends on the number of nodes and the shape of the elements and user-define requirements. Then, in Equation 6 nodal head vector can be solved by giving some boundary conditions.

The seepage quantity that flows across a section can be computed from the nodal heads and the coefficients of the finite element equation. The global set of finite equations for one element is as follows:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \begin{Bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} \tag{8}$$

From Darcy's Law the total flow between two points is:

$$Q = kA\frac{\Delta H}{l} \tag{9}$$

The coefficients c in Equation 8 are a representation of $\frac{kA}{l}$ in Equation 9. Therefore, the flow from Node i to Node j is:

$$Q_{ij} = c_{ij}\left(H_i - H_j\right) \tag{10}$$

The total flow quantity through any section can be computed.

3. Methodology

3.1 Procedure

This project is a study of the quantity of seepage under the influence of sheet piles by the finite element method. The project uses a variety of soil that has been collected over some time by many researchers. This project is divided into 2 steps:

Step One, the finite element model called, case study 1, is specified to be a simple reference model with one thickness layer of permeable soil, a single row of sheet piles and a fine element mesh, as shown in Figure 2 and 3, respectively. Here, the preliminary study of appropriate model is carried out.

Last step, the so-called, case study 2, is specified to be a reference model where two permeable soil layers are located. In this case study, the proportion of the upper layer and total thickness of permeable soil layer is varied within zero to one. Soil types and hydraulic conductivity for both layers in Table 1 are presented. In addition, the simplified geological configuration the upstream and downstream of the total permeable soil layer has a depth of 20 m. in Figure 4. Non-dimensional values are calculated to create the normalized graphs.

3.2 Resources of information

The information in this study has been collected by various researchers [7] to study each case as follows:

Table 1 Hydraulic conductivity of each soil type.

Soil types	Hydraulic conductivity (m/s)
L-1. Very soft silty clay	$5 \times 10^{-8} - 1 \times 10^{-10}$
L-2. Medium stiff silty clay	$1 \times 10^{-8} - 5 \times 10^{-11}$
L-3. Hard clay	$5 \times 10^{-8} - 3 \times 10^{-11}$
L-4. Dense sand	$1 \times 10^{-4} - 1 \times 10^{-6}$

3.3 Discretization and Boundary condition

The finite element modeling techniques can be adopted for the analysis of the mentioned case studies in many different ways.

The steady state seepage of each case study is analyzed by using SEEP/W software [8]. In this study, the model idealizes the flow areas as quadrilateral 4 node elements, and the mesh generation was performed with an optimized state. The discretization and boundary conditions for each case are as follows:

3.4 Case study 1

Figure 2 shows the dimensions of the problem. A simplified geological configuration of a 80 m-Distance and the depth of 20 m. (T) below ground surface is configured.

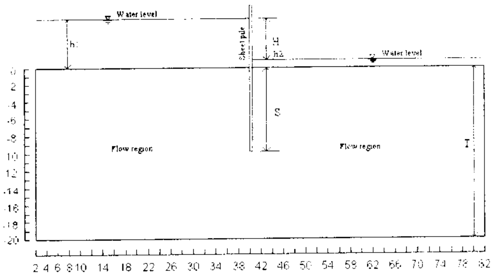

Figure 2 Schematic of the problem

The finite element mesh used for the analysis is shown in Figure 3. It is noted that the boundary node of the upstream and downstream surfaces are designed with different head boundary with various total head (H) within 2 to 10 m. Default boundary conditions (no flows) are assumed for other boundaries. The hydraulic conductivity of the homogeneous material applied can be varied within any range.

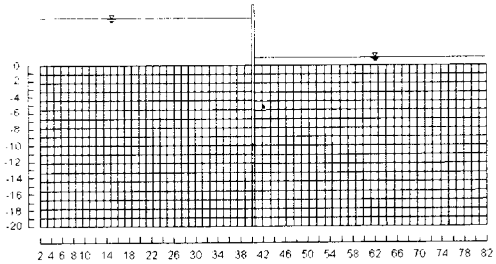

Figure 3 Finite element mesh

3.5 Case study 2

Figure 4 shows the dimensions of the problem. It is noted that the dimensions are the same as those specified in case study 1. In addition, two different permeable soil layers and various thicknesses of both layers are considered.

