

# Fuzzy Analytical Hierarchy Process Part Routing in FMS

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## Abstract

Routing flexibility provides the ability of FMS to efficiently encounter traffic problems caused by machine breakdown, excessive workload, etc. The advantages of imposing routing flexibility can be fully obtained by a competent part routing rule. In this study, Fuzzy Analytical Hierarchy Process (Fuzzy AHP) is applied to form part routing rules from the attributes of alternate machines: workload on machine buffer, processing time, and the probability that the part being routed to the alternate machine can be processed before the machine fails. By means of Fuzzy AHP, the burdensome mathematical model can be avoided by extracting the relationship between the attributes from human experience and knowledge instead. The relationships between the attributes are dynamic which are changed according to the urgency of the part being routed. Three proposed fuzzy-based rules, FuzzyAHP, FuzzyNF and FuzzyWINQ are compared with WINQ, NINQ, SPT and RAN. The measures of performance are mean flow time, mean tardiness, mean lateness, proportion of tardy jobs, and system utilization. For tardiness and system utilization, FuzzyWINQ performs significantly better than other rules.

## 1. Introduction

As competition in industry becomes more intense, a novel production concept named Flexible Manufacturing System (FMS) has been developed to replace conventional production systems. Since FMS uses highly productive and flexible computer-controlled machines as well as automated material handling systems, it is capable of producing a variety of part types, increasing productivity, and reducing production cost.

One important property inherent in FMS is its flexibility, which is the capability to encounter and react efficiently to changes in the environment and process requirements. Routing flexibility is one of the various types of flexibility provided by FMS. Unlike conventional job shops where routing decisions are always made before parts enter the system, FMS always provides alternate processing routes for each part.

Shmilovici and Maimon [1] define routing flexibility as the ability of a system to route parts to alternate machines in case of

breakdowns, or as a response to the prevailing machine loading situation. In this case, parts can be routed to alternate paths to avoid traffic problems, i.e. congestion, or bottlenecks. Part routing can also increase system utilization and decrease makespan. More benefits of routing flexibility are reported by [2-3].

Yao [4] developed routing entropy to evaluate routing flexibility relating to the availability of an individual workstation. Kumar [5] extended Yao's results to include additional technological constraints of the system. Chandra and Tombak [6] employ Linear Programming model to maximize expected contribution of the system which reflects the system routing flexibility. They also point out that routing flexibility depends on a number of alternate machines and their reliability.

Flexible machines, tool transport systems, and routing strategy are the main ingredients of routing flexibility. The routing strategy is very important because inappropriate application of conventional fixed routing to FMS can impede FMS from its full potentiality.

There are a number of researchers attempting to develop routing strategy for FMS.

Analytical methods such as queuing network, Linear Programming and Optimization are also applied to part routing problems. Jiang et al. [7] and Avonts and Van Wassenhove [8] combine the LP model with the queuing network model where the throughput of an individual station and total cost saving are considered in their objective functions respectively. Liu [9] solves the routing problem by Dynamic Programming aiming to minimize the total expected in-process inventory costs.

It is known that routing problem is NP-complete. Thus, it consumes considerable time to solve real-life problems by analytical methods. As a result, a number of researchers turn to heuristic methods. Although heuristics can not guarantee the optimal results, good and acceptable solutions are often obtained.

Yao and Pei [10] use routing flexibility to construct Least Reduction in Entropy (LRE) principle for parts routing. Chen and Alfa [11] developed the algorithm based on the method of successive averages (MSA) to minimize the completion time of an individual batch of parts. Shmilovici and Maimon [1] present minimum flow resistance (MFR) to minimize the time duration that a part spends in the system.

An application of Artificial Intelligence in part routing is also demonstrated by Ben-Arieh and Lee [12]. They apply Fuzzy Logic Controller (FCL) where humans' experience and knowledge are collected as the control rules. The control rules translate the attributes of the alternate machines to the selection indices. The attributes of alternate machines considered are processing time, machine breakdown rate, slack time (based on individual machine) and workload in queue.

In this paper, the goal is to develop part routing policies that consider several attributes of alternate machines in order to improve the system performance and also to impose the ability to avoid routes with a high probability of machine failures. The attributes mentioned are aggregated by Fuzzy Analytical Hierarchy Process (Fuzzy AHP).

The paper is organized as follows. In section 2, the concept of Fuzzy set, Fuzzy AHP, and its application with part routing problem are

discussed. The experimental design is in section 3. The following section demonstrates the experimental results. Finally the conclusion is drawn in section 5.

## 2. Methodology

As previously reviewed, most part routing policies employ precise mathematical models to represent their objective functions. Those mathematical models are developed from the behavior and nature of the system. For many complex systems, the construction of a realistic routing selection model is very difficult. To ease this problem, an Analytical Hierarchy Process (AHP) is applied. By means of AHP, the attributes of each alternative are scored and aggregated according to their weights. The aggregated score is the selection index for the alternative. Thus, several alternatives can be ranked. In this sense, several attributes which are expected to improve system performance and to create the ability to avoid the routes with high probability of machine failure are considered. The weights given for the attributes indicate the relationships between the individual attribute and the selection index without a burdensome mathematical model.

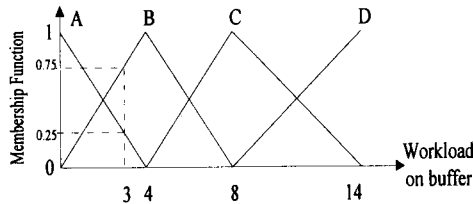
Expert humans can use their experience and knowledge to determine such relationships. However, the humans' experience and knowledge cannot be easily presented in a precise form. Thus, the control policy drawn from humans is in 'Linguistic' or 'Fuzzy' value. To translate a precise (crisp) value to a linguistic one, Zadeh [13] employs the 'Fuzzy Set' concept. The fuzzy set concept is applied with analytical hierarchy process to provide the more flexible way for humans to express their experience and knowledge.

### 2.1 Fuzzy set concept

Fuzzy set is a collection of elements in a universe of discourse where the boundary of the set may overlap with other sets. Each fuzzy set corresponds to a linguistic value. Fuzzy set can be defined as a mathematical formulation known as 'membership function' which represents a degree or membership within the set. If  $X$  is a universe of discourse,  $x$  is the element of  $X$ , the membership function of a fuzzy set  $A$  is denoted by  $\mu_A(x)$ . The

membership function is within the range of 0 and 1.

The isosceles triangle fuzzy sets are shown in Figure 1.



A = very small, B = small, C = moderate and D = large

**Figure 1:** Isosceles triangle fuzzy sets.

In Figure 1, the attribute named 'Workload on buffer' can be classified into four fuzzy sets (A, B, C and D). Each set represents a linguistic value: very small, small, moderate, and large, respectively.

The overlapping at the boundary of each set shows the ambiguity of human perception. For example, nobody can define exactly whether one range of workload on a buffer should be defined as 'very small' or 'small'. Sets of 'very small', 'small', 'moderate' and 'large' are composed of workload on the buffer in the ranges of [0,4], [0,8], [4,14] and [8,14] respectively. They can also be defined in triangular fuzzy number form as (0,0,4), (0,4,8), (4,8,14) and (8,14,14) respectively.

The relationship between crisp value and linguistic value can be explained in the following example. It can be seen that workload on buffer of 3 (crisp value) is a member of set 'very small' and 'small' with the membership of 0.25 and 0.75 respectively (see Figure 1).

