# Approach to Apply Fuzzy Analytical Hierarchy Process on Part Routing Problems in Flexible Manufacturing System

**Parames Chutima and Pattita Suwanruji**
Department of Industrial Engineering, Chulalongkorn University, Thailand

**Abstract**

In this paper part routing problem is modeled as multi-criteria decision making in which the attributes of alternative machines, i.e., workload on machine buffer, processing time, and the probability that the part being routed to the machine can be processed before the machine fails are contemplated. To solve multi-criteria decision making problem, the systematic method, Fuzzy Analytical Hierarchy Process (FuzzyAHP) is applied. The structure of FuzyAHP-based part routing rules is composed of four core components, i.e., Flexible Manufacturing System, Database, Selection Index Evaluation, and Alternative Machine Selection. All components work coordinately in calculating selection index. This paper demonstrates the construction and the function of those components. Finally, the proposed FuzzyAHP-based part routing rules, e.g., FuzzyAHP-WINQ, FuzzyAHP-NF, and FuzzyAHP are compared with the other existing rules, i.e., WINQ, NINQ, SPT, and RAN. The results from the simulation show that for tardiness and system utilization, FuzzyAHP-WINQ performs significantly better than FuzyAHP-based rules and the other part routing rules.

## 1. Introduction

Under the current economic pressure and the hardships encountered by manufacturing companies, managers are increasingly looking for approaches that can be employed to enhance and strengthen the production processes and management. In this respect, an important step lies in devising efficient production planning and control methods matching the company's goals. In addition, the adoption of novel manufacturing technologies, e.g., Flexible Manufacturing Systems (FMS), is also equally important.

FMS has emerged in recent years as a viable answer to the dynamic and uncertain product demand for increased product variety and short product life cycles. Routing flexibility is among the major types of flexibilities that enables FMS to demonstrate its promised potential benefits. A system with routing flexibility can provide alternative production routes for jobs while confronting changes in production environment or process requirements, e.g., machine breakdown, excessive workload, etc.

The key ingredients of routing flexibility include flexible machine tools, tool transportation systems, and routing control strategy. It is obvious that a proper routing strategy plays an important role in utilizing routing flexibility since inappropriate application of the conventional fixed routing to FMS can impede FMS from its full potentiality.

Several methods can be applied to solve part routing problems. It is clear that part routing problems are in a class of NP-Complete problems. As a result, for practitioners and operators, solving such problems through analytical approaches, i.e., Queuing Network, Linear Programming, and Optimization, seems difficult in practice because of the complexity in terms of mathematical modeling and solution searching, especially for such a complex system like FMS. On the other hand, heuristic methods, i.e., Least Reduction Entropy (LRE) [1], Minimum Flow Resistance (MFR) [2], and Number of Jobs in the Next Queue (NINQ) [3], can solve part routing problems with acceptably good solutions. However, their decision indices take into account only one property or attribute of the available alternatives. For instance, NINQ considers only the number of jobs in the queues of the alternatives while not taking into consideration some other crucial information, e.g., machine workload or status. Another drawback of these heuristics is that their performances decline in the situation where the difference among the alternatives is small. For example, NINQ shows diminutives differences among the alternatives when applied to the system with high workload and limitted local buffers. Formulating a part routing rule from several attributes of the alternatives can compensate for such difficulties. Furthermore, a good part routing strategy is supposed to enhance several system performances simultaneously (both inventory related and due-date related performances). These performances are usually a effected by several attributes. Thus, the more performances need to be improved, the more attributes should be taken into account in the decision index.

As mentioned previously, good results could be obtained by modelling part routing problem as a multi-criteria decision-making problem where the attributes of the alternatives are set as the criteria. One well-known approach to solve this problem is Analytical Hierarchy Process (AHP). In this paper, Fuzzy Logic is also employed to strengthen the capability of conventional AHP, The resulting process is thus named FuzzyAHP. This process allows human's knowledge and experiences to be exercised while making routing. decisions about which attribute of the system is deemed more important. The architecture and the key components of FuzzyAHP-based part routing rules developed in this research will be elaborated in detail in the following sections.

## 2. Structure of FuzzyAHP

The process of AHP can be explained as follows. The attributes of each alternative are contemplated as the criteria. These criteria are scored by human's experience and knowledge. A higher score will be given to the attribute that is more preferable. The normalized scores of all attributes are aggregated according to their weights. These aggregated scores, called selection index, will be used to compare the preference among all alternatives. As a result, several alternatives will be ranked and the best will be selected.

In this study, three main attributes of the system are employed to derive the selection index for each alternative including:

1. Part processing time (P): If several machines can be used to process the same operation, the machine with faster operation time should be a better alternative.

2. Workload on the buffer of the alternative machine (W): W refers to the total processing time of all parts residing in the input buffer of the machine. The alternative with lower W is more preferable since the part has a higher probability of spending less time in the queue and thus the system.

3. The probability that the part being routed to the alternative machine can be processed before the machine fails (Pr): Denote $f_i$(MTBF,t) as the probability function that alternative machine i will fail at time t after the last failure, where MTBF is mean time between failure. $T_{now}$ and $T_f$ represent the moment that the decision is taken and the last time that machine i failed respectively. If the part is chosen to be processed on machine i, an amount of time that the part will be processed after the last failure is $T_{now}+W_i-T_f = t$ (appear in equation (1)). $W_i$ is the workload on machine i. As a result, $Pr_i$ can be formulated as follows.

$$Pr_i = 1 - \int_{t=0}^{t} (f(MTBF,t))\,dt \qquad (1)$$

The reason that W and P are selected as the criteria in this study is because they affect the time that parts will be spent in the system. Furthermore, considering Pr as another criterion can boost the system's ability in avoiding the routes with high probability of machine failure.

