ตัวแบบกำหนดการเชิงจำนวนเต็มสำหรับลดจำนวนชนิดของกล่องบรรจุภัณฑ์

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บทคัดย่อ

ในกระบวนการบรรจุภัณฑ์อาจมีการใช้กล่องหลายชนิดถ้าโรงงานอุตสาหกรรมมีสินค้าหรือผลิตภัณฑ์หลายชนิด ค่าใช้จ่ายสำหรับการใช้กล่องหลายชนิดอาจทำให้ต้นทุนในการผลิตสูงขึ้น แต่ต้นทุนการผลิตสามารถลดลงได้หากทำการลดจำนวนชนิดของกล่องที่ต้องใช้ด้วยการแทนกล่องชนิดหนึ่งด้วยกล่องชนิดอื่นที่มีอยู่แล้วภายใต้เกณฑ์ที่เหมาะสม ในงานวิจัยนี้เราใช้เกณฑ์ที่ว่ากล่องชนิดหนึ่งสามารถถูกแทนด้วยกล่องที่ใหญ่กว่า (ซึ่งหมายถึงใหญ่กว่าในทุกๆด้าน) ได้ถ้าเปอร์เซ็นผลต่างความยาวด้านแต่ละด้านของกล่องเมื่อเทียบกับกล่องที่ใหญ่กว่ามีค่าไม่เกินค่าค่าหนึ่ง เรานำเสนอตัวแบบกำหนดการเชิงจำนวนเต็มสองตัวแบบซึ่งเกี่ยวข้องกับการลดจำนวนชนิดของกล่องภายใต้เกณฑ์ดังกล่าว ตัวแบบแรกคือกำหนดการเชิงจำนวนเต็มแบบผสมซึ่งต้องการทำให้ขอบเขตของเปอร์เซ็นผลต่างความยาวของแต่ละด้านของกล่องเมื่อเทียบกับกล่องที่ใหญ่กว่ามีค่าน้อยที่สุดเมื่อกำหนดจำนวนกล่องที่ต้องการเลิกใช้มาให้ ตัวแบบที่สองคือกำหนดการเชิงจำนวนเต็มแบบไบนารีซึ่งต้องการทำให้จำนวนชนิดของกล่องลดลงให้มากที่สุดเมื่อกำหนดขอบเขตของเปอร์เซ็นผลต่างความยาวของแต่ละด้านของกล่องเมื่อเทียบกับกล่องที่ใหญ่กว่ามาให้ โดยตัวแบบทั้งสองนี้สามารถเป็นทางเลือกในการลดจำนวนชนิดของกล่องในการบรรจุภัณฑ์ในอุตสาหกรรมได้ นอกจากนี้มีการนำเสนอตัวอย่างเชิงตัวเลขเพื่อแสดงให้เห็นการใช้งานของตัวแบบและแสดงให้เห็นถึงการนำผลเฉลยที่ได้ไปใช้ในกระบวนการตัดสินใจเพื่อลดจำนวนชนิดของกล่อง

คำสำคัญ: ชนิดของกล่อง, การบรรจุ, กำหนดการเชิงจำนวนเต็ม

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Integer Linear Programming Models for Reduction of Number of Types of Packing Boxes

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Abstract

In packing process, many types of boxes may be used if a factory has several kinds of goods or products. The costs of having many types of boxes can contribute to higher manufacturing costs. However, the costs can be lowered if the number of types of boxes is reduced by substituting one type of box with another available type of box within reasonable criteria. In this paper, we use a criterion that a particular type of box can be replaced by a bigger box (bigger in all dimensions) if the difference in percentage of the length of each side of the box compared with a bigger box does not exceed a specific value. We propose two integer linear programming models which relate to reducing the number of types of boxes under this criterion. The first model is a mixed integer linear programming model which is to minimize the value of the bound of the difference in percentage of the length of each side of the box compared with a bigger box, with the specified number of boxes to be discarded. The second model is a binary integer linear programming model which is to minimize the number of types of boxes when the bound of the difference in percentage of the length of each side of the box compared with a bigger box is specified. Each model can be an option to reduce the number of types of boxes for any industrial packing. Moreover, the numerical examples are given to illustrate the use of the models and to illustrate how their findings can be used in the decision process to reduce the number of the types of boxes.