The arithmetic of fuzzy numbers depends on the range that fuzzy set is defined. The fuzzy set considered in this paper is defined in  $R^+$ . The important algebraic operations of triangular fuzzy number over  $R^+$  can be shown as follows:

Define two triangular fuzzy numbers A and B as  $A=(a_1,a_2,a_3)$ ,  $B=(b_1,b_2,b_3)$ . Assume that c is a constant.

Addition:  $A (+) B = (a_1,a_2,a_3) (+) (b_1,b_2,b_3)$

$$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

$$\text{Subtraction: } A (-) B = (a_1, a_2, a_3) (-) (b_1, b_2, b_3) \\ = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$$

The symmetric (image):

$$-(A) = (-a_3, -a_2, -a_1)$$

$$\text{Multiplication: } A (\cdot) B = (a_1, a_2, a_3) (\cdot) (b_1, b_2, b_3) \\ = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3)$$

$$c(\cdot)A = (c \cdot a_1, c \cdot a_2, c \cdot a_3)$$

$$\text{Inverse: } A^{-1} = (a_1, a_2, a_3)^{-1} \\ = \left( \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right)$$

$$\text{Division: } A (:) B = (a_1, a_2, a_3) (:) (b_1, b_2, b_3) \\ = \left( \frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1} \right)$$

## 2.2 Fuzzy AHP

Analytical hierarchy process (AHP) involves ranking several alternatives according to their weights. The hierarchical pairwise comparison is employed to induce the relative weights of alternatives through pairwise comparison. By means of hierarchy, the importance of the alternatives according to the objective can be viewed. The levels of the hierarchy define the priority classes of the objectives (primary, secondary, etc.) in those levels. AHP is a systematic approach for the decision making. AHP also provides an easier way for the decision maker to assign the importance of the alternatives by pairwise comparison according to the level of hierarchy instead of an absolute comparison according to the primary objective.

The structure of 2-hierarchy decision making with 3 secondary objectives and 2 alternatives is shown in Figure 2.

Saaty's scale [14] as shown in Table 1 is widely used in a pairwise comparison. The scale is chosen to express the relative significance of one alternative (or objective) over another. Nevertheless, in some complex decision problems, it is difficult for the decision maker to compare between alternatives (or objectives) with crisp value (because of the ambiguity in human experience and knowledge). Thus Saaty's scale adjusted for Fuzzy AHP appears to be more reasonable and appealing.

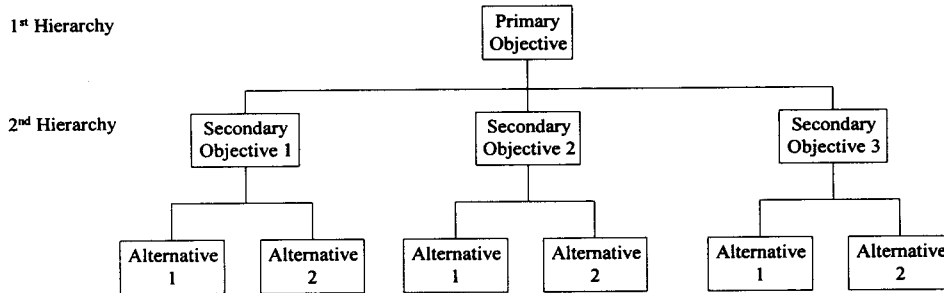
All of the pairwise comparison values can be summarized in a comparison matrix

(Figure 3) from which relative weights of all alternatives or objectives can be extracted.

Given the fuzzy number of a pairwise comparison  $i, j$  in the comparison matrix, the

The left spread:

$$r_{iL} = \left[ \prod_{j=1}^n a_{ijL} \right]^{1/n} \quad (1)$$



**Figure 2** The structure of 2-hierarchy decision making with 3 secondary objectives and 2 alternatives

The right spread:

$$r_{iR} = \left[ \prod_{j=1}^n a_{ijR} \right]^{1/n} \quad (2)$$

The medium value:

$$r_{iM} = \left[ \prod_{j=1}^n a_{ijM} \right]^{1/n} \quad (3)$$

Then the normalized fuzzy weight of criterion or alternative  $i$  becomes:

The left spread:

$$w_{iL} = r_{iL} / \sum_{j=1}^n r_{jR} \quad (4)$$

The right spread:

$$w_{iR} = r_{iR} / \sum_{j=1}^n r_{jL} \quad (5)$$

The medium value:

$$w_{iM} = r_{iM} / \sum_{j=1}^n r_{jM} \quad (6)$$

Suppose there are  $n$  alternatives and  $m$  secondary objectives which are composed to be the primary objective in the decision making problem. Let  $(k_{ijL}, k_{ijM}, k_{ijR})$  denote the normalized rating assigned to alternative  $i$  for

arithmetic mean for each criterion or alternative can be expressed as follows:

the secondary objective  $j$  and  $(w_{jL}, w_{jM}, w_{jR})$  denote the normalized weight of the secondary objective  $j$ . The final score  $(Sc_{iL}, Sc_{iM}, Sc_{iR})$  of alternative  $i$  compared with primary objective is

$$Sc_{iL} = (k_{i1L} \otimes w_{1L}) \oplus \dots \oplus (k_{imL} \otimes w_{mL}) \quad (7)$$

$$Sc_{iM} = (k_{i1M} \otimes w_{1M}) \oplus \dots \oplus (k_{imM} \otimes w_{mM}) \quad (8)$$

$$Sc_{iR} = (k_{i1R} \otimes w_{1R}) \oplus \dots \oplus (k_{imR} \otimes w_{mR}) \quad (9)$$

The final score of alternative  $i$  is a fuzzy number. There are several methods to rank fuzzy numbers [15-16]. The method used in this paper is adapted from Adamo[17]. Adamo uses the concept of  $\alpha$ -level set to obtain an  $\alpha$ -preference index  $(F_\alpha)$ . The element of each fuzzy set within the medium value and the right spread range which corresponds to membership of  $\alpha$  is defined as an  $\alpha$ -preference index. The fuzzy set with the maximum  $\alpha$ -preference index is the most preferable. The method is demonstrated in Figure 4.

If one fuzzy number dominates the other (Figure 4 a), their ranking can be distinguished. Otherwise, it cannot be concluded from their differences. Therefore,

their rankings are the same (Figure 4 b). From Figure 2.3 a,  $\alpha$  is 0.9 and  $F_{0.9}(A) = 0.3$ ,  $F_{0.9}(B) = 0.6$ .  $F_{0.9}(B)$  is significantly larger than  $F_{0.9}(A)$ . Thus, it can be concluded that fuzzy number B is more preferable than fuzzy number A.

### 2.3 Application of fuzzy AHP on part routing problems

Two components are needed to form fuzzy AHP part routing:

1) The attributes of alternate machines

Normally, to select the proper machines, the following attributes should be considered:

- Part processing time (P): If several alternate machines can process the same operation of the part in different amounts of time, the machine which processes the operation in the shortest time should be the best alternative.
- Workload on the buffer of alternate machine (W): It means the summation of the processing time of all parts in the buffer in front of the machine.

