Analytical Model of Tide and River Flow Interaction

Suphat Vongvisessomjai
Professor, Water Engineering and Management Program
School of Civil Engineering
Asian Institute of Technology
P.O. Box 4 Klong Luang, Pathumthani 12120, Thailand

Somchai Rojanakamthorn
Graduate Student, Water Engineering and Management Program
School of Civil Engineering
Asian Institute of Technology
P.O. Box 4 Klong Luang, Pathumthani 12120, Thailand

Abstract

The Chao Phraya River experienced widespread flooding in 1995 which resulted in extensive flood damage and loss of human life. There is a need for data on flood levels along the Chao Phraya River for design of dikes to protect the city of Bangkok as well as assimilative capacity of the river for design of waste treatment plants. The lower reach of the Chao Phraya River is strongly influenced by the tides from the Gulf of Thailand when interact with river flows resulted in a complicated pattern which is simplified to its interaction with individual constituents of tides obtained from harmonic analysis. This study employs the perturbation method to solve analytically the interaction of tides and river flow, taking into account the nonlinear convective inertia and bottom friction. It is found that the interaction of tide and river flow of the Chao Phraya River is well described by the mean water level plus the four constituents of tides. The raising of the mean water level during a high discharge is more than the damping of the tide, causing flooding at high water level. The resulting velocity fields can be used in the analysis on assimilative capacity of the river.

1. Introduction

Flood in the Chao Phraya delta occurred quite often in the past, for example, in 1980 and 1983. In these years, the direct damage to Bangkok city were in the order of 3 and 6 billion baht. After 1983 flood, the Bangkok city was partially protected from flooding by construction of dikes around the city with some pumping capacity to drain rainwater precipitated in the city. The 1995 flood is considered as a severe flood event in Thailand due its widespread extent over the entire country, especially in the Chao Phraya River with a flood damage of about 12 billion baht.

In the past, the Asian Institute of Technology (AIT) has worked for Thai governmental agencies in various projects on flood protection of Bangkok city and has recommended various measures to mitigate floods from the Chao Phraya River [1, 2, 3, 4, 5]. In these studies, AIT developed a mathematical model entitled AIT River Network Model [6] which was used as a tool to compute characteristics of floods before and after the implementation of mitigation measures. This helped in selection of the best mitigation measures such that flood damage was minimized. Through these long periods of work on these flood projects helped us in understanding the interaction of tide and river flow in the Chao Phraya River. The first analytical solution was then published in 1989 with some assumed forms of the solutions [7]. The present solution is derived from the Navier Stokes equation by a more general method instead of using assumed form of solution of Ippen and Harleman [8].

2. Theoretical Considerations

The unsteady one-dimensional equation of continuity and the equation of motion are used to describe the interaction of tide and river flow taking into account the convective inertia force and bottom frictional force. The perturbation method is used to linearize this nonlinear problem.

2.1 Governing equations

Continuity Equation

Consider the lower reach of a river as shown in Fig. 1 and a definition sketch of coordinates and variables used as shown in Fig. 2. We adopt a coordinate system in which the x -axis is horizontal along the still water level (SWL), the origin represents the river mouth, and the upstream direction is positive. The z -axis is perpendicular to x with its origin at SWL and is positive above the origin. If there is no lateral inflow, the equation of continuity can be written in the form

$$\frac{\partial \eta}{\partial t} + uS_b + u\frac{\partial \eta}{\partial x} + (h+\eta)\frac{\partial u}{\partial x} = 0 \quad \text{.........(1)}$$

in which η = instantaneous displacement of water surface above mean water level; u = instantaneous flow velocity; h = water depth; and S_b = bed slope of the reach.

Equation of Motion

Applying Newton's law of motion to one-dimensional flow through an element of water the equation of motion can be written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} + gS_f = 0 \quad \text{..............(2)}$$

in which the flow velocity in the river u is made up of a steady component, u_0 created by the discharge of the freshwater and of a time-dependent component, $u(x,t)$, contributed by the tide; and g = gravitational acceleration. The frictional slope S_f for this unsteady flow is assumed to have the same form as the steady flow and hence can be evaluated from the Chézy's equation as

$$S_f = \frac{u|u|}{C_z^2(h+\eta)} \quad \text{....................(3)}$$

The Chézy's C_z can be expressed in terms of the Manning's n or the friction factor f as

$$\frac{g}{C_z^2} = \frac{gn^2}{h^{1/3}} = \frac{f}{2} \quad \text{....................(4)}$$