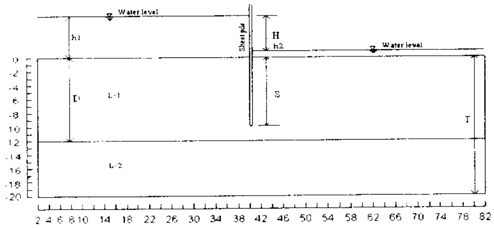

Figure 4 Schematic of the problem

The finite element mesh used for the analysis and the characteristics of the problem are the same specified in the previous case, namely the boundary nodes, number of nodes and elements and the flux section. The hydraulic conductivity of the permeable soil layers follow the types of soil as shown in Table 1. The range of permeability value covers the conditions of Bangkok subsoil.

4. Results

4.1 Preliminary study of an appropriate model

A preliminary study of an appropriate model was set and carried out in order to investigate the excellent results, two different simulations were used in the modeling, namely the number of the elements used and effects of the proportion of models.

4.1.1 Number of elements

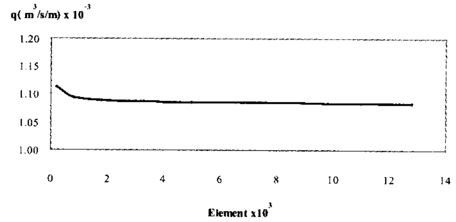

Figure 5 Seepage and number of elements

The influence of the number of elements affected the seepage results as shown in Figure 5. Obviously, the quantity of seepage varied depending on the intensity of number of elements. In Figure 5, the number of elements is at least 4000 elements that give the constant results.

4.1.2 Proportions of the model

Figure 6 shows the quantity of seepage with different the proportions of the model. The proportion larger than three gives accurate results. In this analysis the ratio of distance per depth equal to 4:1 is selected.

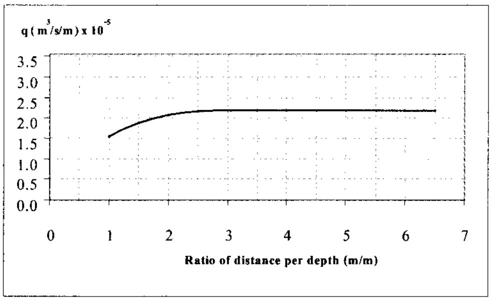

Figure 6 Seepage and proportion of models

4.1.3 Verification of Data Results

The values of quantity of seepage with S/T for these problems are computed and presented in Table 2. The percentage differences in the results are compared to the values computed from the mathematical method as given by Harr [9]. It can be seen that the values of the quantity seepage showed an excellent agreement. The differences were within 7% on average for the problem.

Table 2 Comparison of the results from SEEP/W ; and given by Harr[9].

S/T	Q/(kH)		% Error
	SEEP/W	Harr	
0.1	0.95	1.05	9.52
0.2	0.76	0.84	9.52
0.3	0.64	0.70	8.57
0.4	0.55	0.60	8.33
0.5	0.47	0.51	7.84
0.6	0.41	0.45	8.89
0.7	0.35	0.36	2.79
0.8	0.29	0.30	3.33
0.9	0.23	0.24	4.2

4.2 Normalized Graphs

Figure 7 represents the geometry of problem. The normalized quantity of seepage (q/kH per m width) and ratio of sheet pile depth penetration to depth of soil layer (S/T) are calculated. The values of quantity of seepage with S/T for double soil layer problems are presented in Figures 8 through 12. Figure 8 represents q/kH against S/T in one soil layer. Figures 9-12 present q/k_1H against S/T in double soil layers. The curves labeled k1/k2 apply to the two layer systems with different soil conductivities.

4.3 Example

4.3.1 Example No.1

A soil layer in Figure 2 which has hydraulic conductivity $k = 1\times10^{-6}$ m/sec, different water head between two sides of sheet pile equal to 6 m, if sheet piles are fixed down into soil layer 5 m from ground level and soil layer thickness is 20 m. What is the flow rate of water through sheet piles if sheet piles are 30 m long?