**Table 1** The numerical scale of relative judgment proposed by Saaty and adjusted fuzzy number scale for Fuzzy AHP

Numerical Value	Linguistic Definition	Explanation	Adjust Fuzzy Number Scale
1	Equal Importance	Two activities contribute equally to the objective	(1,1,1)
3	Moderate Importance	Experience and judgment slightly favor one activity over another	(2,3,4)
5	Strong Importance	Experience and judgment strongly favor one activity over another	(4,5,6)
7	Demonstrated Importance	An activity is favored very strongly over another; its dominance demonstrated in practice	(6,7,8)
9	Extreme Importance	The evidence favoring one activity over another is of the highest possible order of affirmation	(8,9,10)
2,4,6,8	Intermediate Values	To reflect the compromise between the two adjacent judgments	(1,2,3), (3,4,5), (5,6,7), (7,8,9)

	A	B	C
A	( $a_{11L}$ , $a_{11M}$ , $a_{11R}$ )	( $a_{12L}$ , $a_{12M}$ , $a_{12R}$ )	( $a_{13L}$ , $a_{13M}$ , $a_{13R}$ )
B	( $a_{21L}$ , $a_{21M}$ , $a_{21R}$ )	( $a_{22L}$ , $a_{22M}$ , $a_{22R}$ )	( $a_{23L}$ , $a_{23M}$ , $a_{23R}$ )
C	( $a_{31L}$ , $a_{31M}$ , $a_{31R}$ )	( $a_{32L}$ , $a_{32M}$ , $a_{32R}$ )	( $a_{33L}$ , $a_{33M}$ , $a_{33R}$ )

(A, B and C represent criteria or objective or alternative)

**Figure 3** Comparison Metrix

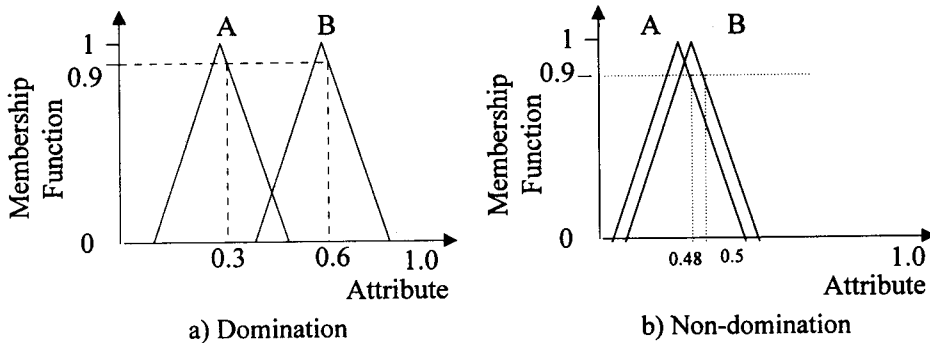


Figure 4 0.9-preference index.

- The probability that the part being routed to the alternate machine can be processed before the machine fails (Pr): Denote  $f_i(\text{MTBF}, t)$  as the probability function that alternate machine  $i$  will fail at time  $t$  after last failure, where MTBF is the mean time before failure.  $T_{\text{now}}$  and  $T_f$  represent the time at decision point and the latest time that machine  $i$  failed respectively. If the part chooses machine  $i$ , the time after latest that part will be processed is  $T_{\text{now}} - T_f + W_i$  which equals to  $t$  failure (if no failure happened).

$$Pr_i = 1 - \int_{t=0}^t (f_i(\text{MTBF}, t)) \quad (10)$$

If machine is out of order, then  $Pr = 0$ .

The difference among each attribute's values may not appear in linear scale as presented by crisp scale. Thus the values of each attribute from different ranges may affect the main objective unequally. The experience and knowledge of humans are the tools used to define such ranges. Thus, crisp value of the input variables will be changed into fuzzy set. To change crisp values into fuzzy set, the membership function must be constructed. There are several methods to assign membership function [18]. The method of 'Inductive Reasoning' which based on the data provided is applied in this paper.

The data for each attribute are obtained by conducting a pilot simulation. Before being transformed to fuzzy set, the data are changed into the scores of 0 to 100 (where 100 means the best). 95% of data are transformed to crisp score by linear function. The extraordinary data (5%) are transformed to the score of 0 or 100

according to their values. This algorithm is imposed to eliminate the deviation between the extraordinary and normal values. The score representing the attribute of alternative  $i$  is denoted as  $\text{Score}(\text{attribute}_i)$ .

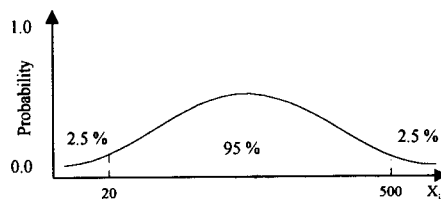


Figure 5 Distribution curve of attribute X

Suppose that attribute  $X$  has a distribution as shown in Figure 5. Ninety five percent of data have  $X$  in the range of  $[20, 500]$ . The data in the range of  $[20, 500]$  are transformed to  $\text{Score}(X)$  with linear function. If the alternative with high  $X$  is desirable, such an alternative yields a high score.  $X$  value of 500 means the best score ( $\text{Score}(500)=100$ ) whereas  $X$  value of 20 means the worst score ( $\text{Score}(20)=0$ ). The linear function is:

$$\text{Score}(X) = (0.208 * X) - 4.16$$

The data which are lower than 20 and higher than 500 are changed into the score of 0 and 100 respectively.

2.) The relationship between the attributes.

The relationship between the attributes is presented by the weights deriving from equations (1) to (6). The relationship between the attributes is the main concept of part routing since it controls routing selection. To increase the robustness of the part routing policy, routing selection should not rely solely on static

weights. The dynamic weights which change corresponding to the condition of part being routed are developed. The urgency of the part is a suitable condition to control the relationship between the attributes. The attribute used to reflect the urgency of the part is slack time (S).

Slack time of the part is:

$$\text{Slack time} = \text{Due date} - \text{Tnow} - \text{time for remaining operation} \quad (11)$$

When slack time is large, the attributes P and Pr should contribute to the main objective significantly since the part routing policy operates in a way that prevents system congestion in the future. The part routing policy attempts to reduce workload in the machine buffer and to avoid the routes which tend to fail by assigning short P and high Pr machine with a high score. On the other hand, when slack time is small or negative, the machine with short W is assigned a high score since it is believed to expedite the part and to reduce the part tardiness.

Similar to the attributes of the alternative, slack time is defined as fuzzy sets, each corresponding to one weighting group. The weights applied to part routing policy are the average of two weighting groups from two fuzzy sets with which slack time is associated.

The examples of fuzzy sets designed for  $W_i$ ,  $Pr_i$  and  $S_i$  are shown in Figures 6 to 8. For the attribute  $P_i$  in this study, the difference in processing time among the alternate machines is not adequate to define fuzzy sets. However non-fuzzy numbers can be defined in the form of, for example, (2,2,2), (5.6,5.6,5.6).

The normalized weights for individual fuzzy sets of slack time are shown in Table 2. Suppose that the part being routed with slack time of 200 min. has 3 alternative machines Machine 1,  $W_1 = 10$ ,  $Pr_1 = 0.85$  and  $P_1 = 20$  Machine 2,  $W_2 = 60$ ,  $Pr_2 = 0.40$  and  $P_2 = 18$  Machine 3,  $W_3 = 5$ ,  $Pr_3 = 0.00$  and  $P_3 = 20.5$

The numerical example for part routing policy by Fuzzy AHP method is demonstrated in Table 3.