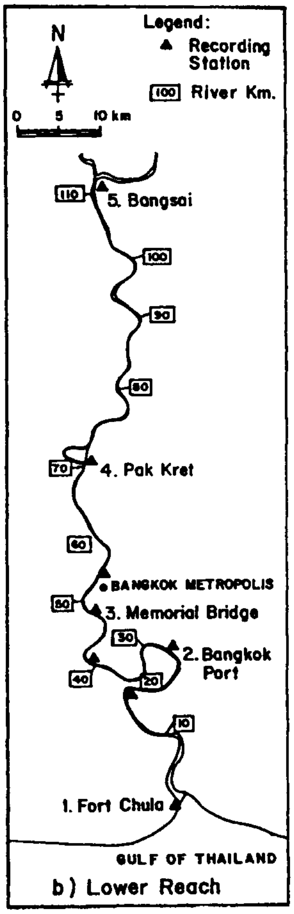

Fig.1 The Chao Phraya River

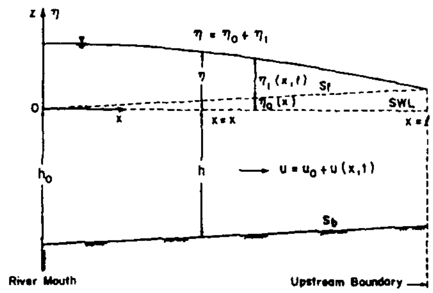

Fig.2 Definition Sketch of Coordinate and Variable

2.2 Perturbation method

Since the equations that have been derived are nonlinear, hyperbolic, partial differential equations, it is not easy to find an analytical solution by the usual mathematical means. Therefore, the perturbation method is applied.

The perturbation technique, or small parameter method, is a common analytical tool to find approximate solutions to nonlinear problems. In essence, it consists in developing the solution for a nonlinear boundary or initial value problem (usually in ascending powers of a parameter) that either appears explicitly in the original problem or is introduced in some artificial manner. A perturbed system is one that differs slightly from a known standard system. The expansion in terms of the perturbation parameter provides a means of obtaining a solution to the perturbed system by utilizing the known properties of the standard system. The method has often been used to investigate the behavior of nonlinear systems.

Suppose that certain equations, and possibly the boundary conditions, depend upon a parameter ε. The general perturbation problem is that of finding the solution for small value of ε, given the linear solution for $\varepsilon = 0$ (the standard system). The parameter ε can then be thought of as a measure of how much the system is perturbed from the standard one that is series expansion of the parameter.

From the governing equations, Eqs. 1 and 2, it is assumed that the solutions for η and u can be expressed in terms of a power series of the small parameter ε as follows:

$$\eta = \eta_0 + \varepsilon\eta_1 + \varepsilon^2\eta_2 + \ldots \qquad \ldots\ldots\ldots\ldots\ldots(5)$$

$$u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \ldots \qquad \ldots\ldots\ldots\ldots\ldots(6)$$

where u_0 is treated as the freshwater velocity, which is considered to be constant for a particular time interval. For another time interval, it will possess different constant magnitude.

The frictional slope, Eq. 3, is expanded in terms of the small perturbation parameter using Eqs. 5 and 6 for the interaction of tide from the river mouth with river flow from upstream coming from the opposite direction

$$S_f = S_{fo} + \varepsilon S_{f1} + \varepsilon^2 S_{f2} + \ldots \qquad \ldots\ldots\ldots\ldots\ldots(7)$$

where $S_{f0} = -\dfrac{u_0^2}{C_z^2(h+\eta_0)}$

and $S_{f1} = -\dfrac{2u_0u_1}{C_z^2(h+\eta_0)} + \dfrac{\eta_1 u_0^2}{C_z^2(h+\eta_0)^2}$

After substitution of Eqs. 5-7 into Eqs. 1 and 2, a set of two equations can be arranged according to the order of the parameter as follows:

Zeroth order

$$\frac{\partial\eta_0}{\partial t} + u_0 S_b + u_0\frac{\partial\eta_0}{\partial x} + (h+\eta_0)\frac{\partial u_0}{\partial x} = 0 \ldots\ldots(8)$$

$$\frac{\partial u_0}{\partial t} + u_0\frac{\partial u_0}{\partial x} + g\frac{\partial\eta_0}{\partial x} + gS_{f0} = 0 \ldots\ldots\ldots\ldots(9)$$

First order

$$\frac{\partial\eta_1}{\partial t} + u_1 S_b + u_1\frac{\partial\eta_0}{\partial x} + u_0\frac{\partial\eta_1}{\partial x} + (h+\eta_0)\frac{\partial u_1}{\partial x} + \eta_1\frac{\partial u_0}{\partial x} = 0 \quad (10)$$

$$\frac{\partial u_1}{\partial t} + u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial x} + g\frac{\partial\eta_1}{\partial x} + gS_{f1} = 0 \ldots\ldots(11)$$

Eliminating u_1 from the above two equations Eqs. 10 and 11 yields the first order governing equation

$$\frac{\partial^2 \eta_1}{\partial t^2} + 2u_0 \frac{\partial^2 \eta_1}{\partial x \partial t} + \left(u_0^2 - gh_0\right)\frac{\partial^2 \eta_1}{\partial x^2} - \frac{2gu_0}{C_z^2 h_0}\frac{\partial \eta_1}{\partial t} - \frac{3gu_0^2}{C_z^2 h_0}\frac{\partial \eta_1}{\partial x} = 0 \qquad \text{.............(12)}$$

in which $h_0 = h + \eta_0$.