The relationships between the attributes are shown by weights assigned to attributes. These weights are employed to control routing selection. Since human can use his/her experience and knowledge in selecting a proper route, these weights will be extracted directly from human experts without using complex mathematics.

To increase the robustness of the developed part routing policy, in this research, routing decision employs dynamic weights which are changed as routing conditions are altered. The key factor used to indicate relative weights of attributes is the urgency of the part. The level of urgency is indicated by part's slack time, which is defined as follows:

Slack time=Due date-$T_{now}$- processing time of the remaining operations (2)

where $T_{now}$ = the moment that the decision is taken.

When slack time is a large positive number (early), Pr will become the significant attribute since the part routing policy will operate to prevent system congestion due to lots of parts being sent to failed machines. In this case, part routing policy will try not only to reduce workload of machines but also avoid the routes with high tendency of machine failure by assigning the machine with higher Pr a higher score. On the other hand, when slack time is a small positive or negative number, the significant attribute will switch to W. In this case, the machine with low W is assigned a higher score since this will expedite the part and reduce the part's tardiness.

As mentioned earlier, the selection index for each alternative is calculated from the score of each attribute and the attribute's weight. Both are extracted from the experience and knowledge of humans. However, the human's experience and knowledge cannot be easily presented in a precise form since:

a) Human evaluates the attribute with the notion of how the value of the attribute affects the preference of the alternative. Thus the values of attribute can not be compared directly. For example, human may evaluate W of 10 and 12 with the same score because in human's opinion the difference between these values is too small to impact the preference of both alternatives.

b) For some attributes, e.g., W, Pr, and S, their values are only estimated. Thus it is not neessarily the case that the better value is the better choice or the less urgent part.

c) Human has ambiguity in assigning attributes weights, i.e., at one level of urgency, it is more reasonable and appealing for human to assign the importance of one attribute over the other in a range rather than given a precise value.

Thus human's knowledge and experience can be extracted more practically by being presented in linguistic form (Fuzzy set). Fuzzy Analytical Hierarchy Process (FuzzyAHP) is the method which combines the fuzzy scaling to AHP. By means of FuzzyAHP, the values of the attributes are transformed to fuzzy values before being aggregated according to the fuzzy weights. The obtained selection indices are in fuzzy forms. The fuzzy selection indices are then ranked by means of Adamo [4].

Figure 1 shows the proposed structure of Fuzzy Analytical Hierarchy Process Part Routing in FMS based on the aforementioned arguments.

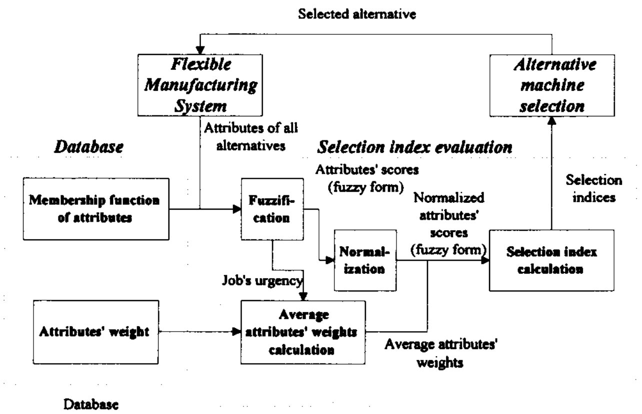


**Figure 1** The structure of FuzzyAHP part routing in FMS

The structure is composed of 4 main components:

**a) Flexible Manufacturing System** As the job is prepared for the next operation, the system searches for and sends the attributes of alternative machines (W, Pr, and P) and attribute of the job (S: Slack time) to the succeeding component.

**b) Database** The database is constructed to maintain the salient characteristics of the system. Also included in the database are the membership function of attributes and attributes' weight.

**c) Selection Index Evaluation** There are 4 procedures to evaluate selection index of the alternatives: Fuzzification, Average attributes' weight calculation, Normalization, and Selection index calculation.

**d) Alternative Machine Selection** The selection index (Fuzzy values) is transformed into a preference value. The alternative machine with highest preference value is selected. The selected alternative is sent to the Flexible Manufacturing System for activating actions.

The following sections (sections 3, 4, and 5) demonstrate the detailed construction of Database, Evaluation of Selection Index, and Alternative Machine Selection components, respectively.

## 3. Database

As can be seen in Figure 1, Database provides the supporting data for Selection Index Evaluation component. Database keeps crucial information related to the characteristics of parts and the system, i.e., membership functions of attributes and attributes' weights. Both are created from human's experience and knowledge to befit the individual system. The approach to create both parts of Database can be explained as follows:

### 3.1 Membership Functions

The method of membership assignment used in this research is Inductive Reasoning. It is necessary that FuzzyAHP has to transform the attributes' values to corresponding score values. Thus the additional procedure, standardized attributes' range, has to be included.

