Stability Analysis of a Fuzzy Control System

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Abstract

Fuzzy controllers are now widely used as intelligent controllers in industrial applications. Intelligent control is understood in the sense that these controllers mimic an experienced human operator. To enhance the robustness of the fuzzy controller, the stability of the fuzzy control system needs to be analyzed. In the stability analysis, the fuzzy controller is replaced with an appropriate describing function which represents the nominal model of the fuzzy controller. The Nyquist stability criteria is then performed to determine the stability of the fuzzy control system. Additionally, this analysis can predict the existence of the limit oscillation. The simulation results have shown that the presented methodology can be used to analyze the stability of a fuzzy feedback control system and predict the limit cycle.

1. Introduction

A class of a fuzzy control system is illustrated in Figure 1.

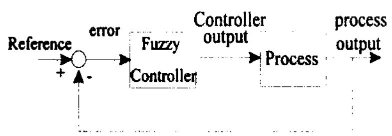

Figure 1 Fuzzy control system

Generally, a set of fuzzy control rules implemented by the fuzzy controller can be acquired from an experienced operator. The acquired knowledge is defined in terms of the fuzzy linguistic rules to achieve control of the process. The fuzzy control laws are considered nonlinearity. This makes it difficult to analyze stability. There are methodologies for investigating the stability of fuzzy control systems: the "Expert Lyapunov Function"[1], the " Energetistic Stability"[2], and the circle criterion[3], [4]. This study proposes a methodology for the stability analysis of a fuzzy feedback control system using describing function analysis and the conventional Nyquist stability analysis.

2. Preliminary Background

Fuzzy-control laws

Figure 2 shows how to define the reference fuzzy sets and membership functions [5].

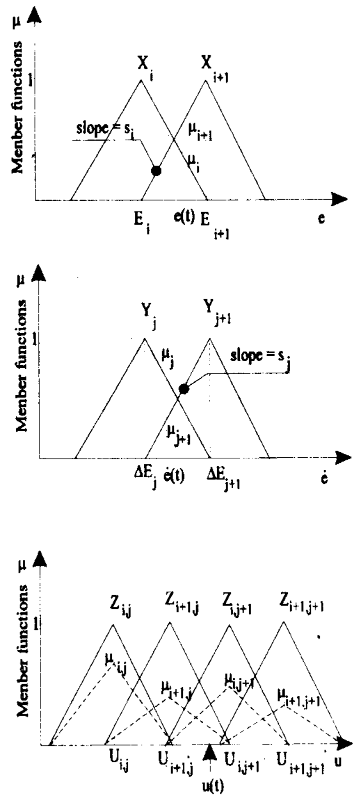

Figure 2 Membership functions associated with a set of rules for fuzzy control law.

Proportional plus derivative rule

$Rule_{i,j}$: If the error e is X_i and the rate of change in error $\dot{e}$ is Y_j,

then the controller output u is $Z_{i,j}$.

where X_i is the i^{th} set for the fuzzy variable e, Y_j is the j^{th} set for the fuzzy variable $\dot{e}$ and $Z_{i,j}$ is $(i,j)^{th}$ set for the fuzzy variable u.

Illustratively, if the error e is medium positive and the rate of change in error $\dot{e}$ is small positive then the controller output u is medium positive.

The controller output is then determined by the compositional rule of inference [6]. For proportional plus derivative rule, the mathematical expression for the fuzzy controller law u can be written as a function of the error and the rate of change in error: [7]:

$$
\begin{aligned}
u(e(t),\dot{e}(t)) = & [1-s_i(e-E_i)][1-s_j(\dot{e}-\Delta E_j)]U_{i,j} \\
& +[1-s_i(E_{i+1}-e)][1-s_j(\dot{e}-\Delta E_j)]U_{i+1,j} \\
& +[1-s_i(e-E_i)][1-s_j(\Delta E_{j+1}-\dot{e})]U_{i,j+} \\
& +[1-s_i(E_{i+1}-e)][1-s_j(\Delta E_{j+1}-\dot{e})]U_{i+1,j+}
\end{aligned}
\quad (1)
$$

Description of fuzzy logic controller

The fuzzy control laws represent a non-linear controller as illustrated in (1). Describing-function analysis[8] is now applied to transform the non-linear control laws to a corresponding describing function in order to analyze the stability. Assuming the fuzzy controller is subjected to a sinusoidal signal, $e(t) = A\sin(\omega t)$, the describing function is defined as:

$$D(A,\omega) = \frac{S_1}{A}\exp(i\phi_1) \quad (2)$$

where A is the amplitude of sinusoidal signal, ω is angular velocity, $i = \sqrt{-1}$, $S_1 = \sqrt{a_1^2 + b_1^2}$, $\phi_1 = \tan^{-1}\left(\frac{a_1}{b_1}\right)$, a_1 and b_1 are the coefficients of cosine and sine in the fundamental harmonics of Fourier series respectively.