The zeroth order equations (Eqs. 8 and 9) represent the standard or unperturbed system, and each succeeding equation has the same form except for being unhomogeneous with different forcing terms. The methods of solution are standard linear methods.

3. Analytical Solution

The zeroth order solution (Eq. 13) is the steady state solution in which the raising of the mean water surface along the river above the mean water level, η_0 is expressed in terms of $S_{f0} = -u_0^2 / \left[C_z^2 (h + \eta_0)\right]$ as

$$\eta_0 = -S_{f0} x \qquad \text{and river flow } u_0 = \text{constant} \qquad \text{.............(13)}$$

When there is no river flow $(u_0 = 0)$, the zeroth order solution (Eq. 13) gives $\eta_0 = 0$ and its first order equation (Eq.12) is the same as that used by Ippen and Harleman [8], its solution showed that the amplitude of the tide was exponentially damped with its progress into the river

$\eta = a_0 \exp(-\mu x)\cos(\sigma t - kx)$

in which μ = damping modulus; the angular velocity $\sigma = 2\pi / T$;T = period of tide; the tide number $k = 2\pi / L$; and L = length of tide.

The standard form of solution of the first order governing equation (Eq. 12) is solved by a more general method expressing η_1 in terms of x and t as $\exp\left[-(\mu + ik)x\right]$ and $\exp(i\sigma t)$ respectively and substituting in Eq. 12 to obtain its solution (Eq. 14) with its corresponding velocity of Eq. 15 and their parametric equations, Eqs. 16 and 17, to define k and μ.

$$\eta_1 = a_0 \exp(-\mu x)\cos(\sigma t - kx) \qquad \text{...........(14)}$$

$$u_1 = \frac{a_0}{h_0} c_0 \exp(-\mu x) \frac{k_0^2}{k^2 + \mu^2}\left\{\left[\frac{k}{k_0} - \frac{k^2 + \mu^2}{k_0^2}\left(\frac{u_0}{c_0}\right)\right]\cos(\sigma t - kx) - \frac{\mu}{k_0}\sin(\sigma t - kx)\right\} \text{......(15)}$$

$$\frac{k}{k_0} = \frac{c_0}{c}$$

$$= \frac{\sqrt{1 + \left(\frac{\mu}{k_0}\right)^2 - 2\left(\frac{\mu}{k_0}\right)^2\left(\frac{u_0}{c_0}\right)^2 + \left(\frac{\mu}{k_0}\right)^2\left(\frac{u_0}{c_0}\right)^4 - \frac{3g}{C_z^2 k_0 h_0}\left(\frac{\mu}{k_0}\right)\left(\frac{u_0}{c_0}\right)^2\left[1 - \left(\frac{u_0}{c_0}\right)^2\right]} - \frac{u_0}{c_0}}{1 - \left(\frac{u_0}{c_0}\right)^2} \text{......(16)}$$

$$\frac{\mu}{k_0}=\frac{\frac{g}{C_z^2k_0h_0}\left[\frac{3}{2}\frac{k}{k_0}\left(\frac{u_0}{c_0}\right)^2-\frac{u_0}{c_0}\right]}{\frac{k}{k_0}-\frac{k}{k_0}\left(\frac{u_0}{c_0}\right)^2+\frac{u_0}{c_0}}\quad\text{.........(17)}$$

Eqs. 16 and 17 will be used to compute k/k_0 and μ/k_0 by trial and error in order to define the assumed solution η_1 (Eq. 14) for known values of u_0/c_0 and $g/(C_z^2k_0h_0)$. For the ranges of $|u_0/c_0|$ <0.10 and $g/(C_z^2k_0h_0)$ = 27, 26, 52 and 56 for the constituents M_2, S_2, K_1 and O_1, respectively, of the Chao Phraya river obtained from the following analysis, $h_0 = 9.2$ m and Chézy's $C_z = 52$ m$^{1/2}$/s or Manning's $n = 0.028$ s/m$^{1/3}$ or the friction factor $f = 7.3\times10^{-3}$. The effects of u_0/c_0 on the ratio of the tide number k/k_0, Eq. 16, are shown in Fig. 3(a), while the effects of u_0/c_0 on the dimensionless damping modulus μ/k_0, Eq. 17, is shown in Fig. 3(b).