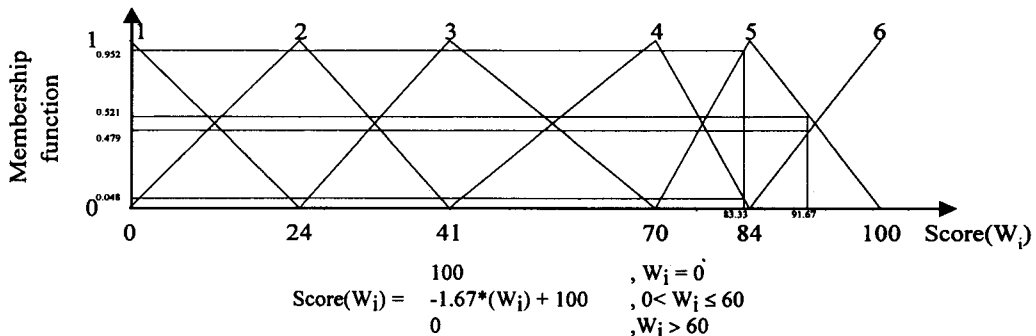
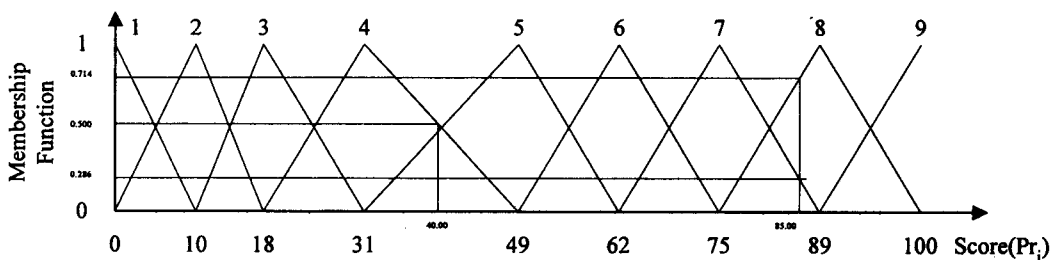
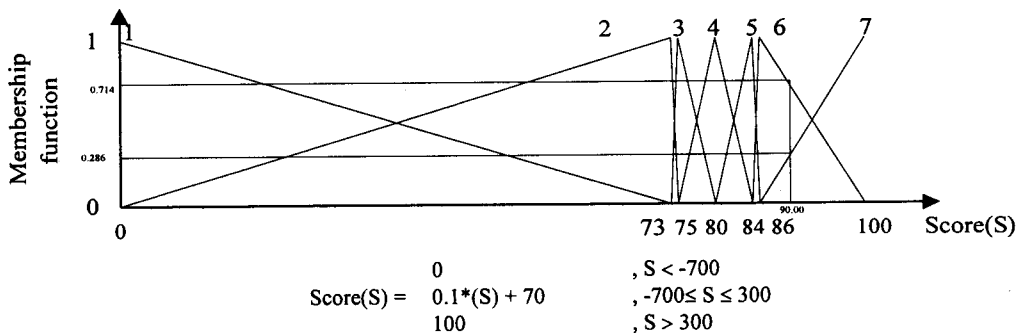


Figure 6 The fuzzy sets definitions for Score of  $W_i$  ( $\text{Score}(W_i)$ ).



$$\text{Score}(Pr_i) = 100 * Pr_i$$

Figure 7 The fuzzy sets definitions for Score of  $Pr_i$  ( $\text{Score}(Pr_i)$ ).



**Figure 8** The fuzzy sets definitions for Score of S (Score(S)).

Slack time of the part is 200 min. The function used to transform slack time to a score is shown in Figure 8. Thus,  $\text{Score}(S) = (0.1 \cdot 200) + 70 = 90$ . Score(S) of 90 corresponds to 2 fuzzy sets, set 6 and set 7 with the membership functions of 0.714 and 0.286 respectively (Figure 8). As mentioned previously, each set of Score(S) is associated with one weighting group (Table 2). Then, the average weights for the attributes are the average of weighting groups corresponding to set 6 and set 7 as follows:

$$\begin{aligned} W &= 0.714 \cdot (0.080, 0.122, 0.205) + \\ &\quad 0.286 \cdot (0.058, 0.081, 0.119) \\ &= (0.074, 0.110, 0.180) \\ Pr &= 0.714 \cdot (0.313, 0.558, 0.919) + \\ &\quad 0.286 \cdot (0.346, 0.577, 0.809) \\ &= (0.322, 0.563, 0.911) \\ P &= 0.714 \cdot (0.190, 0.320, 0.591) + \\ &\quad 0.286 \cdot (0.199, 0.326, 0.590) \\ &= (0.199, 0.326, 0.590) \end{aligned}$$

For attribute W of alternative 1,  $W_1$  is 10. Then, it can be transformed to Score( $W_1$ ) by the function shown in Figure 6.  $\text{Score}(W_1) = (-1.67 \cdot 10) + 100 = 83.33$ . Score( $W_1$ ) of 83.33 is the member of set 4 and 5 with the membership functions of 0.048 and 0.952 respectively (Figure 6). The average fuzzy set of  $\text{Score}(W_1) = 0.048 \cdot (41, 70, 84) + 0.952 \cdot (70, 84, 100) = (68.61, 83.33, 99.23)$ . The total fuzzy set of W is the summation of the average fuzzy sets of all alternatives. Then the total fuzzy set of  $W = (68.61, 83.33, 99.23) + (0, 0, 24) + (76.71, 91.6, 100) =$

$(144.32, 174.99, 223.23)$ . The normalized Score( $W_1$ )

$$\begin{aligned} &= \left( \frac{68.61}{223.23}, \frac{83.33}{174.99}, \frac{99.23}{145.32} \right) \\ &= (0.307, 0.476, 0.683) \end{aligned}$$

Other values shown in Table 3 can be calculated in the same manner as described in the case of  $W_1$ .

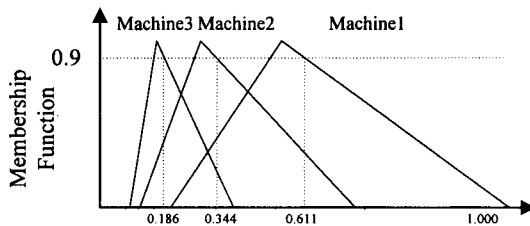
The final scores can be calculated by equation 7, 8 and 9. For alternative #1, the final score is:

$$\begin{aligned} \text{Left} &= (0.074 \cdot 0.307) + (0.322 \cdot 0.439) + \\ \text{Spread} &= (0.199 \cdot 0.325) = 0.229 \\ \text{Medium} &= (0.110 \cdot 0.476) + (0.563 \cdot 0.680) + (0. \\ \text{Value} &= 326 \cdot 0.325) = 0.542 \\ \text{Right} &= (0.180 \cdot 0.683) + (0.911 \cdot 0.680) + (0. \\ \text{Spread} &= 590 \cdot 0.325) = 1.23 \end{aligned}$$

The final scores of all alternate machines are shown in Figure 9.  $\alpha$  is 0.9. Machine1 has maximum F 0.9 of 0.611. Thus machine1 is selected.

It is expected that FuzzyAHP will perform well. However, from preliminary investigations, the performance of FuzzyAHP is quite inefficient and sensitive compared with other existing conventional part routing rules. It can be explained that the fuzzy algorithm used in FuzzyAHP creates an improper selection index which leads to improper decisions. The improper selection index is caused by:





**Figure 9** The final scores of all alternate machines

- The fuzziness of fuzzy sets. In some cases, one attribute dominates others. Since that attributes values are transformed to fuzzy sets, the fuzziness of fuzzy sets reduces the domination. Especially, when combined with other attributes, the alternative with highest value of dominant attribute may not be selected.
- The missing algorithm to define the time left before machine is repaired. There is no attribute which represents the difference between machines which have failed at different times. A machine which

has recently failed will probably have a longer wait before being repaired. Since all out-of-order machines are only assigned equal  $Pr(0)$ . As a result, the machine which has just failed may be chosen.