***Standardized Attributes' Range***

By means of FuzzyAHP, the experts use their experience and knowledge to score the alternatives according to the attribute. It is convenient for human to score the alternatives by comparing each attribute with the maximum and minimum values. The possible maximum and minimum values for each attribute are not difficult to estimate, e.g., the possible maximum value of W is approximately the amount of jobs circulating in the system multiplied by the average operation time. Unfortunately, the possible maximum and minimum values are usually extraordinary thus they are impractical criteria for comparing. For this reason, the values which indicate the specified range of the attributes covering most attribute's values of the interested system must be identified. Furthermore, the specified range can reduce the difference between normal and extraordinary attribute's values.

The distribution of attribute's values can be defined by simulation. By plotting the histogram, the range covering 95% of the simulated attribute's values can be defined. Figures 2 to 5 show the distributions of attributes and their specified ranges.

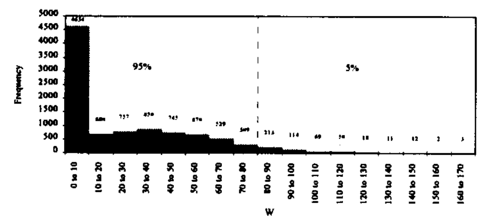


**Figure 2** Distribution of W with specified range of [0,80]

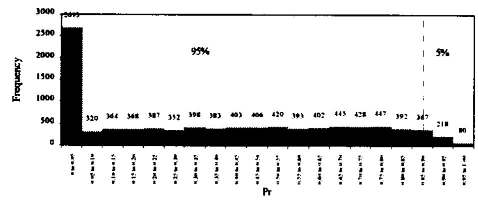


**Figure 3** Distribution of Pr with specified range of [0,0.855]

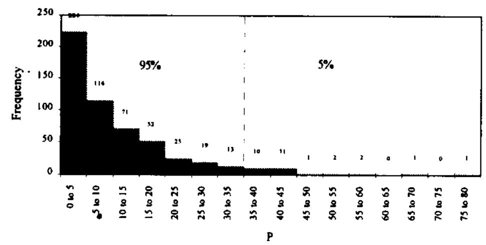


**Figure 4** Distribution of P with specified range of [0,35]

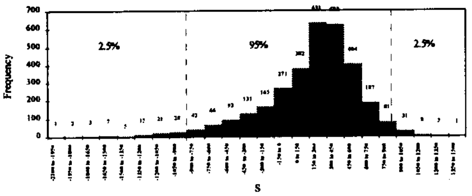

**Figure 5** Distribution of S with specified range of [-900, 825]

The obtained range is standardized to the scale of 0 to 100 for all attributes and attribute's values are then changed to corresponding score by a linear function. The higher score means the more desirable attribute. For example, the alternative with high P is undesirable, resulting in receiving low score.

The most and the least desirable values of the specified range are given scores of 100 and 0, respectively. As can be seen in Figure 2, higher value of W means lower score, thus W value of 80 receives the worst score (Sc(80) = 0). On the other hand, W's value of 0 means the best score (Sc(0) = 100). The value of W which are higher than 80 are truncated to the score of 100. The linear function for transforming W's value to a score is thus:

$$Sc(W) = \begin{cases} 0, W\geq 80 \\ (-1.25*W) + 100, 0<W<80 \\ 100, W=0 \end{cases} \quad (3)$$

By the same procedures, the linear functions that are used to transform other attributes to scores are:

$$Sc(Pr) = \begin{cases} 0, Pr=0 \\ (1.17*Pr), 0<Pr<0.855 \\ 100, Pr\geq 0.855 \end{cases} \quad (4)$$

$$Sc(P) = \begin{cases} 0, P\geq 35 \\ (-1.25*P) + 100, 0<P<35 \\ 100, P=0 \end{cases} \quad (5)$$

$$Sc(S) = \begin{cases} 0, S\leq -900 \\ (0.058*S)+52.174, -900<S<825 \\ 100, S\geq 825 \end{cases} \quad (6)$$

### *Membership Value Assignment*

To transform a precise score to a linguistic value (fuzzy score), the membership function must be assigned. The membership assignment method used in this study is Inductive Reasoning [5]. The method employs the relationship between input and output data and the concept of Least Entropy to classify the range of data in each fuzzy set. Since the method depends on the data collected from the system, this method is suitable for data-abundant, stable and complex systems.

The procedures for assigning membership function by Inductive Reasoning can be explained as follows:

**(a) Collect the data of the attributes** The range from which data is collected must be wide enough and stable so that the obtained membership function can be used in all situations happening in the system. In this research, the attributes W, Pr, and P of all alternatives and attribute S of the job are collected while the jobs are routed. The attributes are collected randomly by simulation from the stable range. Because RANDOM part routing rule (i.e., a part selects a machine to perform its required operation randomly) yields the most fluctuated data, it is applied to the system in data collection procedure. As a result, a wide range of attributes is obtained.

**(b) Define the type of data** The data is classified into two types: type 1 and type 2. There is no existing definite rule to classify these types of data. However, the separation should show the objective of changing input to output data. In this research, the objective of membership assignment to attributes W, Pr, and P is to compare the values of attributes by human's opinion. Thus the input data is the attributes' values of all alternatives, the output data is the best attribute's value and the other values (not the best one). The best value is defined as type 1 and the other values are defined as type 2. Suppose that the data is collected for 3 jobs being routed as Table 1.