3.1 Analysis and results

The analytical solutions of tide and river flow interaction will be compared with the recorded water levels of the five stations along the Chao Phraya River. The harmonic analysis separates effects of river flow and tidal action.

3.2 Description of the Chao Phraya River

The Chao Phraya River is the most important river in Thailand. Its catchment area is 162,000 km^2, occupying most of the northern region and the central part of Thailand, and its length is approximately 1,000 km. It drains into the Gulf of Thailand, where a strong mixed tide prevails at the river mouth.

The geometry in the lower reach of this river is rather uniform, possessing an average width of approximately 400 m, an average depth of 9.2 m and a rather flat slope of approximately 2×10^5. Tides recorded at five stations in this lower reach in 1983 are used in this study. Observed tides at these five stations from April 5-18, 1983 [Fig. 4(a)] reveal a small damping in the dry season, while those from October 5-18, 1983 [Fig. 4(b)] show a large damping in the rainy season. These two figures show that at the river mouth (Station 1), where the river drains into the Gulf of Thailand, the tide is not damped out by the river flow in the rainy season as compared to the dry season. Due to the damping, the range of tide at the upstream station is smaller than at the downstream station at any time that the river flow is the same. When the river flow increases from the dry season to the rainy season, the damping increases while the mean water level rises. As a result, the water level is higher than the river banks and causes flooding. There exists a rating equation at Station 5 to covert the mean daily water level in m above MSL $(\bar{h})$ to the net river discharge in m^3/s (Q_f), i.e.

$$Q_f = 857\bar{h}\quad\text{.....................................(18)}$$

3.3 Harmonic analysis

The resultant tide $\eta_r(t)$, recorded as a function of time t at any station, is composed of a finite number of constituents N, each with its own period T_i, amplitude a_i, and phase angle δ_i for the constituent i

$$\eta_r(t)=\eta_0+\sum_{i=1}^{N}a_i\sin\left(\frac{2\pi t}{T_i}+\delta_i\right)\quad\text{...........(19)}$$

The mean water level η_0, the amplitudes a_i, and phase angles δ_i are evaluated from a discrete hourly $\eta_r(j)$ for $j=1$ to M as follows:

$$\eta_0=\frac{1}{M}\sum_{j=1}^{M}\eta_r(j)\quad\text{.............(20)}$$

$$a_i=2\left(A^2+B^2\right)^{1\,2}\quad\text{.............(21)}$$

and

$$\delta_i=\tan^{-1}\left(\frac{B}{A}\right)\quad\text{.............(22)}$$

where $A = \frac{1}{M}\sum_{j=1}^{M}\eta_r(j)\sin\left(\frac{2\pi j}{T_i}\right)$

and $B = \frac{1}{M}\sum_{j=1}^{M}\eta_r(j)\cos\left(\frac{2\pi j}{T_i}\right)$

Since the tide is composed of various constituents that interact simultaneously with the river flow, the resulting records of water levels