- The improper weights given by humans. The weights applied to each attribute are dynamic and the weights change gradually over the range of slack time. However in some situations, only one attribute dominates other attributes.

To improve the drawbacks of FuzzyAHP, the more sophisticated algorithms which prevent improper selection are created as follows:

- The algorithm to eliminate failed machines from the list of alternatives: The FuzzyAHP with this algorithm is named FuzzyNF (FuzzyAHP with No Failed Alternatives)
- The algorithm to combine FuzzyAHP with Work in Next Queue (WINQ) which chooses the machine with smallest workload on machine buffer:

**Table 2** The normalized weights for individual fuzzy sets of slack time

Fuzzy Set		Normalized Weight								
		$W_i$			$Pr_i$			$P_i$		
		L	M	R	L	M	R	L	M	R
1	(0,0,73)	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
2	(0,73,75)	0.482	0.667	0.900	0.136	0.167	0.213	0.136	0.167	0.213
3	(73,75,80)	0.245	0.500	0.872	0.170	0.25	0.419	0.170	0.25	0.419
4	(75,80,84)	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
5	(80,84,86)	0.170	0.250	0.419	0.245	0.500	0.871	0.170	0.25	0.419
6	(84,86,100)	0.080	0.122	0.205	0.313	0.558	0.919	0.190	0.320	0.591
7	(86,100,100)	0.058	0.081	0.119	0.346	0.577	0.890	0.223	0.342	0.586

(L = Left Spread, M = Medium Value and R = Right Spread)

**Table 3 Numerical Example**

		Alternative # 1	Alternative # 2	Alternative # 3
S	Crisp Value Score (S)	200 90		
	Fuzzy Set 1 : $\mu$	(84,86,100) : 0.714		
	Associated Weight	$W_i = (0.080, 0.122, 0.205)$ $Pr_i = (0.313, 0.558, 0.919)$ $P_i = (0.190, 0.320, 0.591)$		
	Fuzzy Set 2 : $\mu$	(86,100,100) : 0.286		
$W_i$	Associated Weight	$W_i = (0.058, 0.081, 0.119)$ $Pr_i = (0.346, 0.577, 0.890)$ $P_i = (0.223, 0.342, 0.586)$		
	Average Weight	$W_i = (0.074, 0.110, 0.180)$ $Pr_i = (0.322, 0.563, 0.911)$ $P_i = (0.199, 0.326, 0.590)$		
	Crisp Value Score ( $W_i$ )	10 83.33	60 0	5 91.67
	Fuzzy Set 1 : $\mu$	(41,70,84) : 0.048	(0,0,24) : 1.000	(70,84,100) : 0.521
$Pr_i$	Fuzzy Set 2 : $\mu$	(70,84,100) : 0.952	(0,24,41) : 0.000	(84,100,100) : 0.479
	Average Fuzzy Set	(68.61,83.33,99.23)	(0,0,24)	(76.71,91.66,100)
	Total Fuzzy Set	(145.32,174.99,223.23)		
	Normalized Fuzzy Set	(0.307,0.476,0.683)	(0,0,0.165)	(0.344,0.524,0.688)
$P_i$	Crisp Value Score ( $Pr_i$ )	0.85 85	0.40 40	0.00 0
	Fuzzy Set 1 : $\mu$	(62,75,89) : 0.286	(18,31,49) : 0.500	(0,0,10) : 1.000
	Fuzzy Set 2 : $\mu$	(75,89,100) : 0.714	(31,49,62) : 0.500	(0,10,18) : 0.000
	Average Fuzzy Set	(71.28,85.00,96.85)	(24.50,40.00,55.50)	(0,0,10)
$P_i$	Total Fuzzy Set	(95.78,125.00,162.35)		
	Normalized Fuzzy Set	(0.439,0.680,1.011)	(0.151,0.320,0.579)	(0,0,0.104)
	Crisp Value Score ( $P_i$ ) = $1/P_i$	20 (20,20,20) (0.05,0.05,0.05)	18 (18,18,18) (0.055,0.055,0.055)	20.5 (20.5,20.5,20.5) (0.049,0.049,0.049)
	Total Fuzzy Set	(0.154,0.154,0.154)		
	Normalized Fuzzy Set Form	(0.325,0.325,0.325)	(0.357,0.357,0.357)	(0.318,0.318,0.318)
	Final Score	(0.229,0.542,1.236)	(0.120,0.297,0.768)	(0.089,0.161,0.406)
	0.9-Preference Index	0.611	0.344	0.186

**Result : Alternative 1 (machine 1) is selected.**

This algorithm can eliminate the fuzziness of FuzzyAHP. The pilot run indicates that the W attribute dominates the others. Thus the algorithm that allows only alternatives which have the differences of W attribute within the specified range using FuzzyAHP for part routing is developed. If there are no such alternatives, WINQ is applied. FuzzyNF which applies this algorithm is called FuzzyWINQ. The specified range is obtained from simulation.

### 3. Experimental Design

In this study, the factors which are of interest include system configurations, workload in the system, and part routing rules. The experiments are conducted via computer simulation. The system configuration has 2 levels: simple and complex. The layout of the system is modified from the literature [19-20]. The layout of both systems is depicted in Figures 10 and 11. The complex system is composed 11 machines, whereas the simple one comprises 4 machines. The configuration of the system depends on the complexity of the part produced. In a complex system, the number of operations per part is larger. For simple and complex systems, the number of operations per part are uniformly distributed with the range of [2,3] and [4,6] respectively.

The workload in the system relates to the number of parts in the system. For each configuration level, the number of parts in the system generating high (87%) and low (70%) system utilization is traced by pilot simulation without machine breakdown.

The proposed part routing rules, FuzzyAHP, FuzzyNF and FuzzyWINQ are compared with other conventional rules.

- Work in Next Queue (WINQ): Chooses the machine with smallest workload on machine buffer.
- Number in Next Queue (NINQ): Chooses the machine with smallest number of parts in machine buffer.
- Shortest Processing Time (SPT): Chooses the machine with smallest processing time.
- Random (RAN): Chooses the machine randomly.

The infinite part types are generated, each having a different number of operations, processing time and number of machines provided for each operation. The arrival process of part is limited by the number of a parts allowed in the system. When the finished part leaves the system, the new part enters the system immediately.

For due date assignment, the Total Work Content (TWK) method is employed. The value of the multiplier K is used to determine the degree of due date tightness. To maintain the equity of due date tightness in each system with different configuration and preliminary system utilization, K values that give 30% of tardy job are fixed.

The processing time is exponentially distributed with the mean of 10. The different processing time for individual alternative machine is uniformly distributed in the range of 0-15 percent. The setup time is sequence independent and included with processing time. The machine scheduling rule is First Come First Served (FCFS).

The transportation system consists of a guided path as shown in Figures 3.1 and 3.2. Two AGVs with the speed of 47 m/min are employed.

The machine breakdown in the simulation model depends on time before failure and time taken to repair which are exponentially distributed with mean time to failure (MTBF) and mean time to repair (MTTR) of 570 and 200 respectively. The average machine breakdown is 30 percent.

Five system performances based on time, due date, and cost are measured as follows:

- Time-based measure: Mean flow time.
- Due date related measures: Mean tardiness, Proportion of tardy jobs (PT) and Mean lateness.
- Cost-based measure: System utilization (high system utilization means low cost of idle machines).