**Table 1** The data collected for 3 jobs

| Job | Alternative 1 | Alternative 2 | Alternative 3 | Alternative 4 |
|---|---|---|---|---|
| Job1 | Sc(W)=20<br>Sc(Pr)=75<br>Sc(P)=63 | Sc(W)=50<br>Sc(Pr)=20<br>Sc(P)=60 | - | - |
| Job2 | Sc(W)=50<br>Sc(Pr)=20<br>Sc(P)=50 | Sc(W)=85<br>Sc(Pr)=75<br>Sc(P)=48 | Sc(W)=90<br>Sc(Pr)=85<br>Sc(P)=52 | - |
| Job3 | Sc(W)=44<br>Sc(Pr)=28<br>Sc(P)=22 | Sc(W)=70<br>Sc(Pr)=56<br>Sc(P)=24 | Sc(W)=85<br>Sc(Pr)=80<br>Sc(P)=25 | Sc(W)=30<br>Sc(Pr)=70<br>Sc(P)=20 |

From Table 1, Job 1 has 2 alternatives. Sc (W) of alternatives 1 and 2 are 20 and 50, respectively. The alternative with highest Sc (W) is the most preferable, thus Sc(W) of 50 is set as type 1 and Sc(W) of 20 is set as type 2. The classification of data in Table 1 are depicted in Figures 6 to 8.

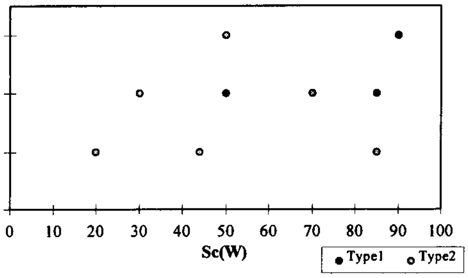


**Figure 6** The classification of Sc(W)

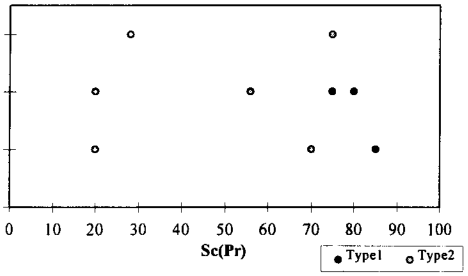


**Figure 7** The classification of Sc(Pr)

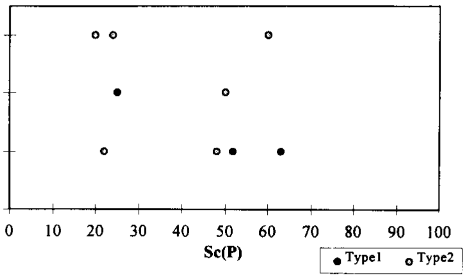


**Figure 8** The classification of Sc(P)

For attribute S of the jobs, the objective of membership assignment is to define the urgency of jobs in human's opinion. The job which has S more than X times of the operation time left is classified as type 1. From pilot simulations, X is set equal to 2. Suppose that 3 jobs in previous example have urgency data as shown in Table 2.

**Table 2** Urgency of Jobs

| Job | Slack Time | Operation Time Left | Score(S)* |
|---|---|---|---|
| 1 | 30 | 10 | 53.91 |
| 2 | 30 | 50 | 53.91 |
| 3 | -100 | 10 | 46.38 |

*Equation (6)

From Table 2, Job 1 has slack time of 30 which is more than 2 times of operation time left (10*2=20). Thus, Score(S) of 53.91 for the first data is set as type 1. Figure 9 shows the types of Score(S) of the example listed in Table 2.

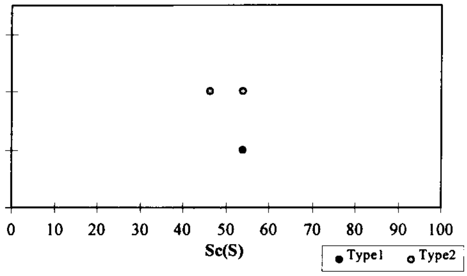


**Figure 9** The classification of Sc(S)

**(c) Define the range of fuzzy set by Least Entropy** The concept of Least Entropy is applied to separate the data to the appropriate fuzzy set. First, the data is separated into 2 areas, p and q by x (Figure 10). The entropy yielded from the separation of data at x in the range of $[x_1,x_2]$, S(x) [6] is:

$$S(x) = p(x)S_p(x) + q(x)S_q(x) \quad (7)$$
$$S_p(x) = -[p_1(x)\ln p_1(x) + p_2(x)\ln p_2(x)] \quad (8)$$
$$S_q(x) = -[q_1(x)\ln q_1(x) + q_2(x)\ln q_2(x)] \quad (9)$$

$p_k(x)$ and $q_k(x)$ are the conditional probability that data of type k are in the range of $[x_1, x_1+x]$ and $[x_1+x, x_2]$, respectively.

p(x) and q(x) are the probability that data are in the range of $[x_1, x_1+x]$ and $[x_1+x, x_2]$, respectively.

$$p_k(x) = [n_k(x)+1]/[n(x)+1] \quad (10)$$
$$q_k(x) = [N_k(x)+1]/[N(x)+1] \quad (11)$$
$$p(x) = n(x)/n \quad (12)$$
$$q(x) = 1-p(x) \quad (13)$$

$n_k(x)$ is the number of data of type k in $[x_1, x_1+x]$ range.

n(x) is the number of all data in $[x_1, x_1+x]$ range.

$N_k(x)$ is the number of data of type k in $[x_1+x, x_2]$ range.