3. Nyquist Stability Analysis

After the fuzzy controller is placed with an appropriate describing function, the closed loop characteristic equation can be written as

$$1 + D(A,\omega)G(i\omega) = 0$$

or

$$G(i\omega) = -1 / D(A,\omega) \quad (3)$$

where G is a transfer function of process. The stability criteria [9] can be summarized as follows.

1. The system is stable, if the plot of $\{-1/D\}$ is not enclosed by G.
2. The system is unstable, if the plot of $\{-1/D\}$ is enclosed by G.
3. The system has stable or unstable limit cycle oscillation, if the plot of $\{-1/D\}$ intersects G. The values of A and ω at intersection points indicate the amplitude and frequency of the limit cycle.

4. Simulation Results

The control of liquid level in a tank is shown in Figure 3.

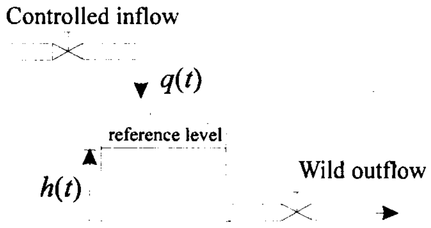

Figure 3 Control of liquid level

The control objective is to maintain a certain liquid level. The dynamics between the liquid level and the controlled flow can be written as:

$$a\frac{dh}{dt} = q - c\sqrt{h} \quad (4)$$

where h is the level of liquid, q is the controlled inflow rate, c is a constant and a is the cross-sectional area of the tank.

By applying linearization approach and Laplace transformation, the linearized transfer function is obtained as

$$\frac{H(s)}{Q(s)} = G_p(s) = \frac{1\times 10^4}{100s+1} \quad (5)$$

where the normal operating point [h=0.5 m., $q=1\times10^{-4}$ m^3/sec. a=100 cm^2 and $c=1.41\times10^4$]. The membership functions corresponding to the rule sets in Table 1 are defined in Figure 4.

Table 1 A set of fuzzy logic rules.

ė \ e	NL1	NL2	NS	Zero	PS	PL1	PL2
NL1	lnc	lnc	lnc	snc	mpc	lpc	lpc
NL2	lnc	lnc	lnc	snc	mpc	lpc	lpc
NS	lnc	lnc	mnc	cc	mpc	lpc	lpc
Zero	lnc	lnc	mnc	cc	mpc	lpc	lpc
PS	lnc	lnc	mnc	cc	mpc	lpc	lpc
PL1	lnc	lnc	mnc	spc	lpc	lpc	lpc
PL2	lnc	lnc	mnc	spc	lpc	lpc	lpc

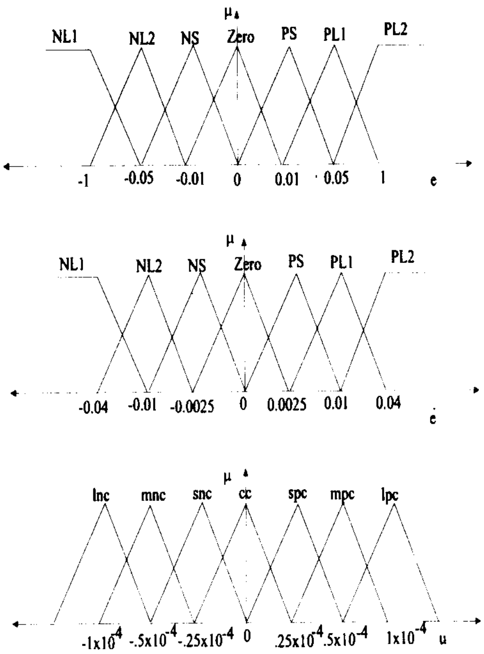

Figure 4 Membership functions of fuzzy controller

From Figure 5, the plot of {-1/D} is not enclosed by G_p. This indicates that the system will be stable according to Nyquist stability criteria.