Keywords: Types of boxes, Packing, Integer Linear Programming

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1. Introduction

Many kinds of products or goods in a factory are packed into boxes every day. Most industries which have several kinds of goods usually use many types of boxes for packing, which leads to a high packing cost. In fact, there are several mathematical problems which relate to boxes and can be considered in product packing to reduce the packing cost such as the packing optimization problem and the container loading problem.

The packing optimization problem is to seek a minimal container which can hold a given number of smaller rectangular boxes. It relates to product packing if we view the products or goods as these smaller rectangular boxes and view the minimal container as a box with optimal dimension for packing the products or goods. There are many research studies on the packing problem such as [1 – 3]. In these papers, the objective is to minimize container volume which can hold a given number of smaller rectangular boxes.

The container loading problem is to arrange a given number of rectangular boxes into a large rectangular container so as to meet some objectives. It also relates to product packing if we view the products as smaller boxes which are arranged optimally into the container which is viewed as a packing box. Examples of research studies on the container loading problem can be found in [4 – 7]. Chen *et al*. [4] provided a mixed integer linear programming formulation to solve the container loading problem. The objective is to select a number of containers to pack a given set of boxes. Huang *et al*. [5] considered the container loading problem which aims to pack a given set of nonuniform-size rectangular boxes into a large rectangular container such that the length of the occupied space in the container is minimized and proposed an effective heuristic algorithm for solving the container loading problem based on this objective function. Other literatures have a different objective which is to pack as many boxes as possible into a container of fixed dimensions or to maximize volume utilization [6 – 7]. This objective is equivalent to minimizing the waste space in the container.

However, the problem which relates to reducing the number of types of boxes when there are too many types of packing box has never been mentioned in the literature. This problem is also important to manufacturers since it would be more efficient in the aspects of the cost reduction if the number of types of packing boxes can be reduced in product packing.

The remaining of this paper is organized as follows. In Section 2, two objectives for reduction of types of packing boxes are introduced. The proposed models are explained in Section 3. Numerical examples are shown in Section 4. The discussions are in Section 5. Finally, the conclusions are in Section 6.

2. Model objectives

Assume that we have a set of different box types for packing goods. For simplicity, we will call “box type i” as “box i” and “boxes of different sizes” as “boxes”. We assume that box i can substitute box j if and only if box i is bigger than box j in all dimensions within a specific bound expressed as a percentage of the larger one. This is assumed to make sure that all goods that fit in the smaller box can also be packed in the larger box. According to this criterion, we consider two objectives as follows.

Objective 1

The first objective is to minimize the percentage bound when a user imposes the number of boxes that he/she wants to reduce or discard as well as to find which boxes could replace these discarding boxes. This can also check whether the reduction is possible and keep the difference of each side between the substituting box and the discarding box to be relatively small so that the substituting box is not much larger than the discarded box.

Objective 2

Another objective is to minimize the number of boxes when the bound percentage is given by the user as well as to find which boxes are discarded and which boxes substitute them. This objective also implies the number of discarded boxes within the given percentage bound.

From Objectives 1 and 2, we aim to model each problem as a mathematical model. We will use the definitions of the height, length, and width of a box according to the definitions in Ministry of Industry Announcement No.4432 (2012) [8]. All other requirements about the qualities of box manufacturing stated in [8] are not relevant to our study since we aim only to reduce the number of existing packing boxes based on their sizes. The definitions of the height, length, and width according to [8] are as follows:

- The height of a box refers to the only dimension without a flap.
- The length of a box is always the longest side of the box that has a flap.
- The width of a box is the second longest side of the box that has a flap.