N(x) is the number of all data in $[x_1+x, x_2]$ range.

n is the number of data in $[x_1,x_2]$ range.

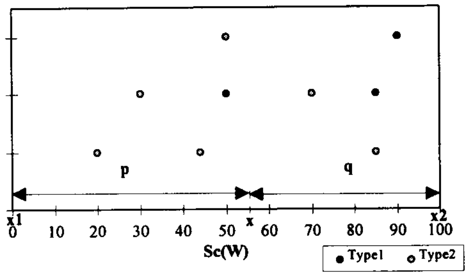


**Figure 10** Two areas obtained by the first separation

The x's value which yields the least entropy is the primary threshold (PRI). The data in p and q areas are separated in the same manner for the second separation. The secondary thresholds, SEC1 and SEC2, are the points which the least entropy are yielded in the range $[x_1,x]$ and $[x,x_2]$, respectively. Similar concept is applied to the tertiary thresholds, i.e., TER1, TER2, TER3, and TER4. Figure 11 shows the fuzzy sets obtained from the first, second, and third separations.

The membership functions of Sc(W), Sc(Pr), and Sc(S) are demonstrated in Figures 13 and 14.

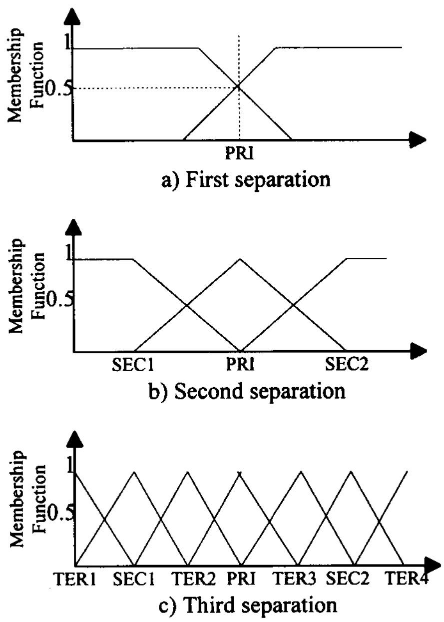


**Figure 11** The separations of data

### 3.2 Average Attributes' Weight

As mentioned previously, humans use experience and knowledge to define the importance of each attribute. The importance of attributes is prioritized from the job's urgency. Thus, the attributes' weight for each range of Sc (S) must be defined. The Sc(S) is separated into several ranges (fuzzy sets); for this reason, the attributes' weights are established for each fuzzy set of Sc(S). To find the attributes' weight, the pairwise comparison is exercised. Since humans are ambiguous when making a comparison, the importance of one attribute over another is represented in fuzzied Saaty's scale [7].

| j=1..3 \ i=1..3 | W | Pr | P |
|---|---|---|---|
| W | $(a_{11L},a_{11M},a_{11R})$ | $(a_{12L},a_{12M},a_{12R})$ | $(a_{13L},a_{13M},a_{13R})$ |
| Pr | $(a_{21L},a_{21M},a_{21R})$ | $(a_{22L},a_{22M},a_{22R})$ | $(a_{23L},a_{23M},a_{23R})$ |
| P | $(a_{31L},a_{31M},a_{31R})$ | $(a_{32L},a_{32M},a_{32R})$ | $(a_{33L},a_{33M},a_{33R})$ |

*$(a_{ijL}, a_{ijM}, a_{ijR}) = 1/(a_{ijR}, a_{ijM}, a_{ijL})$

**Figure 12** Comparison matrix

*($a_{ijk}$ is the fuzzy number that stands for the comparison of the $i^{th}$ to the $j^{th}$ elements on the k side (where k =left (L), middle (M), or right (R))*

All of the pairwise comparison values can be summarized in a comparison matrix, (Figure 12) attributes' weight can be extracted according to the following equations.

$$Wt_{iL}=\{[\prod_{j=1}^{n} a_{ijL}]^{1/n}\}/\{\sum_{j=1}^{n}[\prod_{j=1}^{n} a_{ijR}]^{1/n}\} \quad (14)$$

$$Wt_{iM}=\{[\prod_{j=1}^{n} a_{ijM}]^{1/n}\}/\{\sum_{j=1}^{n}[\prod_{j=1}^{n} a_{ijM}]^{1/n}\} \quad (15)$$

$$Wt_{iR}=\{[\prod_{j=1}^{n} a_{ijR}]^{1/n}\}/\{\sum_{j=1}^{n}[\prod_{j=1}^{n} a_{ijL}]^{1/n}\} \quad (16)$$

Table 3 presents the example of attributes' weight associated with fuzzy set of Sc (S).

**Table 3** Attributes' weight*

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*See Figure 14

## 4. Evaluation of Selection Index

There are 4 procedures to evaluate the selection index of the alternatives:

**(a) Fuzzification** The values of attributes acquired from a Flexible Manufacturing System are transformed to crisp scores and then fuzzy scores. The supporting data used in fuzzification are membership functions selected from database.

Figure 13 shows the membership function of Sc(W) and Sc(Pr). It can be seen that one crisp score can be a member of one or two fuzzy set(s). Thus, average fuzzy score is averaged from the fuzzy sets with which attributes' crisp score is associated. The average is weighted by the memberships of the associated fuzzy sets.