### 4. Experimental Result and Discussion

There are three factors of interest involved in the experiment as described in the previous section. The effects of each factor, and interaction between factors on

the performances of the systems are investigated by ANOVA.

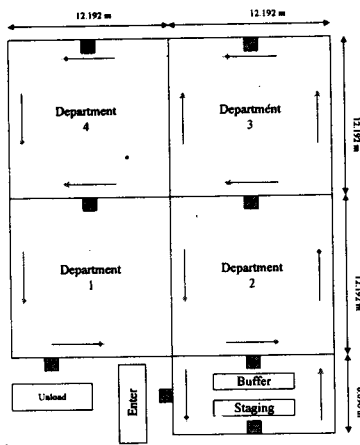


Figure 10 Simple system configuration

The results of ANOVA with 5% of significance level are shown in Table 4. It is clear that all measures of performance are significantly effected by all factors and their interactions except system utilization which is not significantly effected by system configuration. Mean flow time, mean

tardiness, mean lateness and proportion of tardy jobs are mainly effected by system configuration since its F-Ratio values are maximum compared with other main effects and interactions. For system utilization, it is mainly effected by workload in the system.

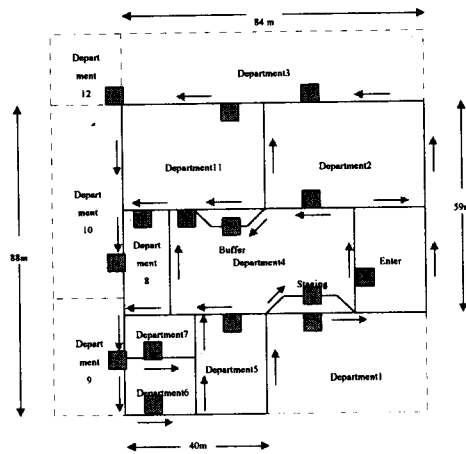


Figure 11 Complex system configuration

Table 4 The results of ANOVA against several measures of performance.

Variation Sources	F-Ratio				
	Mean Flow Time	Mean Tardiness	Mean Lateness	Proportion of Tardy Job	System Utilization
C	9527.602*	1553.385*	738.385*	1552.821*	0.405
W	5507.149*	369.263*	10.305*	275.036*	1033.527*
R	626.305*	437.664*	618.342*	1469.136*	823.715*
CXW	766.734*	31.577*	15.612*	6.345*	4.572*
CXR	139.938*	100.679*	136.355*	143.145*	2.655*
WXR	23.487*	8.953*	21.749*	50.327*	4.062*
CXWXR	7.227*	4.244*	5.979*	56.821*	3.019*

(Note: C = Configuration of the system, W = Workload in the system, R = Part routing rule, \* = significance at 5% significance level.)

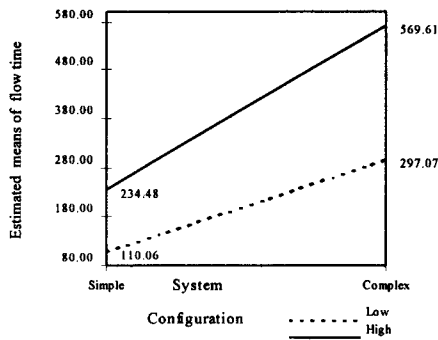
The measures of performance influenced by main effects and the interactions between them are described by the estimated means. The estimated mean is the average value of system performance according to main effect or the interaction between main effects.

#### 4.1 Effect of system configurations and workloads

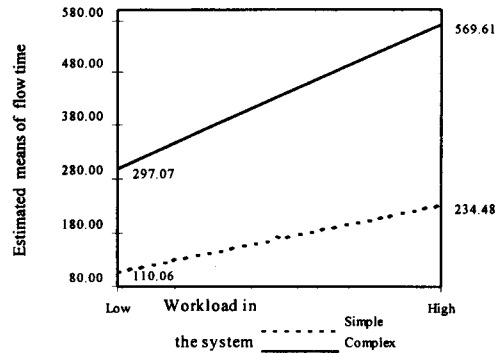
The increasing workload and complexity cause high system congestion. Since the configuration of the system is changed from 4 machines into 11 machines, the average number of operations per part is increased and the part has a tendency to spend longer time in the system to finish all assigned operations.

When the system congestion is boosted, the average waiting time in an individual machine buffer is increased. Furthermore, some entities which can not enter machine buffers must be transported to a central buffer area

where they wait for the availability of the proper machines. As a result, mean flow time, mean tardiness and proportion of tardy job are increased (Figures 12 - 14).

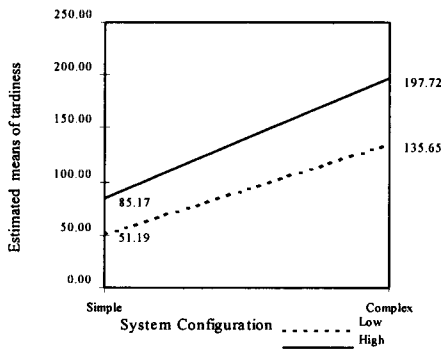


a)

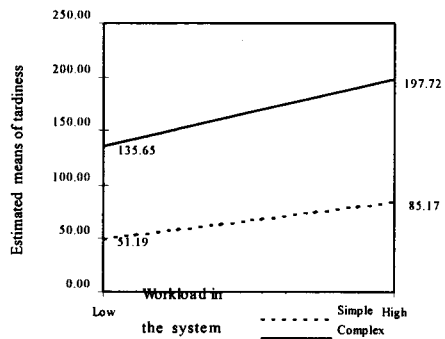


b)

**Figure 12** Estimated means of flow time  
a) against system configuration. b) against workload in the system.

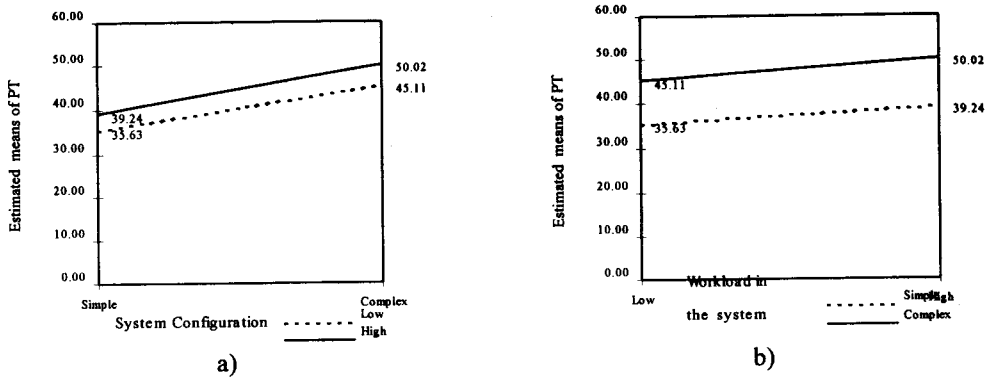


a)

