Production Efficiency Analysis with a Farm-Level Average-Cost Characteristic Curves

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Abstract

Farm program analysis often involves the measurement of production efficiency. This study presents a theoretical model that can be used to measure farm-level long-run and short-run production efficiency. It also provides unbiased estimates of the minimum cost of producing an output within an enterprise. The model is a computational feasible approach that can be applied to agricultural data from primary and secondary sources.

1. Introduction

Estimates of production costs for farm enterprises are conventionally estimated from average values of observed data from farm records. The reliability of cost estimates obtained from farm records data usually depends on the specific managerial decisions made by farmers, and thus may not represent the minimum averge production costs for the enterprises.

Measures of production efficiency were introduced by Farrell [1]. Contemporary models are presented by Burely [2] and Fare et. al. [3,4]. This paper presents a theoretical model that can be easily used in the empirical estimates of production efficiency based on standard farm records data. The study provides an estimation of minimum average-cost characteristic curves that can be utilized to measure the overall efficiency of the underlying production technology ; and it also provides unbiased estimates of minimum average production costs which can be used in individual farm decision-making and written budgeting context.

2. Theoretical Derivation

Neoclassical production theory states that the firm (farm) minimizes costs subject to the output and technology constraints. A serious problem concerning this analysis is the consistency of the data to be used in the empirical analysis. For example, the observed cost data obtained from farm records may not be the minimum cost to produce an output. A nonparametric approach to production analysis, based on the work of Afriat [5,6,7,8], Hanoch and Rothschild [9], Diewert and Parkan [10,11,12], and Varian [13,14,15], provides the conditions necessary and sufficient to assure that a specific cost-data series is consistent with the theoretical concept. Similar work in consumer demand analysis can be found in Afriat, Diewert, Diewert and Parkan, and Varian. In the following section, the necessary and sufficient conditions provide a test for determining if the observed data are consistent with the cost-minimization model. Varian (p. 60-74), and Hanoch and Rothschild (p.259-260, 266-267) provide detailed discussions of the theorem and its proof. This paper illustrates how these necessary and sufficient conditions can be utilized in an anlysis of farm records data.

Suppose that a farm produces outputs from various combinations of factor inputs. Let the list $(\mathbf{W}^k, \mathbf{X}^k, y^k)$ be the observed data for the farm. That is, the product produced by the farm in amount y^k utilizing the mix of factor inputs $\mathbf{X}^k$ based on a specific technology. And factor prices are represented by $\mathbf{W}^k$. The $\mathbf{X}$ and $\mathbf{W}$ are nonnegative i-vectors of factor-inputs and prices, respectively. Thus, y^k is a k*1 column vector of outputs associated with a k*i matrix of factor inputs and factor prices. The conditions that assure that the ($\mathbf{W}^k$, $\mathbf{X}^k$, y^k) are consistent with the cost-minimization model follow.

Technologically-feasible choices for the farm are represented by the production possibilities set **Y**, a subset of $\mathbf{R}^n$. A restricted production-possibility set can be described by an input-requirement set V(y). The input-requirement set V(y) contains all factor-input vectors **X** that can produce at least y units of output due to the farm's technology. Suppose the farm produced only one output of the amount y^k from the factor-input vectors $\mathbf{X}^k$. Thus the netput bundle for the farm is written as (y^k, $-\mathbf{X}^k$). Then the restricted, technologically-feasible choices for the farm are written as

(1) $V(y^k) = \{\mathbf{X}^k \text{ in } R_+^n : (y^k, -\mathbf{X}^k) \text{ is in } Y\}$

Furthermore, if the decisionmaker's objective is to minimize the cost of producing an output level y^k when factor prices are $\mathbf{W}^k$. Then there exists a population of input-requirement sets

(2) $\{V(y)\} = \{\mathbf{X}^k \text{ in } R_+^n : \min \mathbf{W}^k\mathbf{X}$ subject to $\mathbf{X}$ is in $V(y^k)\}$

based on the decisionmaker's objective for k = 1,..., n. Therefore, a necessary condition for the population of input-requirment sets {V(y) to rationalize (in the sense of cost minimization model) the cost-data is

(3) $\mathbf{W}^k\mathbf{X} \geq \mathbf{W}^k\mathbf{X}^k$ for all **X** in V (y^k).

Furthermore, {V(y)} are linked in the following sense :

(4) If **X** is in V(y) and $y \geq y'$, then
X is in $V(y')$.