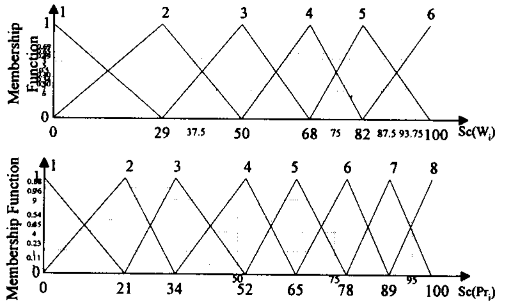



**Figure 13** Membership functions of Sc(W) and Sc(Pr)

For attribute P in this research, the difference among the alternative machines is not adequate to define fuzzy sets; nevertheless, non-fuzzy scores can be defined in the form of, for example, (3,3,3), (6,6,6), etc. [8]

**(b) Normalization** The normalized scores of alternative i associated with the attributes (Sc $(Attribute_i)_N$) are calculated by:

$$Sc(Attribute_{,i})_{N,L}=Sc(Attribute_i)_L / \sum_{i=1}^{n} Sc(Attribute_i)_R \quad (17)$$

$$Sc(Attribute_i)_{N,M}=Sc(Attribute_i)_M / \sum_{i=1}^{n} Sc(Attribute_i)_M \quad (18)$$

$$Sc(Attribute_i)_{N,R}=Sc(Attribute_i)_R / \sum_{i=1}^{n} Sc(Attribute_i)_L \quad (19)$$

where n is the number of the alternatives.

**(c) Average Weight Calculation** The attributes' weights ($Wt_{attributeL}$, $Wt_{attributeM}$, $Wt_{attributeR}$) depend on the urgency of the job represented by S. By being coordinated with attributes' weights from database, the average attributes' weights can be calculated.

Figure 14 shows the membership function of Sc(S). As the membership function of Sc(W) and Sc(Pr), it can be seen that one Sc(S) can be the member of one or two fuzzy sets(s). Each fuzzy set of Sc(S) represents one group of attribute's weights. Thus average attribute's weight is averaged from the groups of attribute's weights with which Sc(S) is associated.

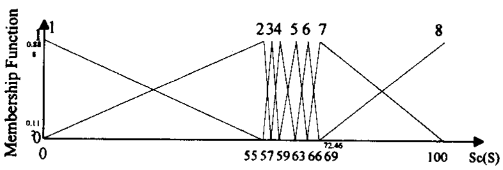



**Figure 14** Membership function of Sc(S)

**(d) Selection Index (SI) Calculation** The selection index is calculated by means of FuzzyAHP. The input data for this procedure are normalized fuzzy scores of attributes and the average attributes' weights which are obtained from the previous procedures. Selection index of alternative i is a fuzzy value which can be calculated as follows:

$$SI_{i,L}=(Sc(W_i)_{N,L}*Wt_{W,L}) + (Sc(Pr_i)_{N,L}*Wt_{Pr,L}) + (Sc(P_i)_{N,L}*Wt_{P,L}) \quad (20)$$

$$SI_{i,M} = (Sc(W_i)_{N,M}*Wt_{W,M}) + (Sc(Pr_i)_{N,M}*Wt_{Pr,M}) + (Sc(P_i)_{N,M}*Wt_{P,M}) \quad (21)$$

$$SI_{i,R} = (Sc(W_i)_{N,R}*Wt_{W,R}) + (Sc(Pr_i)_{N,R}*Wt_{Pr,R}) + (Sc(P_i)_{N,R}*Wt_{P,R}) \quad (22)$$

Table 4 shows the numerical example.

**Table 4** Numerical Example

| | | Alternative#1 | Alternative#2 | Alternative#3 | Alternative#4 |
|---|---|---|---|---|---|
| S | Crisp Value | 350 | | | |
| | Sc(S) (Equation (6)) | 72.46 | | | |
| | Fuzzy Set1:$\mu$* | (66,69,100):0.888 | | | |
| | Associated Weights1** | $Wt_W$=(0.080,0.122,0.205) $Wt_{Pr}$=(0.313,0.558,0.919) $Wt_P$=(0.190,0.320,0.591) | | | |
| | Fuzzy Set2:$\mu$* | (69,100,100):0.112 | | | |
| | Associated Weights2** | $Wt_W$=(0.058,0.081,0.119) $Wt_{Pr}$=(0.346,0.577,0.890) $Wt_P$=(0.223,0.342,0.586) | | | |
| | Average Weight | $Wt_W$=(0.0775,0.1174,0.1954) $Wt_{Pr}$=(0.3167,0.5601,0.9158) $Wt_P$=(0.1937,0.3225,0.5904) | | | |
| $W_i$ | Crisp Value | 20 | 10 | 50 | 5 |
| | Sc($W_i$) (Equation (3)) | 75 | 87.5 | 37.5 | 93.75 |
| | Fuzzy Set1:$\mu$*** | (50,68,82):0.5 | (68,82,100):0.694 | (0,29,50):0.595 | (68,82,100):0.347 |
| | Fuzzy Set2:$\mu$*** | (68,82,100):0.5 | (82,100,100):0.306 | ((29,50,68):0.405 | (82,100,100):0.653 |
| | Average fuzzy Sc($W_i$) | (59,75,91) | (72.28,87.51,100) | (11.75,37.51,57.29) | (77.14,93.75,100) |
| | $\Sigma$Sc($W_i$) | (220.17,293.77,348.29) | | | |
| | Sc($W_i$)$_N$ | (0.169,0.255,0.413) | (0.208,0.298,0.454) | (0.034,0.128,0.26) | (0.221,0.319,0.454) |
| $Pr_i$ | Crisp Value | 0.812 | 0.641 | 0 | 0.428 |
| | Sc($Pr_i$) (Equation (4)) | 95 | 75 | 0 | 50 |
| | Fuzzy Set1:$\mu$*** | (78,89,100):0.454 | (52,65,78):0.231 | (0,0,21):1 | (21,34,52):0.111 |
| | Fuzzy Set2:$\mu$*** | (89,100,100):0.546 | (65,78,89):0.769 | - | (34,52,65):0.889 |
| | Average fuzzy Sc($Pr_i$) | (84,95,100) | (62,75,86.46) | (0,0,21) | (32.56,50,63.56) |
| | $\Sigma$Sc($Pr_i$) | (178.56,220,271.02) | | | |
| | Sc($Pr_i$)$_N$ | (0.31,0.432,0.56) | (0.229,0.341,0.484) | (0,0,0.118) | (0.12,0.227,0.356) |
| $P_i$ | Crisp Value | 20 | 22 | 21 | 23 |
| | Sc($P_i$)**** | 0.05 | 0.045 | 0.048 | 0.043 |
| | Fuzzy Sc($P_i$) | (0.05,0.05,0.05) | (0.045,0.045,0.045) | (0.048,0.048,0.048) | (0.043,0.043,0.043) |
| | $\Sigma$Sc($P_i$) | (0.187,0.187,0.187) | | | |
| | Sc($P_i$)$_N$ | (0.268,0.268,0.268) | (0.244,0.244,0.244) | (0.255,0.255,0.255) | (0.233,0.233,0.233) |
| | Selection Index (Equation (20)-(22)) | (0.163,0.358,0.752) | (0.136,0.305,0.676) | (0.052,0.097,0.309) | (0.1,0.24,0.552) |