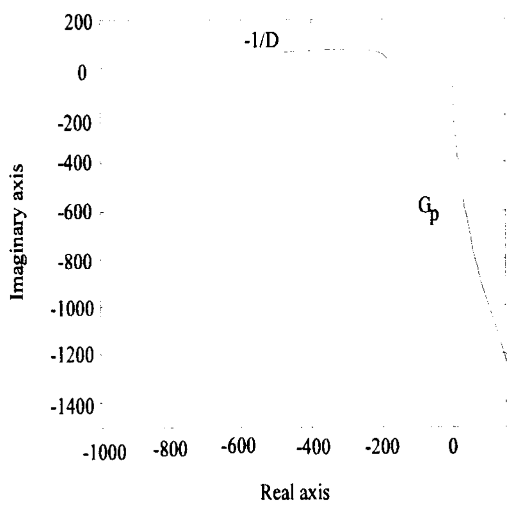

Figure 5 Nyquist plot between {-1/D} and {G_p}

For simulation as illustrated in Figure 6, the fuzzy controller tries to bring the initial level (0.2 m. higher from reference level) to the reference level. It is shown that the system is stable.

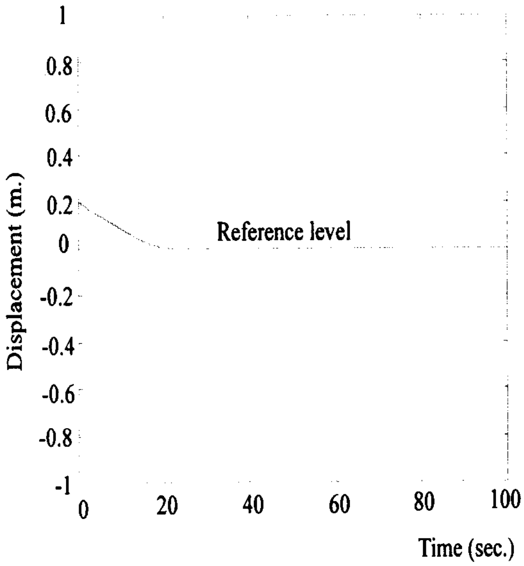

Figure 6 Stable fuzzy control system

To demonstrate the methodology in the case of limit cycle, a delay is introduced to the transfer function and its gain is increased. The new transfer function is given as

$$G(s) = \frac{2.0 \times 10^4 e^{-3s}}{100s + 1} \qquad (6)$$

By implementing the Nyquist criteria, Figure 7 indicates that the system will have a stable limit cycle with an amplitude 0.036 m. and frequency 0.095 Hz. Figure 8 shows that the limit cycle occurs with an amplitude 0.039 m. and frequency 0.098 Hz. Note that the source of error is due to the replacement of the fuzzy controller with only the describing-function.

However, the results yield some practical information to analyze stability and predict the limit cycle.

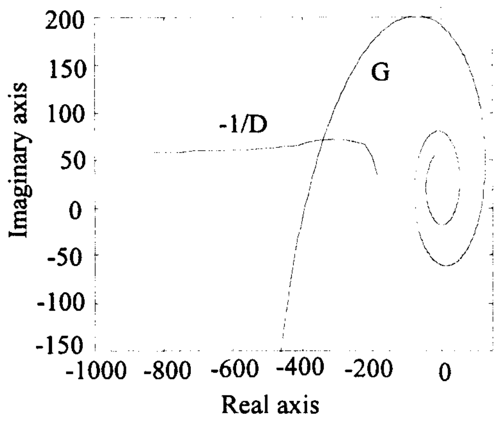

Figure 7 Nyquist plot between {-1/D} and {G}

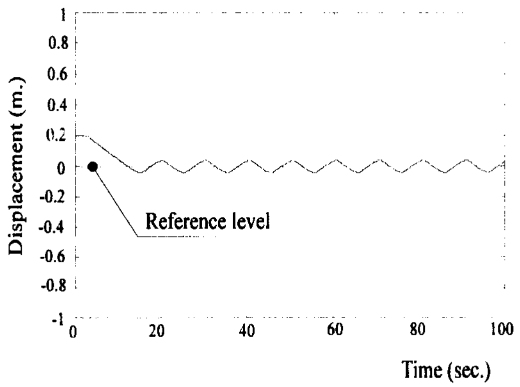

Figure 8 Limit cycle with amplitude 0.039 m.

and frequency 0.098 Hz.

5. Conclusion

The example demonstrates that the stability of the fuzzy control system can be analyzed and the occurrence of the limit cycle can be predicted by the presented methodology.

6. References

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