Fig. 1 displays an example of a box with the height, length, and width indicated.

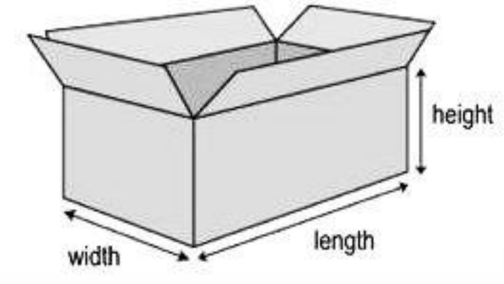

Fig. 1. Height, length, and width of a box.

3. Proposed Models

Given a set of n different box types $N=\{1,2,...,n\}$, let $L_i, W_i,$ and H_i be the length, width, and height of box i, $i \in N$, respectively. From both objectives, if we want to reduce the number of boxes by discarding a box, we have to use another box for substituting it. Box i can substitute box j if box i is bigger than box j in all dimensions and the size difference in percentage of the larger one on each box side does not exceed a specific percentage, say $100x\%$ ($x \in [0,1]$). It means that if $L_i - L_j \geq 0$, $W_i - W_j \geq 0$, and $H_i - H_j \geq 0$ as well as $\frac{L_i - L_j}{L_i} \leq x$, $\frac{W_i - W_j}{W_i} \leq x$, and $\frac{H_i - H_j}{H_i} \leq x$, then we can use box i in place of box j.

3.1 Model for Objective 1

In this subsection, we propose a mixed integer linear programming (MILP) model for Objective 1 (Model 1). The parameters and variables used in this model are defined below.

Parameters:

n	The total number of given boxes.
N	The set of all boxes $N=\{1,2,...,n\}$.
M	An arbitrary large positive number.
ε	An infinitesimally small positive number.
L_i, W_i, H_i	Parameters indicating the length, width, and height of box i, respectively.

m The total number of boxes to be discarded.

Variables:

x Decision variable implying the bound of the difference percentage for each side between of a discarded box and its substituting box.

T_{ij} Decision binary variable which is equal to 1 if box i substitutes box j.

$y_{ij}, a_{ij}, b_{ij}, c_{ij}, d_{ij}$ Auxiliary binary variables.

In this model, the variable T_{ij} is defined only when $i \neq j$. The proposed MILP model can be stated as follows.

Model 1

$$Min \quad x \tag{1}$$

$$s.t. \quad x \le 1 \tag{2}$$

$$[|L_i - L_j| + |W_i - W_j| + |H_i - H_j|] - [(L_i - L_j) + (W_i - W_j) + (H_i - H_j)] \le M(1 - y_{ij}) \quad \forall i, j \in N,\ i \neq j \tag{3}$$

$$T_{ij} \le y_{ij} \quad \forall i, j \in N,\ i \neq j \tag{4}$$

$$(L_i - L_j) - x(L_i) \le M(1 - a_{ij}) \quad \forall i, j \in N,\ i \neq j \tag{5}$$

$$T_{ij} \le a_{ij} \quad \forall i, j \in N,\ i \neq j \tag{6}$$

$$(W_i - W_j) - x(W_i) \le M(1 - b_{ij}) \quad \forall i, j \in N,\ i \neq j \tag{7}$$

$$T_{ij} \le b_{ij} \quad \forall i, j \in N,\ i \neq j \tag{8}$$

$$(H_i - H_j) - x(H_i) \le M(1 - c_{ij}) \quad \forall i, j \in N,\ i \neq j \tag{9}$$

$$T_{ij} \le c_{ij} \quad \forall i, j \in N,\ i \neq j \tag{10}$$

$$\sum_{\substack{\forall i \in N \\ i \neq j}} T_{ij} \le 1 \quad \forall j \in N \tag{11}$$

$$T_{ij} - 1 \le M(1 - d_{ij}) - \varepsilon \quad \forall i, j \in N,\ i \neq j \tag{12}$$