Given: $k = 1\times10^{-6}$ m/sec, H = 6 m., S = 5 m., T = 20 m. Sheet piles are 30 meter long. S/T ratio equal to 5/20 = 0.25

Solution: from graph in Figure 8, for S/T = 0.25, obtained q/(kH) = 0.67. So, total flow rate through sheet piles = $0.67\times k\times H\times 30$ (m)

$= 0.67\times1\times10^{-6}$ (m/sec) $\times$ 6 (m) $\times 30$ (m)

= 0.4342 (m^3/hr) ***Ans***

4.3.2 Example No. 2

Two soil layers in Figure 4 which have hydraulic conductivity $k_1 = 1\times10^{-8}$ m/sec, $k_2 = 10\times10^{-8}$ m/sec , different water head between two sides of sheet pile equal to 8 m , if sheet piles are fixed down into soil layers 5 m from ground level, thickness of upper soil layer is 8 m and the soil layers' total thickness is 20 m. What is flow rate of water through sheet piles if sheet piles are 30 m long?

Given: $k_1 = 1\times10^{-8}$ m/sec , $k_2 = 10\times10^{-8}$ m/sec, H = 6 m., S = 5 m., D = 8 m, T = 20 m., Sheet piles are 30 meter long. S/T ratio equal to 5/20 = 0.25

Solution: $k_1/k_2 = 0.1$, D/T = 0.4 use graph in Figure 10, S/T = 0.25 and $k_1/k_2 = 5$, obtained $q/(k_1H) = 1.67$. So, total flow rate through sheet piles = $1.67\times k_1\times H\times30$ (m)

= $1.67\times1\times10^{-8}$ (m/sec) × 8 (m) ×30 (m)

= 4.0×10^{-6} (m^3/sec)

= 0.0144 (m^3/hr) ***Ans***

Interpolations of values are needed in case that values fall between the lines of graph.

5. Conclusion and Recommendation

The following conclusion can be drawn from the present study. A two-dimensional finite element analysis of the quantity of seepage can be effectively calculated using the computer package called SEEP/W. In the proposed finite element model of the problem, the flow regions are discretized by quadrilateral plate elements. Two case problems selected in the study, consisting of one permeable soil layer and two layered systems were presented.

The study confirms the successful numerical modeling of ground water flow through the single row of sheet pile. This was evident as the good results depended on both the number of elements and proportional model of problem, which was more than or equal to 4000 elements and 4:1 (Distance per Depth) proportional. The quantity of seepage is verified to the quantity of flow curve obtained from the mathematical solution as given by Harr[9].

The quantity of flow for different hydraulic conductivity in double soil layers are presented in normalized graphs. The major advantage of the graphs is that it gives simple and quick solutions to the seepage problems without sacrificing the accuracy. The method is very useful in preliminary designs and feasibility studies. However, there is limitation of the prediction of boiling situation in the downstream side.

Finally, some following studies are recommended: the problems on three or more layered soil layers, the problems of three-dimensional case, both in steady state and transient state, particularly interest in pore water distribution, and ground subsidence.

6. Acknowledgments

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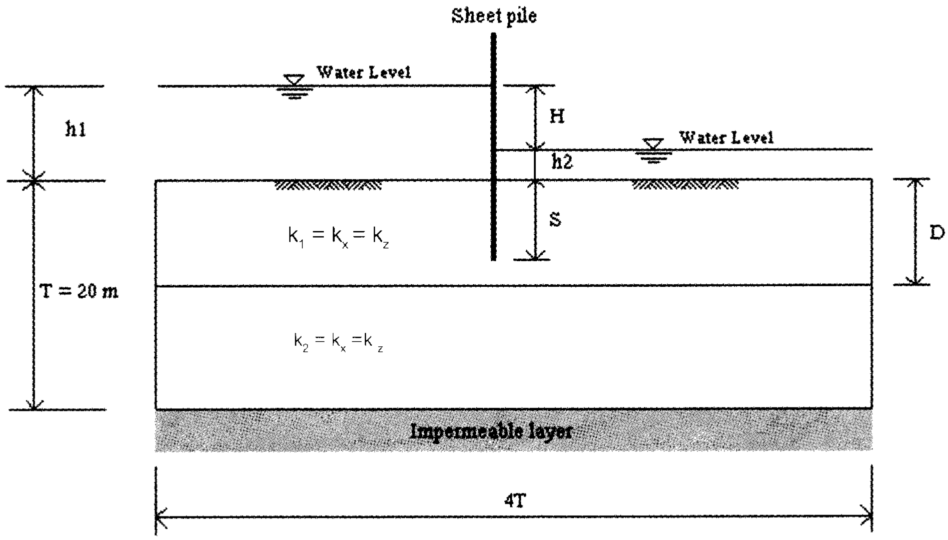