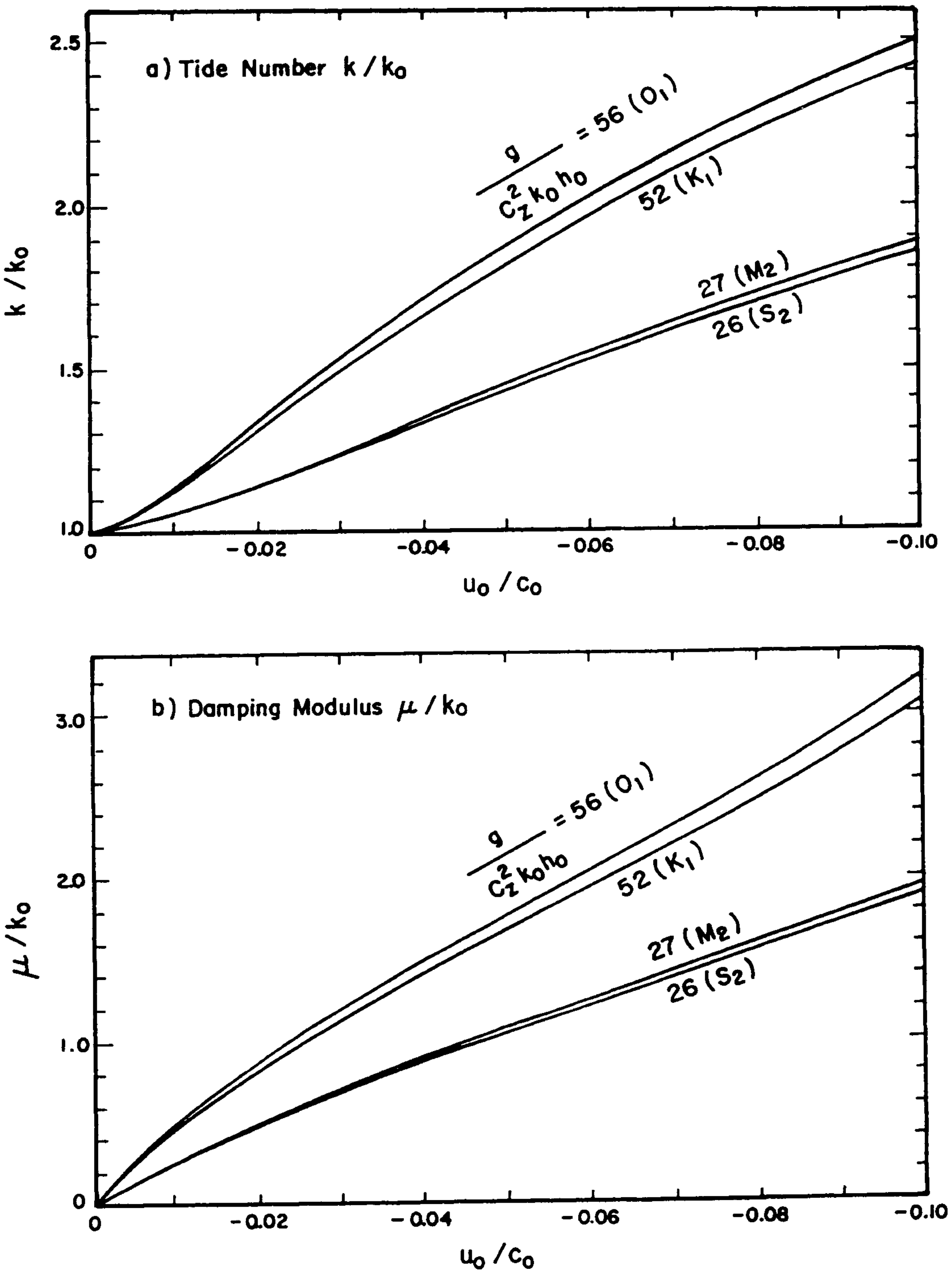

Fig.3 Effects of Froude Number u_0/c_0 on a) Tide Number k/k_0 and b) Damping Modulus μ/k_0