In addition, if free disposal is assumed, then the input-requirement set for the farm should be positive monotonic in the following sense :

(5) If **X** is in V(y) and $\mathbf{X}' \geq \mathbf{X}$, then
$\mathbf{X}'$ is in V(y).

Furthermore, in {V(y)}, if $y^l \geq y^k$, then equation (4) implies $\mathbf{X}^l$ is in $V(y^k)$. Since $V(y^k)$ is the cost-minimization imput-requirement set. It follows that

(6) $\mathbf{W}^k\mathbf{X}^l \geq \mathbf{W}^k\mathbf{X}^k$.

Thus the population of linked input-requirement sets from the cost-minimization model implies

(7) If $y^l \geq y^k$, then
$\mathbf{W}^k\mathbf{X}^l \geq \mathbf{W}^k\mathbf{X}^k$ for all k and l.

[Given {Zi} ε $\mathbf{R}^n$, the convex positive monotonic hull (com^+ {Zi}) is the convex hull of {Zi + ei} for all ei ≥ 0. And the convex negative monotonic hull (com^- {Zi}) is defined as the convex hull of {Zi + ei} for all ei ≤ 0.]

Suppose the cost-data ($\mathbf{W}^k$, $\mathbf{X}^k$, y^k) satisfy the condition in equation (7). Let V(y) be the convex positive monotonic hull of the $\mathbf{X}^k$ such that $y^k \geq$ y. Therefore, $V(y) = com^+ \{\mathbf{X}^j : y^j \geq y\}$. If there are no $y^k \geq y$ then let V(y) = 0. Thus, $V(y^k)$ is a convex set (polytope), and the vertices of $V(y^k)$ are some subset of $\{X^l: y^l \geq y^k\}$. Therefore, these X^l's satisfy

(8) $\mathbf{W}^k\mathbf{X}^l > \mathbf{W}^k\mathbf{X}^k$

by equation (7). Since $V(y^k)$ is a convex set, and the vertices of $V(y^k)$ satisfy the condition described by equation (8). It follows that for any $\mathbf{X}^k$, $\mathbf{W}^k\mathbf{X} \geq \mathbf{W}^k\mathbf{X}^k$ for all **X** in $V(y^k)$. Hence, if the cost data satisfy equation (7), then the cost-data series were generated by a cost-minimization model for a population of linked input-requirement sets $\{V(y^k)\}$. Varian refers to this condition equation (7) as the **Weak Axiom of Cost Minimization (WACM).**

In short, if the cost data were generated by a cost-minimization model, then these cost data can be rationalized by the population of input requirement sets $\{V(y^k)\}$ thus satisfying (7). Furthermore, the sufficient condition is that {V(y)} consists of all V(y) that are nontrivial, closed, convex, and monotonic in each y.

The conceptual model suggests that a family of (minimum) average cost curves can be estimated from a population of input-requirement sets. However, in the farm program analysis, the minimum average-cost characteristic curves of an enterprise are especially interesring. This is because the relationship between (minimum) average-cost and production-output may be obtained empirically for any scale of operation of a given enterprise. Consequently, a family of minimum average-cost characteristic curves can be sketched from this relationship for the given enterprise.

A typical family of (minimum) average-cost characteristic curves for an enterprise is shown in Figure 1. The shape of the average-cost curves are obtained from the neoclassical production theory (see Varian p. 43-44). In Figure 1, the curves are sketched for several different scale of production operation for grain farms enterprise. The lower envelope of these curves is also a part of the long-run average-cost curve for the enterprise.

The characteristic curves in Figure 1 can be mathematically described by a natural exponential function. That is, the (minimum) average cost can be defined as