$$T_{jk} \le d_{ij} \quad \forall i, j, k \in N,\ i \neq j \neq k \tag{13}$$

$$T_{ki} \le d_{ij} \quad \forall i, j, k \in N,\ i \neq j \neq k \tag{14}$$

$$\sum_{\forall i \in N} \sum_{\substack{\forall j \in N \\ i \neq j}} T_{ij} = m \tag{15}$$

$$x \ge 0;\ T_{ij}, y_{ij}, a_{ij}, b_{ij}, c_{ij}, d_{ij} \in \{0,1\}.$$

The objective function (1) is to minimize the bound percentage and still allows the total number of discarded boxes to equal m. Inequality (2) ensures that $100x\%$ is a valid percentage. Constraints (3) – (4) ensure that if box i is not bigger than box j in all dimensions, then T_{ij} is forced to be zero. This is because if box i is bigger than box j, it means that $L_i - L_j \ge 0$, $W_i - W_j \ge 0$, and $H_i - H_j \ge 0$, then the left-hand side (LHS) of constraint (3) will be 0 and the variable y_{ij} in constraint (3) can be 0 or 1. Hence, by constraint (4), T_{ij} can be 0 or 1. On the other hand, if box i is not bigger than box j, it means that $L_i - L_j < 0$ or $W_i - W_j < 0$ or $H_i - H_j < 0$, then LHS of constraint (3) will be positive and the variable y_{ij} in constraint (3) must be 0. Hence, by constraint (4), $T_{ij} = 0$.

Constraints (5) – (6) enforce that if the length of box i is greater than the length of box j and the difference exceeds $100x\%$ of the larger, then $T_{ij} = 0$. This is because if LHS of constraint (5) is positive, then a_{ij} in constraint (5) can be only 0. Hence, by constraint (6), $T_{ij} = 0$. On the other hand, if LHS of constraint (5) is negative or zero, then a_{ij} in constraint (5) can be 0 or 1 and so can T_{ij} by constraint (6). However, if LHS of constraint (5) is negative but $L_i - L_j < 0$, it means the length of box i is not large enough to replace box j. Therefore, by constraints (3) – (4), T_{ij} is forced to be 0 eventually.

Constraints (7) – (8) and (9) – (10) are similar to constraints (5) – (6), except that they are considering the width and the height, respectively, instead of the length. Constraint (11) ensures that if a box is to be discarded, it can be substituted by only one box. Constraints (12) – (13) ensure that if a box is discarded, then it cannot substitute other boxes. This is because if T_{ij} is equal to 1 in constraint (12) (i.e. box j is to be discarded), then the variable d_{ij} in constraint (12) must be only 0. Hence in constraint (13) $T_{jk}=0, \forall k \neq j$. Constraints (12) and (14) ensure that if box i substitutes box j, then box i cannot be substituted by other boxes. This is because if T_{ij} is equal to 1 in constraint (12), then the variable d_{ij} in constraint (12) must be only 0. Hence in constraint (14) $T_{ki}=0, \forall k \neq i$. Both constraints (12) and (14) are needed to prevent double replacement, which can violate the size difference percentage bound. For example, if box i substitutes box j and box k substitutes box i, it potentially means box k substitutes box j but the length difference in percentage between box k and box j may exceed $100x\%$. Constraint (15) requires that the total number of discarded boxes must be equal to the parameter m, which is given by the user.

The solution to Model 1 provides the minimum percentage bound that allows the total number of discarded boxes to equal m as well as the values T_{ij} that tell which box is discarded and which box substitutes it, or return infeasibility if the problem is impossible for the current set of parameters.