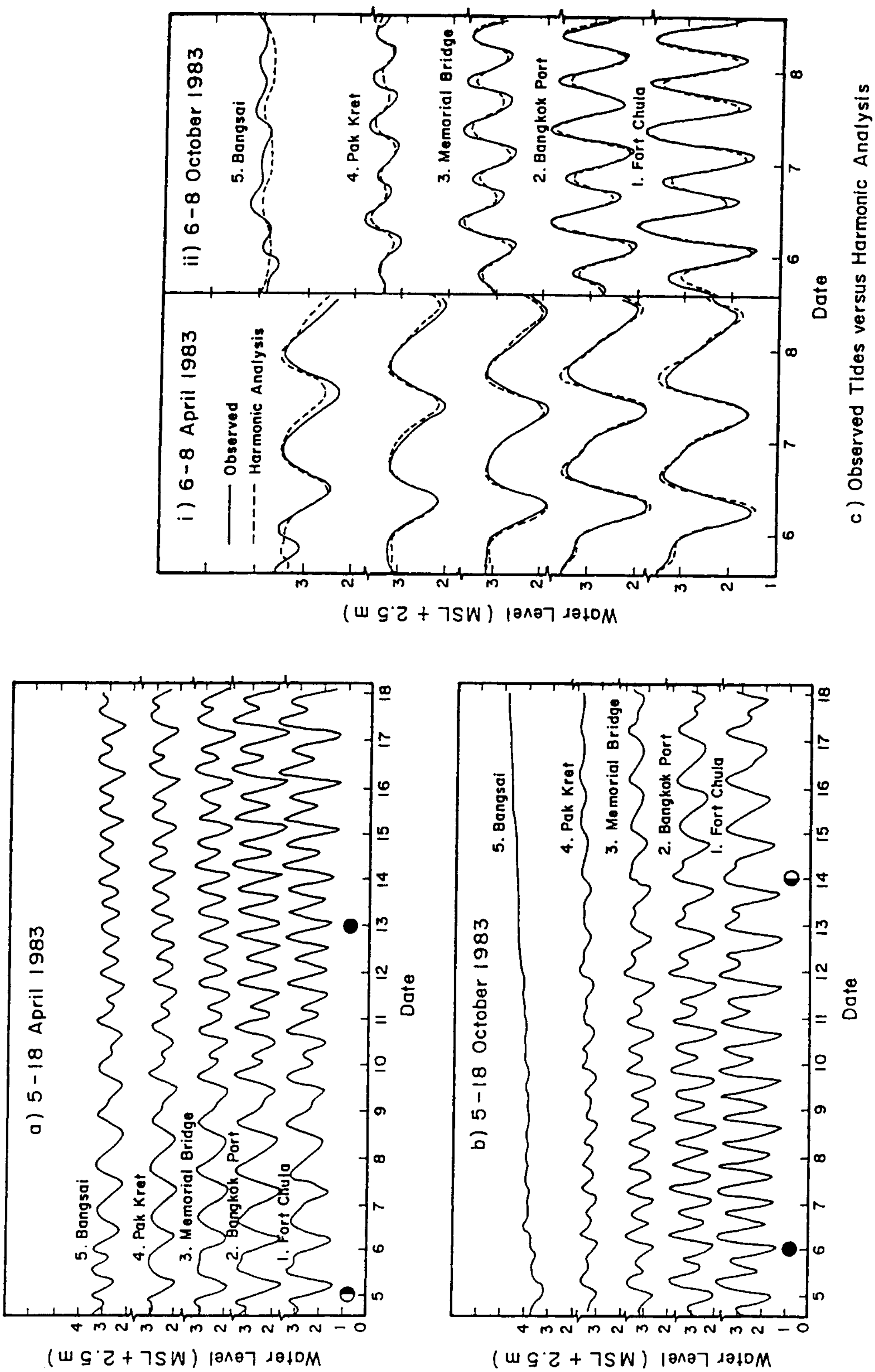

Fig.4 Observed Tides in a) Dry Season , b) Rainy Season and c) Observed Tides versus Harmonic Analysis

in the river show complicated patterns. The individual interaction of each constituent of the tide in each month with the river flow is obtained by a harmonic analysis of the hourly water levels. Four predominant constituents of the tides are considered in this study. The results for each constituent are then compared to the analytical solution.

Table 1 Results

Tide (1)	Period Month/yr (2)	a_0 (St.1) (m) (3)	η_0 (St.5) (m MSL) (4)	u_0 (m/s) (5)	$u_0/\sqrt{gh}$ (6)	μ (km^{-1}) (7)	μ/k_0 (8)	k/k_0 (St.5) (9)
M_2	4/1983	0.634	0.515	-0.165	-0.0174	0.0110	0.74	1.55
M_2	8/1983	0.595	0.763	-0.299	-0.0315	0.0115	0.78	1.74
M_2	9/1983	0.627	0.935	-0.363	-0.0382	0.0142	0.96	1.77
M_2	10/1983	0.608	2.145	-0.650	-0.0684	0.0258	1.74	1.74
S_2	4/1983	0.369	0.515	-0.165	-0.0174	0.0110	0.72	1.46
S_2	8/1983	0.323	0.763	-0.299	-0.0315	0.0132	0.86	1.42
S_2	9/1983	0.376	0.935	-0.363	-0.0382	0.0138	0.90	1.54
S_2	10/1983	0.336	2.145	-0.650	-0.0684	0.0262	1.71	1.46
K_1	4/1983	0.556	0.515	-0.165	-0.0174	0.0072	0.94	1.80
K_1	8/1983	0.624	0.763	-0.299	-0.0315	0.0085	1.10	1.66
K_1	9/1983	0.585	0.935	-0.363	-0.0382	0.0087	1.13	1.96
K_1	10/1983	0.614	2.145	-0.650	-0.0684	0.0212	2.75	1.89
O_1	4/1983	0.379	0.515	-0.165	-0.0174	0.0067	0.94	1.73
O_1	8/1983	0.403	0.763	-0.299	-0.0315	0.0087	1.23	1.47
O_1	9/1983	0.513	0.935	-0.363	-0.0382	0.0095	1.34	1.68
O_1	10/1983	0.514	2.145	-0.650	-0.0684	0.0213	3.00	0.24

The four constituents of tide (M_2, S_2, K_1 and O_1 with periods of 12.4206, 12.0000, 23.9346 and 25.8194 hr, respectively) adequately represent the recorded tides as shown in Fig. 4(c).