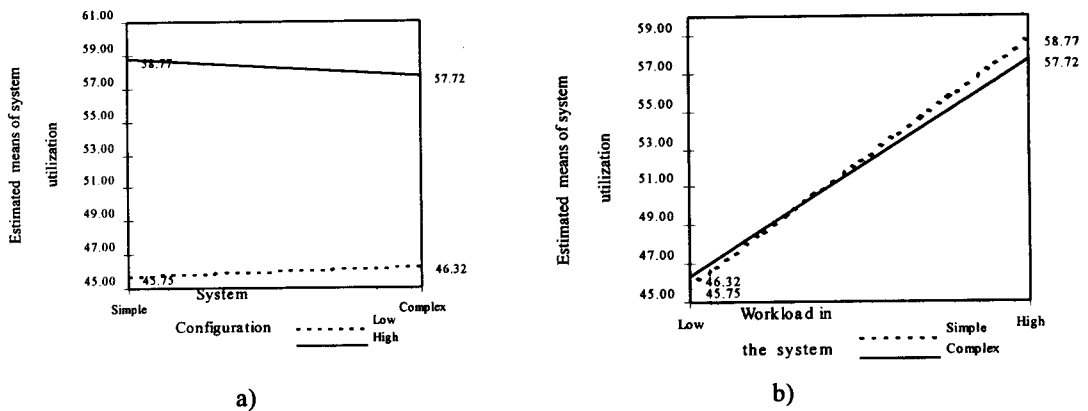


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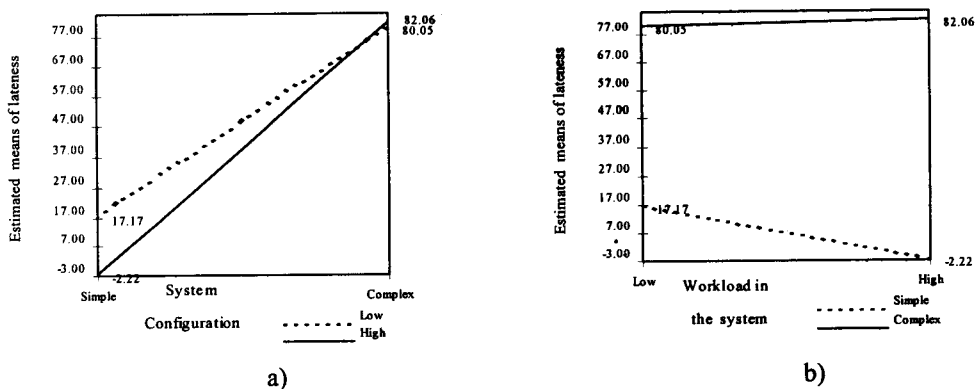
**Figure 13** Estimated means of tardiness  
a) against system configuration. b) against workload in the system.



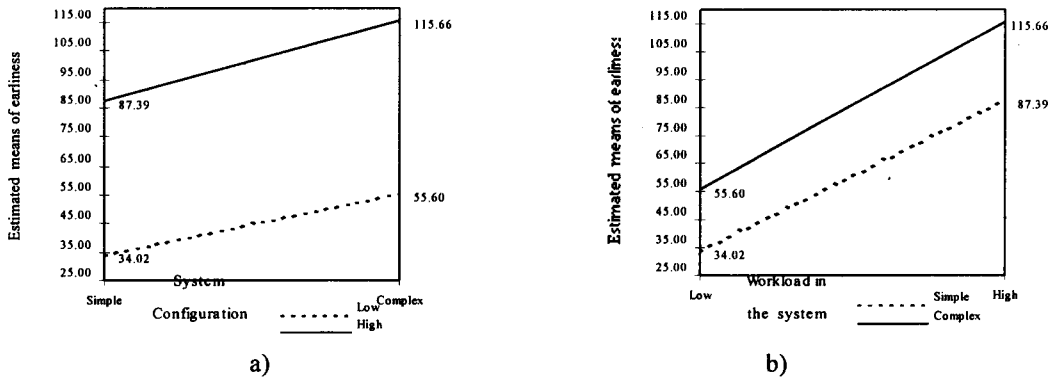
**Figure 14** Estimated means of PT  
a) against system configuration. b) against workload in the system.



**Figure 15** Estimated mean of system utilization  
a) against system configuration. b) against workload in the system.



**Figure 16** Estimated means of lateness  
a) against system configuration. b) against workload in the system.



**Figure 17** Estimated mean of earliness  
a ) against system configuration. b) against workload in the system.

The effects of system configuration and workload in system to mean lateness and system utilization show interesting results. While workload in the system is increasing, both simple and complex systems gain higher system utilization, but complex system yields lower system utilization than the simple one at high workload in system (Figure 15). The reason can be explained that, when there is excessive workload in the system, blocking and locking can easily happen and this occurrence can reduce machine utilization. The complex system fails to encounter these situations sooner than the simple system. Since the complex system has a larger number of parts circulating in the system, the congestion boosted in the complex system is more than in the simple one at the same level of workload in the system.

Figure 16 shows the relation of system configuration and workload in system to mean lateness which can be implied in terms of mean earliness as in Figure 17. It is interesting that at high workload in system and high system complexity, mean earliness is increased despite mean tardiness and the proportion of tardy jobs is high. It can be implied that the system with higher congestion tends to create early jobs with higher difference between due date and completion time.

#### 4.2 Effect of part routing rules

Figures 18 to 22 show the effects of part routing rules to the measures of performance. The part routing rules considered should be classified into 3 groups.

- FuzzyWINQ, WINQ and NINQ. The rules in this group have the best

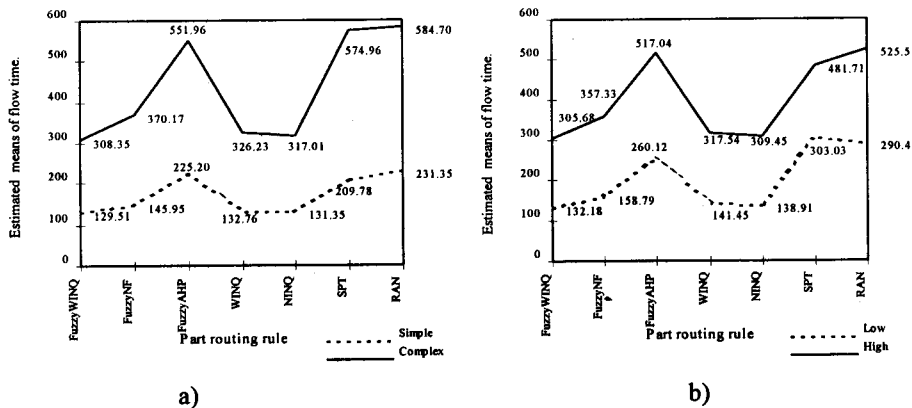
measures of performance and they maintain the best performances despite the changes of workload in the system and system configuration. FuzzyWINQ and NINQ still yield higher system utilization even though the complexity of the system is increasing. That means FuzzyWINQ and NINQ have good ability to encounter congestion in the system. For mean lateness, the rules in this group have negative mean lateness which can imply that the amount of tardy jobs is small and early jobs spend short time in the system compared with their due dates. Among the rules in the group, FuzzyWINQ shows the best performance against various performance measurements except proportion of tardy jobs.