3.2 Model for Objective 2

In this subsection, we propose a binary integer linear programming (BILP) model for Objective 2 (Model 2). This model is essentially modified from Model 1. The variable x which is the bound of the difference percentage for each side between of a discarded box and its substituting box in Model 1 becomes a parameter given by a user in Model 2. In addition, the objective function is changed to minimize the number of types of boxes. Other parameters used in Model 2 include n, N, M, ε, L_i, W_i, H_i which have the same meaning as in Model 1. The variables used in Model 2 include $T_{ij}, y_{ij}, a_{ij}, b_{ij}, c_{ij}$, and d_{ij} which are defined in the same fashion as in Model 1. The proposed BILP model for Model 2 can be stated as follows.

<u>*Model 2*</u>

$$Min \quad n-\sum_{\forall i\in N}\sum_{\substack{\forall j\in N\\ i\neq j}} T_{ij} \qquad (1')$$

$$s.t. \qquad (3)-(14)$$

$$T_{ij}, y_{ij}, a_{ij}, b_{ij}, c_{ij}, d_{ij} \in \{0,1\}.$$

The objective function (1′) is to minimize the number of types of boxes. The constraints in Model 2 include constraints (3) – (14) from Model 1 and can be interpreted in the same fashion.

An optimal solution of Model 2 provides the minimum number of boxes as well as the values T_{ij}'s that indicate which boxes are discarded and which boxes substitute them.

The large positive number M in both Model 1 and Model 2 can be set to the value $12\max_{i\in N}\{L_i, H_i\}$ to ensure the proper requirements in constraints (3), (5), (7), (9), and (12). This value is derived from the following analysis. Let $A=\max_{i\in N}\{L_i, H_i\}$. Since the worst case or maximum value of LHS of constraint (3) will occur when $L_i-L_j<0$, $W_i-W_j<0$, and $H_i-H_j<0$. Then,

$$\text{LHS} = 2|L_i-L_j|+2|W_i-W_j|+2|H_i-H_j|,$$
$$\leq 2(|L_i|+|L_j|)+2(|W_i|+|W_j|)+2(|H_i|+|H_j|),$$
$$\leq 2(A+A)+2(A+A)+2(A+A),$$
$$= 12A.$$

If we use $M=12\max_{i\in N}\{L_i, H_i\}$, it is easy to see that this value is also large enough to eliminate constraints (5), (7), (9), and (12) when the binary variable a_{ij}, b_{ij}, c_{ij}, or d_{ij} is equal to 0.

As for the value of ε in constraint (12), we want ε to be large enough so that whenever the variable T_{ij} is equal to 1, the binary variable d_{ij} is 0 (which, in turns, forces the variable T_{jk} in constraint (13) and T_{ki} in constraint (14) to be 0). Hence, we need ε to be positive. In addition, we want ε to be small enough so that whenever $T_{ij} = 0$, the variable d_{ij} can be either 0 or 1. Hence, ε must be less than 1. Therefore, in both models we can set the value of ε to a positive number so that $0 < \varepsilon < 1$.

4. Numerical Examples

To validate the proposed models, two generated sets of boxes in Tables 1 and 2 are used with Model 1 and Model 2 and both models are solved by CPLEX version 12.6.3. There are 12 and 40 different boxes in Tables 1 and 2, respectively.

Table 1 Sizes of boxes in Dataset 1.

Box type number	Dimension (cm.)		
	Length	Width	Height
1	100.5	33.8	19.5
2	71.5	49.5	14.5
3	50.3	50.3	30
4	70.5	48.5	24.5
5	63.5	44	21.5
6	66	42	19.5
7	71	46	24
8	57	46	11
9	69	52	9.3
10	67	50.3	28
11	64	40	28.5
12	70.5	49.5	16

4.1 Results of Using Model 1

In this subsection, we apply the Model 1 in Section 3.1 to Datasets 1 and 2 by varying the values of the parameter m, which is the number of boxes that we want to discard (if possible). The solution results from the model using these two datasets are reported in Tables 3 and 4, respectively.

Table 2 Sizes of boxes in Dataset 2.