The observed hourly water levels of the five stations in April, August, September and October, 1983, are used to compute η_0, a_i and δ_i by Eqs. 20, 21, and 22 respectively. The monthly data are selected for this analysis in order to minimize the errors of η_0, a_i and δ_i, and the mean monthly discharge Q_f, or the river flow velocity u_0 in the entire lower reach of the river could be assumed to be constant. The results obtained from the harmonic analysis are listed in Table 1. The values of a_0 are equal to the values of a_i at the river mouth (Station 1), the river flow velocity u_0 is determined from the rating equation (Eq. 18) of Station 5, and the ratio of k/k_0 is computed from the difference of the phases of Stations 1 and 5, $(\delta_i)_0$ and δ_i with a distance $\Delta x = 112$ km

$$\frac{k}{k_0} = \frac{\sqrt{gh}\left[(\delta_i)_0 - \delta_i\right]T_i}{2\pi(\Delta x)} \quad \text{.....................(23)}$$

Fig. 5 shows that the attenuation of the amplitude of the tide a_i along the river follows the exponential damping of Eq. 14, i.e.

$$a(x) = a_0 \exp(-\mu x) \quad \text{..........................(24)}$$

The damping modulus μ obtained from the fitting is normalized by k_0, as μ/k_0 is also listed in Table 1.

The values of η_0 of Eq. 19 represent the heights of the mean water levels in any month based on Eq. 20. The increases in the mean monthly water levels along the river in 1983 as plotted in Fig. 6 are linearly distributed with distance x along the river, in accordance with Eq. 13, and with slope $s_{fo} = -u_o^2/\left[C_z^2(h+\eta_o)\right]$.

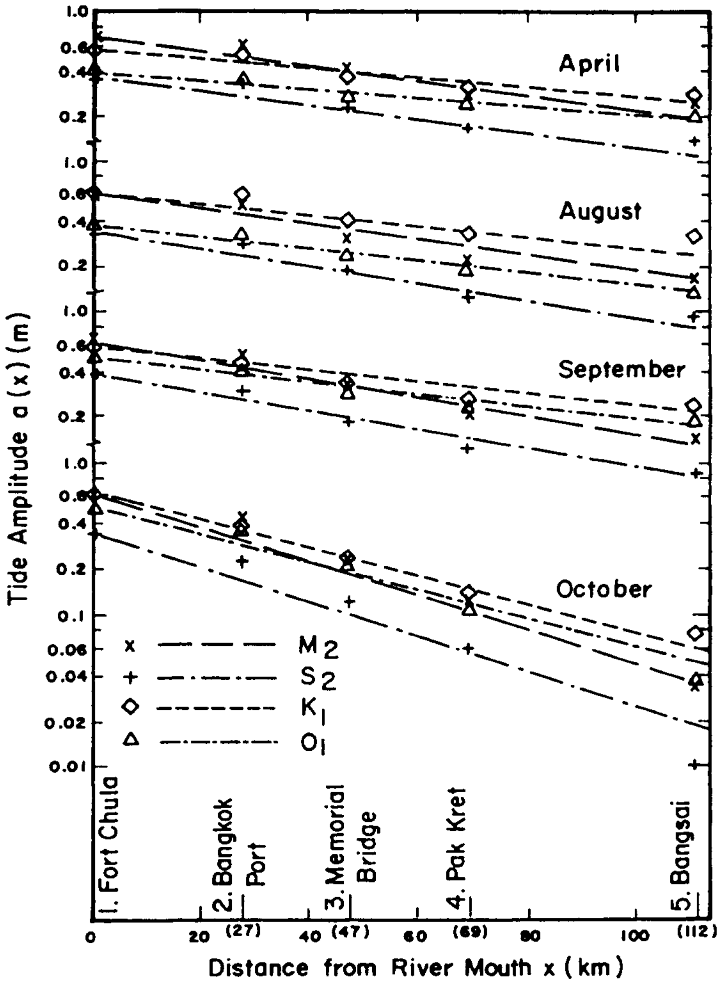

Fig. 5 Attenuation of Tides in 1983

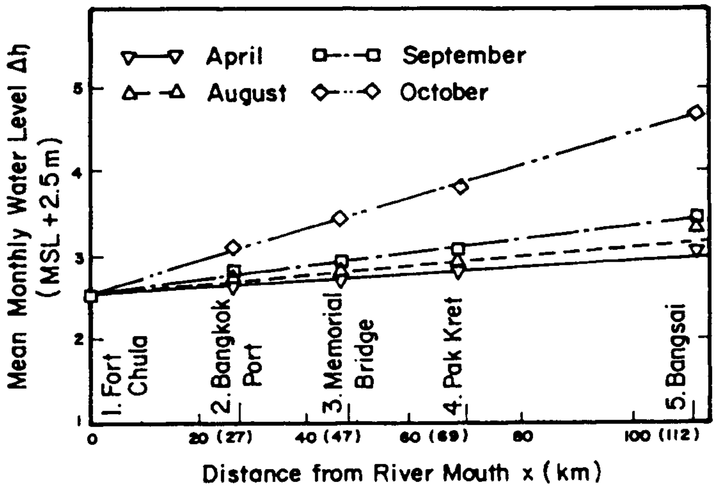

Fig. 6 Variation of Mean Monthly Water Levels along the River

The water level in the lower reach of the Chao Phraya River can be expressed as the sum of the mean water level plus the four constituents of tides as shown in Fig. 4(c) as

$$\eta(x,t) = -S_{fo}\,x + \sum_{i=1}^{4} a_{oi}\exp(-\mu_i x)\cos(\sigma_i t - k_i x) \quad (25)$$

The rising of the mean water level during the high discharge of the river and the high tide from the river mouth are the main causes of flooding of this river, since the rising of the mean water level is more than the damping of the tide.