- FuzzyNF. This rule has quite good performance, especially mean tardiness where it behaves almost as well as the rules in the first group. However, for system utilization performance, FuzzyNF has lower system utilization while the complexity of the system is increasing. Furthermore, FuzzyNF obtains the proportion of tardy jobs insensitive to the changes of system configuration and workload.
- FuzzyAHP, SPT and RAN. The rules in this group give the worst and most sensitive measures of performance compared with other groups. Mostly, all rules in this group give closed performances but there exists some cross over between them. Interestingly,

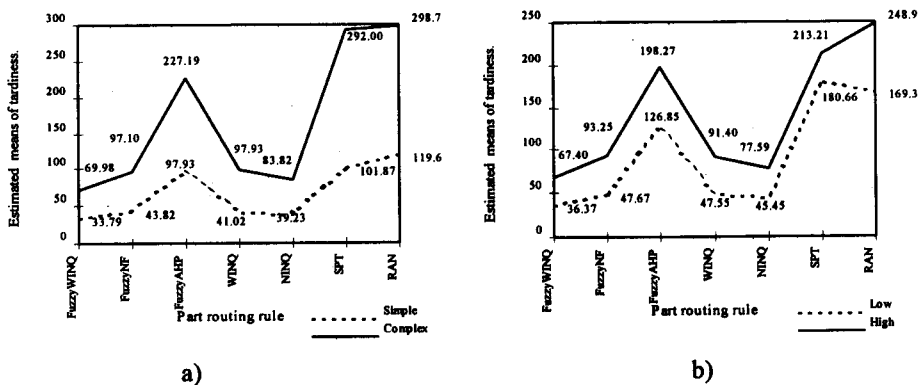
FuzzyAHP gives rather better mean tardiness compared with SPT and Random. The rules in this group give large positive mean lateness and it can be inferred that the proportion of tardy jobs is high and the early jobs finish before their due dates within short time.

Duncan's multiple range test is applied with 5% significance level to trace the best rule for each measure of performance. The results are shown in Table 5. For tardiness and system utilization, FuzzyWINQ performs significantly better than other rules. Although FuzzyWINQ has the best mean flow time and lateness, they are not significantly different from NINQ

(for flow time) and WINQ and NINQ (for lateness). In fact, the failure to reject the null hypothesis does not lead to the conclusion that there is an equality in the means. It is possible that there is not enough difference between the means for the given sample size. For the proportion of tardy jobs, NINQ performs best, but not significantly different compared to FuzzyWINQ and WINQ.

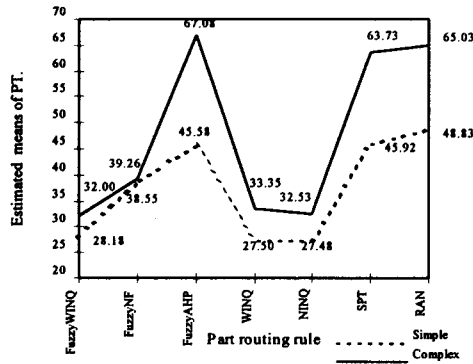


**Figure 18** Estimated means of flow time separated by part routing rules  
a) against system configuration. b) against workload in the system

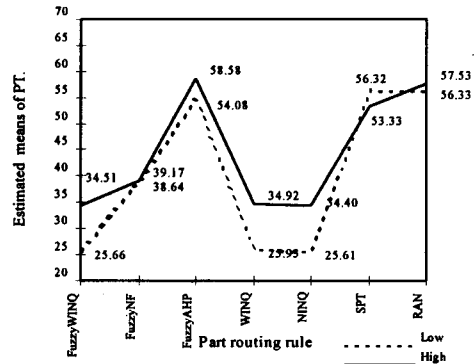


**Figure 19** Estimated means of tardiness separated by part routing rules  
a) against system configuration. b) against workload in the system



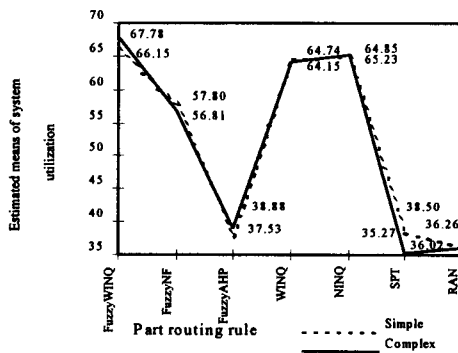


a)

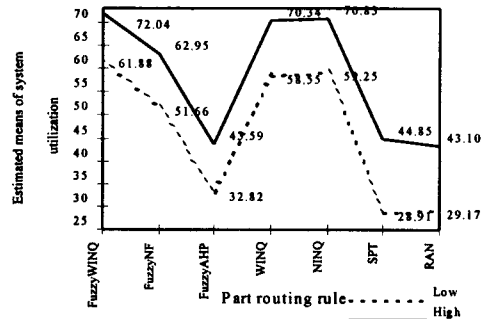


b)

**Figure 20** Estimated means of PT separated by part routing rules  
a) against system configuration. b) against workload in the system

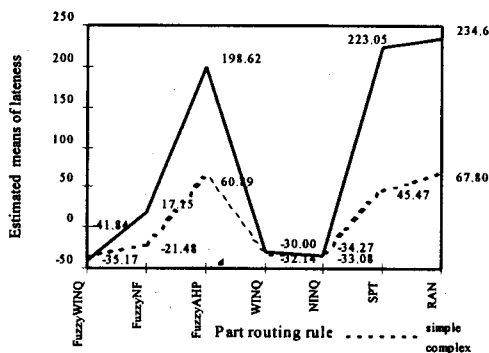


a)

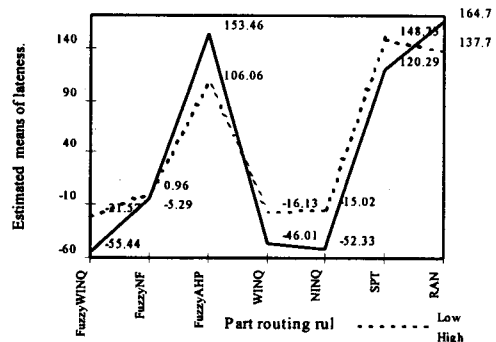


b)

**Figure 21** Estimated means of system utilization separated by part routing rules  
a) against system configuration. b) against workload in the system



a)



b)

**Figure 22** Estimated means of lateness separated by part routing rules  
a) against system configuration. b) against workload in the system

## 5. Conclusions

In this study, Fuzzy AHP is applied to develop part routing rules because of its ability to extract human experience and knowledge. By means of FuzzyAHP the burdensome mathematical model can be avoided in constructing the relationship between interested attributes expected to improve system performance. The interesting attributes are workload on machine buffer, processing time, and the probability that the part being routed to the alternate machine can be processed before the machine fails. The relationship between attributes can be dynamically changed by the urgency of the part being routed. The urgency index is slack time. This increases the robustness of FuzzyAHP.

Nevertheless, FuzzyAHP has some drawbacks since its fuzzy nature may create

improper decisions. To improve these, FuzzyAHP is extended with the more sophisticated algorithms. As a result, FuzzyNF which eliminates failed machines from the list of alternatives and FuzzyWINQ which combines FuzzyNF with WINQ are generated.

From ANOVA, FuzzyWINQ has tardiness and system utilization significantly better than the other rules. For the rest of the measures of performance, FuzzyWINQ performs satisfactorily.

The results also point out that the part routing rules considering the attribute which concerns the probability to fail of machine can improve mean tardiness even for such an inefficient rule as FuzzyAHP.

**Table 5** Duncan's multiple range analysis for various performances by different part routing rules.

Part Routing Rule	Mean Flow Time	Mean Tardiness	Mean Lateness	Proportion of Tardy Job	System Utilization
FuzzyWINQ	218.93a	51.88a	-38.50a	30.09a	66.96a
FuzzyNF	258.06c	70.46b	-2.17b	38.90b	57.30c
FMCD	388.58d	162.56c	129.76c	56.33d	38.20d
WINQ	229.50b	69.48b	-31.07a	30.43a	64.44b
NINQ	224.18a,b	61.52b	-33.67a	30.01a	65.04b
SPT	392.38d	196.93d	134.26c	54.86c	36.88d,e
Random	408.02e	209.18e	151.24d	56.93d	36.14e

(Note: a, b, c, d and e represent homogeneous group.)

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