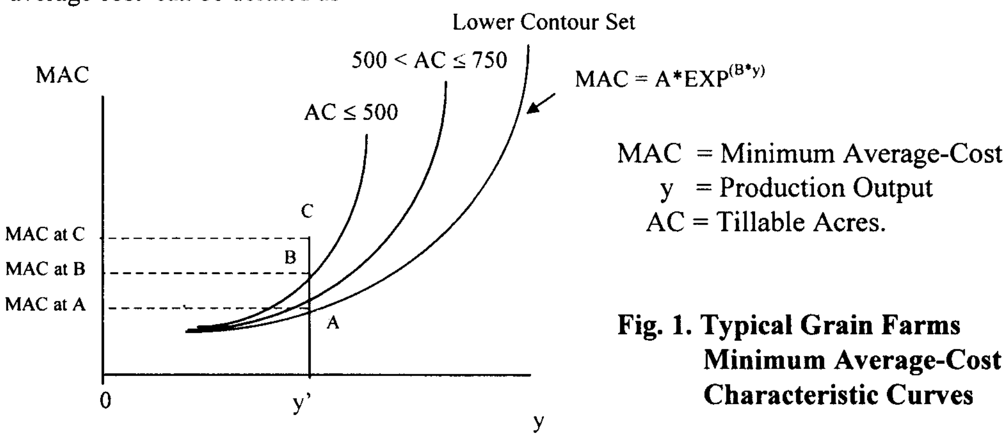

Fig. 1. Typical Grain Farms Minimum Average-Cost Characteristic Curves

(9) $MAC = A*EXP^{(B*y)}$

where :

MAC = the (minimum) average cost,

A = a constant at a point where short-run averge costs curve tangents to a long-run average costs curve,

B = the rate of growth of the (minimum) average cost,

y = the production output.

Since the minimum average-cost function represents a cost-minimizing point on an isoquant, it provides the optimal choice for a specific level of output. Thus, given a specific enterprise, any firm producing an output with average costs greater than the minimum average costs is producing at a point that is less than fully efficient. For example, the optimal production efficiency (100%) of a 500 acre grain farm with a level of production y' is at point A (long-run effciency), and at point B (short-run efficiency) in Figure 1. Suppose the grain farm produced at point C, the production efficiencies are:

(10) PES = (MAC at B) *100/(MAC at C), and

(11) PEL = (MAC at A) *100/(MAC at C)

where :

PES = % of short-run production efficiency, and

PEL = % of long-run production efficiency.

3. Data and Empirical Results

To demonstrate the imple-mentation of the concept detailed in the previous section, an empirical analysis is performed. The empirical analysis utilizes data on 99 grain farms located in the Ohio Valley of Kentucky. The data correspond to the 1981 calendar year as compiled by the Kentucky Farm Business Analysis Program. The four major input categories used in the analyses are soil fertilizer (F), pesticides (P), machinery repairs (M), and fuel and oil (O).

Data from the business management program provide information about tillable acreage, crop production, and expenses of each factor input used by the enterprise. The observed cost data ($\mathbf{W}^k$, $\mathbf{X}^k$, y^k) used in this study are defined as follows :

(12) $y^k = \left(\sum_{j=1}^{4} TR^k j\right) / AC^k$

(13) $TR^k j = PR^k j * Pj$

(14) $X^k i = TE^k i / AC^k$

(15) $W^k i = TR^k / (Ui * AC^k)$

(16) k = 1, 2,...,n ; i = F, P, M, O;
j = 1 (corn), 2(white corn), 3(soybeans), 4(wheat) ;

where :

y^k = the k^{th} production output,

$X^k i$ = amount of factor input i used to produce the k output,

$W^k i$ = amount of factor price j associated with $X^k i$,

$TR^k j$ = total returns of the j product,

$TE^k i$ = total expenses of the factor input i,

$PR^k j$ = total production of the j product,

Pj = average crop price of j received by farmer,

Ui = average cost per tillable acre paid by farmer,

AC^k = total tillable acreages used to produce y^k.

The average crop prices (Pj) and the average cost per tillable acre (Ui) in Ohio Valley are reported in Table 1 and 2 respectively.

Through a simple algorithm, **WACM** reveals that the 99 farms participating in the Kentucky Farm Business Analysis Program in the Ohio Valley could be classified into twenty rationalized population subsets. Each set of the rationalized population is consistent with the cost minimization model. That is, they are the input-requirement sets {V(y)} that satisfy the condition equation (7). Table 3 reports the rationalized population that is the lower contour of the twenty sets.

Estimates of minimum average-cost characteristic curves were obtained for the Ohio Valley grain farms in 1981. The statistical model used in the empirical analysis is

(17) $MAC^k = A*EXP^{(B*yk)} + \varepsilon^k$

where ε^k is a random error with mean 0 and variance σ^2. Since the error term ε^k is uncorrelated with the regressors, a BLUE estimate of the parameters A and B can be obtained via ordinary least squares (OLS) estimation according to Gauss-Markov theory.

Although there exist several OLS estimation techniques, the nonlinear-iterated OLS was used in the empirical analysis. (A detailed discussion of the estimation method can be found in SAS/ETS User's Guide : Econometrics and Time Series Library, 1985 Edition, p 506-534.)