Figure 7 Geometry of problem

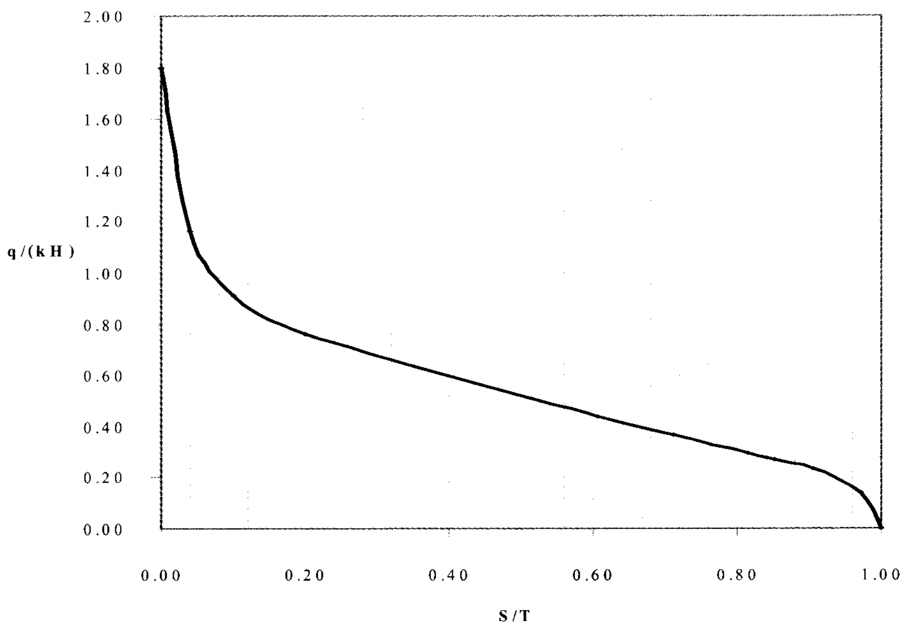

Figure 8 Quantity of seepage and S/T

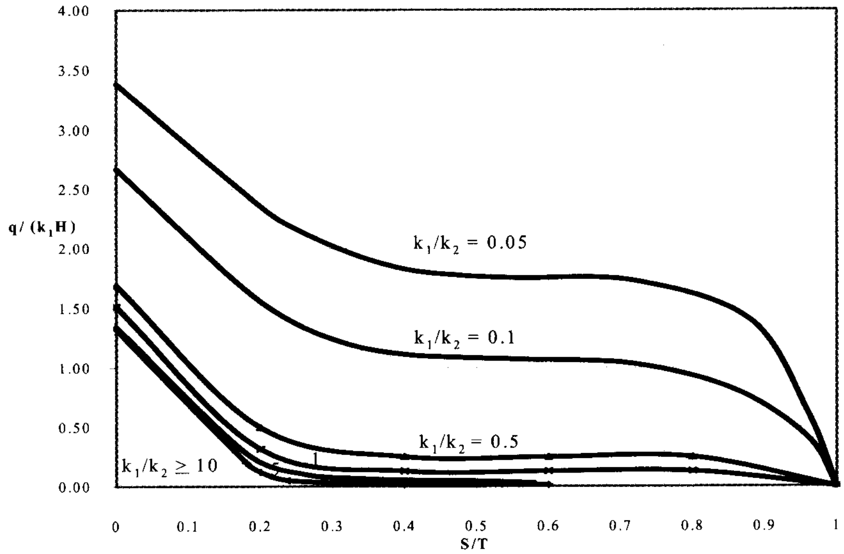

Figure 9 Quantity of seepage and S/T (D=0.2T)