Box type number	Dimension (cm.)		
	Length	Width	Height
1	10	10	10
2	15	15	40
3	17	11	6
4	20	20	60.5
5	20.5	14.5	6
6	25	17	9
7	25	25	25
8	27.3	22.5	30
9	30	20	11
10	35	22	14
11	35.8	30.5	80
12	36	31	26
13	40	40	50
14	40	24	16
15	45	40	35
16	45	30	20
17	49	43	31
18	50	45	30
19	51	43	35
20	53.6	21.7	42
21	55	45	40
22	60	60	20
23	60	20	45
24	63.3	37.5	40
25	66	60	100
26	69	50	44
27	70	40	10
28	74.5	49.3	50
29	78	40	55
30	80	80	40
31	80	40	20
32	84	33	22
33	87.6	74	60.5
34	93	34	20
35	95	50	26
36	100	30	30
37	105	32	37
38	110	70	25
39	112.5	40.5	10
40	120	60	35

The results in Tables 3 and 4 are examples of scenarios that can help making a decision to reduce the number of boxes for any packing

Table 3 Computational results of Model 1 using Dataset 1.

m	Opt. Obj. Val. (x^*)	List of $T_{i,j}=1$
1	10.56%	$T_{7,5}$
2	18.75%	$T_{4,5}$, $T_{7,6}$
3	24.14%	$T_{4,5}$, $T_{7,6}$, $T_{2,8}$
4	Infeasible	-

Table 4 Computational results of Model 1 using Dataset 2.

m	Opt. Obj. Val. (x^*)	List of $T_{i,j}=1$
3	12.50%	$T_{14,10}$, $T_{19,15}$, $T_{19,17}$
5	18.92%	$T_{9,6}$, $T_{14,10}$, $T_{21,15}$, $T_{21,19}$, $T_{37,36}$
7	23.08%	$T_{9,6}$, $T_{14,10}$, $T_{19,15}$, $T_{19,17}$, $T_{26,21}$, $T_{35,31}$, $T_{37,36}$
10	25.00%	$T_{5,3}$, $T_{9,6}$, $T_{16,14}$, $T_{21,15}$, $T_{21,17}$, $T_{21,18}$, $T_{21,19}$, $T_{26,24}$, $T_{35,31}$, $T_{37,36}$

industries. For example, if a manager wants to reduce 1, 2, or 3 boxes in Dataset 1, he/she needs to accept the percentage bound at 10.56%, 18.75%, or 24.14% according to Table 3, respectively. Furthermore, the results also show that it is impossible to discard m boxes when $m \geq 4$ in Dataset 1. The right column of Table 3 indicates possible substitutions. For example, if we want to discard 2 boxes ($m=2$), we can discard the 5^{th} and 6^{th} boxes and use the 4^{th} and 7^{th} boxes instead, respectively. Similarly, if he/she wants to reduce 3 boxes in Dataset 2, for example, he/she needs to accept the bound percentage at 12.50% according to Table 4. He/she also know that he/she can discard the 10^{th}, 15^{th}, and 17^{th} boxes and use the 14^{th}, 19^{th}, and 19^{th} boxes instead, respectively. The results in other cases of m in Table 4 can also be explained in the similar manner.

4.2 Results of Using Model 2

In this subsection, we apply Model 2 in Section 3.2 to Datasets 1 and 2 by varying the values $100x\%$, which are the bound of the difference percentage for each side between a discarded box and its substituting box. The solution results from the model using datasets from Tables 1 and 2 are reported in Tables 5 and 6, respectively.

Table 5 Computational results of Model 2 using Dataset 1.

$100x\%$	Opt. Obj. Val.	List of $T_{i,j}=1$
10%	12	-
15%	11	$T_{7,5}$
20%	10	$T_{4,5}$, $T_{7,6}$

Table 6 Computational results of Model 2 using Dataset 2.