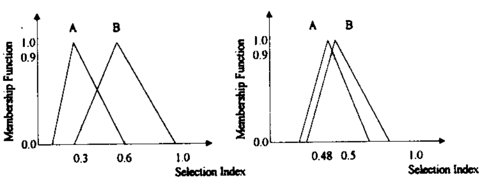
*Figure 14
**Table 3
*** Figure 13
****Sc($P_i$)=1/$P_i$ [8]

## 5. Alternative Machines Selection

The selection indices are fuzzy numbers. The methods used to rank fuzzy numbers are demonstrated by Adamo[4], Dubois and Prade [9] and Bortolan and Degani[10]. In this study, the method of [4] is employed. The steps can be outlined as follows.

First, the element of each fuzzy number within the medium value and the right spread range corresponding to the membership of $\alpha$ is defined as an $\alpha$-preference index ($F_\alpha$). Second, the fuzzy number with the maximum $\alpha$-preference index is the most preferable.

If one fuzzy set dominates the other(s) (Figure 15a), its ranking can be distinguished. On the other hand, in case of non-domination, its ranking is the same as the other(s) (Figure 15b). From Figure 15a, $\alpha$ is 0.9 and $F_{0.9}(A)$ = 0.3, $F_{0.9}(B)$ = 0.6. $F_{0.9}(B)$ is significantly larger than $F_{0.9}(A)$. Thus, it can be concluded that fuzzy number B is more preferable than fuzzy number A.




a) Domination b) Non-domination

**Figure 15** 0.9-Preference Index

The alternative machine selection of the example shown in Table 4 is displayed in Table 5. From Table 5, alternative machine#1 is selected.

**Table 5** The alternative machines selection

| | 0.9- Preference Index |
|---|---|
| Alternative#1 | 0.3974 |
| Alternative#2 | 0.3421 |
| Alternative#3 | 0.1182 |
| Alternative#4 | 0.2712 |

## 6. Improvement of FuzzyAHP Part Routing Rule

This section points out the drawbacks of FuzzyAHP Part Routing Rule and proposes the algorithm to improve its efficiency.

Although FuzzyAHP is expected to perform well, the performances of FuzzyAHP, based on preliminary investigations, are rather inefficient and sensitive compared with other conventional part routing rules, e.g. NINQ. It can be explained that the fuzzy algorithm used in FuzzyAHP occasionally creates an improper selection index which leads to inefficient decision. The improper selection index is caused by:

**(a) The missing algorithm to define the time left before the currently failed machine will be repaired** The attributes studied in this research give no difference among failed machines. In fact, the machine with shorter time after failure has higher probability to spend longer time until the repair will be completed. Since all out-of-order machines are assigned with equal Pr (Pr=0), the machine which has just failed may be chosen.

The simplest algorithm to alleviate this shortcoming is to eliminate failed machines from the list of the alternatives. The FuzzyAHP part routing rule exercising this algorithm will be named FuzzyAHP-NF (FuzzyAHP with no failing alternatives).

**(b) The fuzziness of fussy sets** In some cases, one attribute dominates others. However, it is not necessary that the alternative with highest score of dominant attribute is always selected. This can be explained that transforming a crisp value to a fuzzy value can create fuzziness which reduces the domination. Despite the fact that all attributes encounter fuzziness randomly, the system performances are considerably violated in the case of attribute W.

To limit the fuzziness within the acceptable range, routing decision should be WINQ (select machine with smallest W), providing that there exists the alternative with extraordinarily high Sc(W).

By this reason, the algorithm equipped with the alternate routing decision (WINQ and FuzzyAHP) and the toggle criterion, namely FuzzyAHP-WINQ, is thus established.