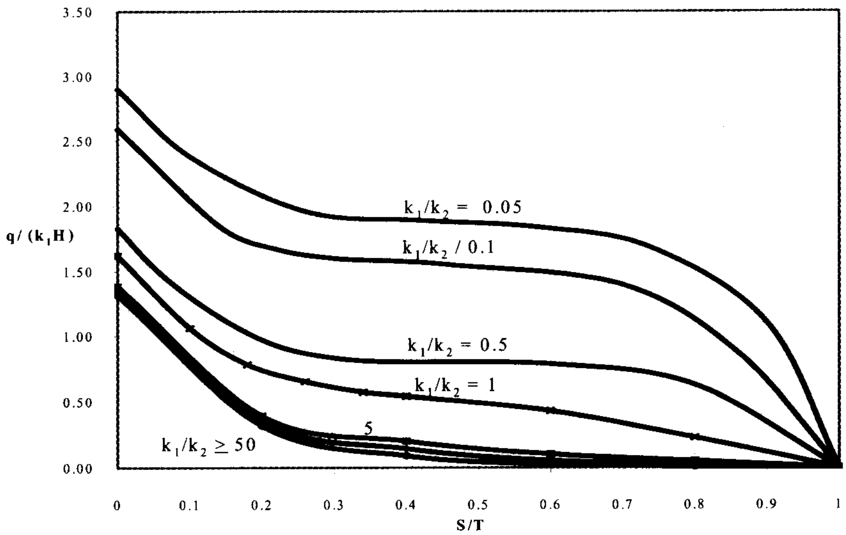

Figure 10 Quantity of seepage and S/T (D=0.4T)

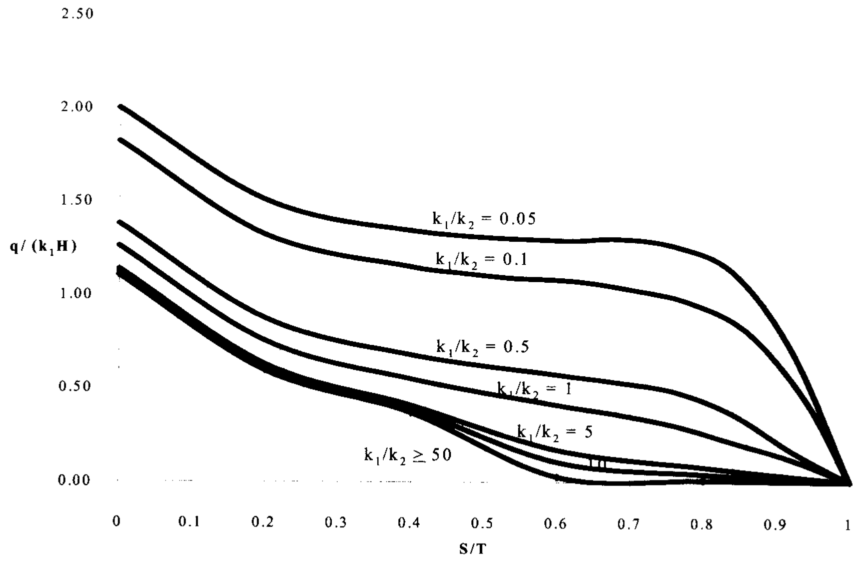

Figure 11 Quantity of seepage and S/T (D=0.6T)

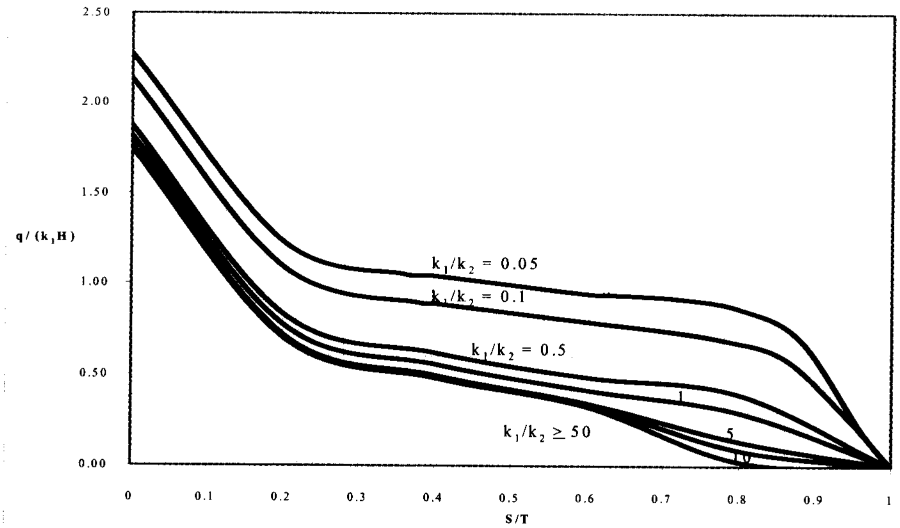

Figure 12 Quantity of seepage and S/T (D=0.8T)