$100x\%$	Opt. Obj. Val.	List of $T_{i,j}=1$
10%	40	-
15%	37	$T_{14,10}$, $T_{19,15}$, $T_{19,17}$
20%	35	$T_{9,6}$, $T_{14,10}$, $T_{21,15}$, $T_{21,19}$, $T_{37,36}$
25%	30	$T_{5,3}$, $T_{9,6}$, $T_{16,14}$, $T_{21,15}$, $T_{21,17}$, $T_{21,18}$, $T_{21,19}$, $T_{26,24}$, $T_{35,31}$, $T_{37,36}$

The results in Table 5 tell that if our acceptable bound percentage is 10%, we cannot discard any boxes in Dataset 1 (we need to accept more the bound percentage so that we can reduce some boxes). But if the acceptable bound percentage is 15% or 20%, we can discard at most 1 or 2 boxes in Dataset 1 and can use only 11 or

10 boxes in product packing, respectively. The results in Table 6 can be explained similarly. For example, if our acceptable bound percentage is 20%, we can discard at most 5 boxes in Dataset 2 and use only 35 boxes in product packing. We can discard the 6^{th}, 10^{th}, 15^{th}, 19^{th}, and 36^{th} boxes and use the 9^{th}, 14^{th}, 21^{st}, 21^{st}, and 37^{th} boxes instead, respectively.

5. Discussions

The first model is useful when the user has a target number of boxes to be eliminated but is not sure if the elimination is possible or acceptable. If the solver returns an infeasible result, that means the target number is unrealistic. This suggests the user should lower his/her target number to obtain possible eliminations. If the problem is feasible, the solution of the model will give box substitution plan as well as the largest size difference in percentage between the substituting boxes and the discarded boxes. It is up to the user to decide whether the largest size difference is acceptable. If not, the user should consider lowering the target number of discarded boxes which may lead to smaller size difference.

The second model is useful when the user wants to discard as many boxes as possible while having a limit of the size difference in percentage in mind. (Certainly, the size difference in this case refers to the size difference between the substituting boxes and the discarded boxes.) Unlike the problem in the first model, the problem in this model is always feasible since a trivial solution (i.e. zero substitution) is feasible. The solution of this model will give a substitution plan as well as the number of boxes it can eliminated. If the user is not satisfied with the number of discarded box, he/she should increase the limit of the size difference to allow more possibility of further eliminations.

The two models are closely related since they share most of the constraints, but an optimal solution from one model does not necessarily imply an optimal solution of another model. This can be explained as follows. Suppose $x = x'$ (the bound of the difference percentage) is the optimal solution to Model 1 using parameter $m = m'$ (number of discarded boxes). If we use $x = x'$ as the parameter in Model 2, the value $n - m'$ may not be the minimum objective value (the number of remaining box types after substitution) of Model 2 because it is possible that, after using the substitution plan from the optimal solution of Model 1, some of the remaining $n - m'$ boxes may still have size difference percentage no larger than $100x'\%$ and therefore can be further eliminated. On the other hand, suppose $n - m''$ is the optimal objective value of Model 2 using the parameter $x = x''$. If we use $m = m''$ as the parameter of Model 1, the value $x = x''$ may not be the smallest bound for Model 1 because it is possible that the actual largest size difference percentage in the optimal substitution plan from Model 2 may be strictly less than $x = x''$ and therefore a smaller bound can be obtained for Model 1.

6. Conclusions

This paper presents two integer linear programming models which relate to reducing the number of types of packing boxes by box type substitution based on their sizes. A particular type of box could be replaced by a bigger box if the size difference in percentage of the larger one on each box side stays within a specific value. The first model has the objective to minimize the bound provided that the number of discarded box types is specified. The second model has the objective to minimize the number of types of boxes when the bound percentage is given by the user. Both models tell a user which box can be discarded and which box can substitute it. They can be used as options to reduce the number of types of boxes when there are several types of boxes to use in the product packing. A limitation of these models is they consider reducing the number of types of boxes based on their sizes only. In fact, there are other factors that may be considered in box type substitution such as the material or color of boxes, which are not considered in these models.

7. References

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