For brevity, estimates of minimum average-cost characteristic curves were obtained using equation (17), and the results for the lower contour average-cost characteristic curve is reported in Table 4. This lower contour average-cost curve is the lower envelope of all the minimum average-cost characteristic curves for the Ohio Valley grain farms for 1981, and is also a segment of the long-run average costs curve of the enterprise (Varian p. 44).

In order to make direct comparison between the estimates resulting from rationalized population and the estimates resulting from unrationalized population which is the population of input requirement that was not generated by the cost minimization model, equation (17) was estimated again with the given observed data series ($\mathbf{W}^k$, $\mathbf{X}^k$, y^k). The estimated results for unrationalized population are presented in Table 5. The estimate of the rate of growth in the average costs is statistically insignificant. In contrast, the signs of the estimated coefficients from the rationalized population (data in Table 3) are consistent with **a priori** expectations and singnificant at the 0.01 level.

4. Summary

The study shows that the cost minimization model can be utilized to determine an unbiased minimum average-cost characteristic curve of a production technology. However, this minimum average-cost curve involves the relationship between the minimum costs of producing a specific output and the level of optimal output produced from a certain technology. Thus, the optimal production efficiency is determined.

Efficiency of a farm operation is a consideration, especially when the actual production costs are greater than the minimum costs of production for the enterprise. If the production efficiency is defined as the percentage of the ratio of optimal production costs to the actual costs of producing an output. Then, this study also shows that individual farm record data can be used for an analysis of farm-level efficiency employing a nonparametric approach, and that unbiased estimates of minimum average production costs obtained from such an approach can be utilized to evaluate the performance of an individual farm.

Item	Price ($/bushel)
Yellow Corn	3.09
White Corn	4.12
Soybeans	7.15
Wheat	3.90

Table 1. Average Crop Prices (Pj), Ohio Valley, 1981.*

Factor Inputs	Cost-per Tillable acre ($)				
	Under 500 (acres)	500-749 (acres)	750-999 (acres)	1000-1499 (acres)	1500-OVER (acres)
Soil fertility	44.45	38.85	41.74	42.76	42.13
Pesticide	19.96	19.96	18.13	21.00	23.93
Machinery repairs	15.83	13.42	15.15	14.93	11.57
Fuel and oil	17.16	16.66	15.68	15.77	12.94

Table 2. Ohio Valley Grain Farms, average cost per tiallable acre (Ui).*

Output (y)	Factor Inputs (Xi)				Factor Prices (Wi)				Avg. Costs
(acres)	(F)	(P)	(M)	(O)	(F)	(P)	(M)	(O)	(MAC)
159	10	1	8	6	4	8	11	10	1.2327
165	6	6	9	6	4	7	14	13	1.6364
166	16	6	8	10	4	7	14	13	2.0964
303	9	5	7	16	7	15	19	18	1.8449
328	20	15	10	11	8	14	28	25	2.8201
351	24	8	13	15	8	18	22	21	2.8054
395	41	16	11	13	9	16	34	31	3.5494
450	55	31	7	12	11	25	30	29	4.3067

Note : **WiXj ≥ WiXi** for all yj ≥ yi (by the **Weak Axion of Cost** Minimization), units of F, P, M, O, and MAC are in $/tillable acre.

Table 3. Inputs and Outputs of a Rationalized cost-data, Lower contour set, Ohio Valley Grain Farms, 1981.*

* Source : The Kentucky Farm Business Analysis, 1981.

Sysnlin Procedure
OLS Estimation
Nonlinear-iterated OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R^2
MAC = EXP^{By}	1	7	0.94682	0.13526	0.36778	0.8737

Nonlinear-iterated OLS Parameter Estimates

Parameter	Estimate	Approx STD. Error	'T' Ratio	Approx Prob>!T!
B	.0031241	.00013188	23.69	0.0001

Table 4. Estimates of Ohio Valley Grain Farms Minimum Average-Cost Characteristic Curve Coefficients, Lower Contour Set in 1981

Sysnlin Procedure
OLS Estimation
Nonlinear-iterated OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root	MSE	R^2
$MAC=A1EXP^{B1y}$	2	97	799.05	8.23761		2.87012	0.0033

Nonlinear-iterated OLS Parameter Estimates

Parameter	Estimate	Approx STD. Error	'T' Ratio	Approx Prob>!T!
A1	5.29581	1.48384	3.57	0.0006
B1	-0.000639	.00095038	-0.67	0.5029

Tables 5. Estimates of Ohio Valley Grain Farms Average-Cost Characteristic Curve Coefficients in 1981

5. References

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