When FuzzyAHP-WINQ is implemented, the currently failed machines are eliminated. The remaining alternatives are then added to the second list and ranked according to Sc(W). The alternative with maximum Sc(W) is in the first order and so forth. The alternative in the first order is added to the third list. The Sc(W) difference between first order's and others' are all compared with the toggle criterion. The alternative whose Sc(W) difference is less than the toggle criterion is added to the third list. Finally, if there exists two or more alternatives in the third list, the routing decision is FuzzyAHP considering only the alternatives in the third list. Otherwise, the routing decision is FuzzyAHP considering the alternatives in the second list (the alternative in the first rank of the third list is selected).

## 7. Experimental Design

In this study, the FuzzyAHP-based rules, e.g. FuzzyAHP, FuzzyAHP-NF, and FuzzyAHP-WINQ are compared with other existing conventional rules as follows:

- Work in Next Queue (WINQ): Select the machine with smallest workload on machine buffer.
- Number in Next Queue (NINQ): Select the machine with smallest number of parts in machine buffer.
- Shortest Processing Time (SPT): Select the machine with smallest processing time.
- Random (RAN): Select the machine randomly.

All part routing rules are tested via computer simulation under various system configurations and workloads in system [11]. Other key parameters are listed in Table 6.

Table 6 Key parameters

| Key Parameters | Value Setting |
|---|---|
| Number of part type | Infinity |
| Arrival process | Set the number of part circulating in the system associated with system workload level |
| Due date assignment | TWK with 30% tardy job |
| Processing time | Primary machine: Exponentially distribute with the mean of 10<br>Others: Uniformly deviate from primary machine's between 1-15% range. |
| Set up time | Sequence independent and included in processing time |
| Transporter | 2 AGVs with the speed of 47m/min |
| Machine breakdown rate | Approximately 30% |

Five system performances based on time, due date, and cost are measured as follows:

- Time-based measure: Mean flow time.
- Due date related measures: Mean tardiness, Proportion of tardy jobs (PT) and Mean lateness.
- Cost-based measure: System utilization (high system utilization means low cost of idle machines).

## 8. Experimental Results

After the analysis of variance indicates the significant impact of the factor, Duncan's multiple range test is applied with 5% significance level to trace the best part routing rule for each system performance. The results are shown in Table 7. It is clear that most of the part routing rules performs significantly differently in each performance. For tardiness and system utilization, FuzzyAHP-WINQ performs significantly better than other part routing rules. Despite FuzzyAHP-WINQ has the best mean flow time and lateness, they are not significantly different from NINQ (for flow time and lateness) and WINQ (for lateness). In fact, the failure to reject the null hypothesis does not lead to the conclusion that there is an equality in the means. It is possible that there is not enough difference between the means for the given sample size. For the proportion of tardy jobs, NINQ performs best but not significantly different compared to FuzzyAHP-WINQ and WINQ. More details can be found in Chutima and Suwanruji [11].

## 9. Conclusions

In this study, FuzzyAHP is applied to develop part routing rules due to its ability to extract human experience and knowledge. FuzzyAHP imitates not only human's process of scoring the alternative machines according to the attributes but also human's opinion about the importance of individual attributes in individual situations.

The structure of the FuzzyAHP-based rules is composed of Flexible Manufacturing System, Database, Evaluation of Selection Index, and Alternative Machine Selection components.

The attributes addressed in this research are: workload on machine buffer, processing time, and the probability the part being routed to the alternative can be processed before the alternative fails. These attributes of all alternatives are examined by the Flexible Manufacturing component when the routing decision takes place. The values of the attributes are the inputs of Selection Index Evaluation component which calculates the selection indices by corporation with Database component.

Finally the selection indices from the previous component are compared at the Alternative Machine Selection component and the selected alternative is sent back to Flexible manufacturing System for activating actions.

Nevertheless, FuzzyAHP has some drawbacks since its fuzzy nature may create improper decisions. To improve these drawbacks, FuzzyAHP is extended with more sophisticated algorithms. As a result, FuzzyAHP-NF which eliminates failing alternatives from the list of alternatives and FuzzyAHP-WINQ which combines FuzzyAHP-NF and WINQ are generated.

From Duncan's multiple range test, FuzzyAHP-WINQ has tardiness and system utilization significantly better than the other rules. For the rest of the measures of performance, FuzzyAHP-WINQ performs satisfactorily well.

**Table7** Duncan's multiple range analysis for various performances by different part routing rules.

| Part Routing Rule | Mean Flow Time | Mean Tardiness | Mean Lateness | Proportion of Tardy Jobs | System Utilization |
|---|---|---|---|---|---|
| FuzzyAHP-WINQ | 218.93a | 51.88a | -38.50a | 30.09a | 66.96a |
| FuzzyAHP-NF | 258.06c | 70.46b | -2.17b | 38.90b | 57.30c |
| FuzzyAHP | 388.58d | 162.56c | 129.76c | 56.33d | 38.20d |
| WINQ | 229.50b | 69.48b | -31.07a | 30.43a | 64.44b |
| NINQ | 224.18a,b | 61.52b | -33.67a | 30.01a | 65.04b |
| SPT | 392.38d | 196.93d | 134.26c | 54.86c | 36.88d,e |
| RAN | 408.02e | 209.18e | 151.24d | 56.93d | 36.14e |

(Note: a, b, c, d, and e represent homogeneous group)

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