The hourly data in the month are also decomposed into shorter durations of 15 days and 7.5 days for the harmonic analysis. The results of these harmonic analyses for the most crucial month of October where flood peak occurred show some deviation; the exponential damping of the tide, $a_i = a_0 \exp(-\mu x)$ for the shorter durations of 15 days and 7.5 days when compared with monthly results, shows a similar tendency while the tidal amplitudes of 15 days and 7.5 days (due to variation in freshwater velocity) fluctuate around the mean monthly values.

4. Conclusion

From the study of the interaction of tides and river flow of the Chao Phraya River, the following conclusions can be drawn:

1. The complicated observed water levels (Fig.4) were separated into mean water level plus various constituents of tides by means of harmonic analysis

2. The perturbation method used to solve nonlinear problems beautifully separated the effects of river flow into the steady component of its zero order solution in describing the rising of the mean water level, and the unsteady component of its higher order solution in describing the attenuation of tidal amplitude and the reduction of its celerity.

3. The river flow u_0 raised the mean water level in accordance with Eq. 13 (Fig. 6) in which $S_{f0} = -u_0^2/\left[C_z^2(h+\eta_0)\right]$, and it damped the tides in accordance with Eq. 14; which matched the recorded tides well (Fig. 5).

4. The water levels in the lower reach of the Chao Phraya River could be expressed as the sum of the mean water level plus the four constituents of tides (Eq. 25) which could compare well with recorded tides at the five stations along the river in Fig. 4 (c).

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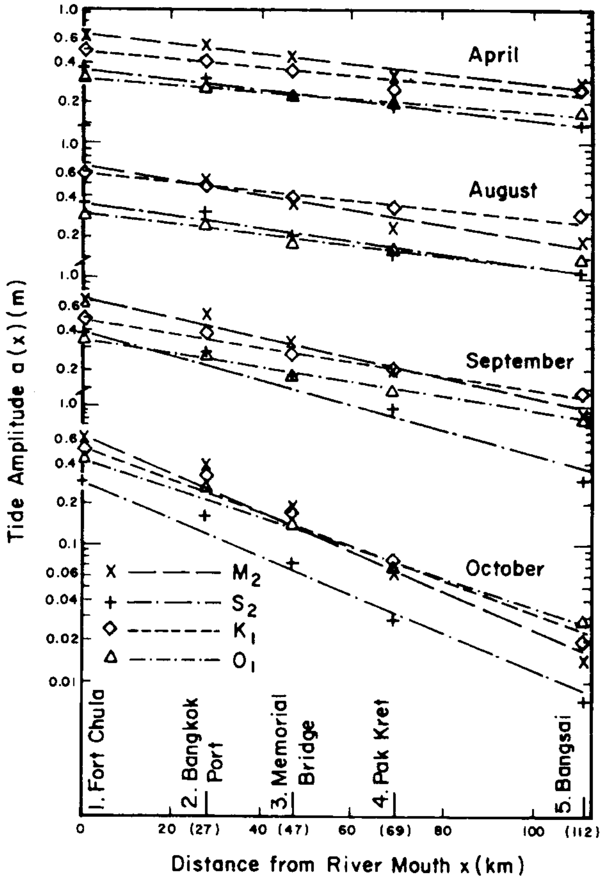

Fig. 7 Attenuation of Tides in 1980

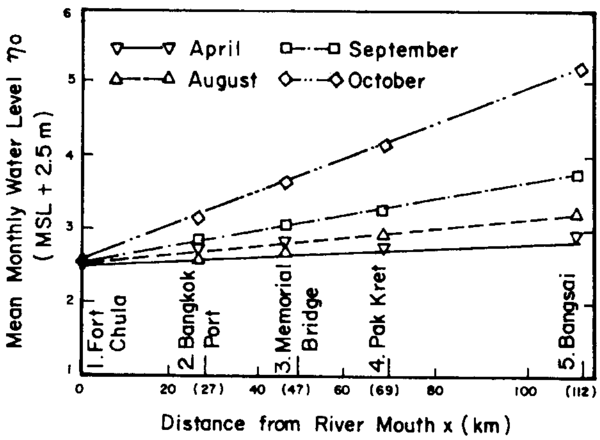

Fig. 8 Variation of Mean Monthly Water Levels along the River