A Mathematical Model for Pollution Travelling Salesman Problem

Supakrit Nanasilp[1*] and Warisa Wisittipanich[2]

Chiang Mai University 239, Huay Kaew Road, Muang District, Chiang Mai Thailand, 50200

Received: 3 February 2020; Revised: 2 March 2020; Accepted: 3 March 2020

Abstract

This paper studies Pollution Travelling Salesman Problem (PTSP). PTSP is a generalization of the well-known Travelling Salesman Problem (PTSP) which aims to find a suitable tour that minimizes fuel consumption in liters. This paper presents a mixed integer linear programming (MILP) for Pollution Travelling Salesman Problem which its objective function considers fuel consumption in term of vehicle engine efficiency, transmission, vehicle load, vehicle speed, road angle, distance and driver wages. The decision variables of the problem include routing of a vehicle, vehicle load and speed level. In this study, the PTSP is solved by the exact algorithm with optimization solver. The experiment uses 15 generated instances with the number of cities varying from 5 to 20 cities. The numerical results obtained by LINGO show that, in small-size instances, optimal solutions are easily found within fast computing time. When the number of cities increases to 15 cities, the model takes 3 to 8 hours to find optimal solutions. Finally, when the number of cities increases to 20 cities, the model cannot find optimal solution within acceptable computing time of 10 hours.

Keywords: Pollution Travelling Salesman Problem, Mathematical Model, Travelling Salesman Problem, Minimization of Fuel Consumption

---

* Corresponding author: E-mail: supakrit_nanasilp@cmu.ac.th

[1,2] Department of Industrial Engineering, Faculty of Engineering, Chiang Mai University.

1. Introduction

Nowadays, the pollution released from vehicles, industrial factories, incinerators, etc. has increased from the past which causing many factories to obey rules and has good standards for environmental management. The overlooked is the air pollution from the transportation of motorized vehicles because it is the source of most polluted air problems that produces carbon monoxide, hydrocarbon oxides of nitrogen and sulfur. Statistically, 55% of the hydrocarbons come out of the exhaust pipe, 25% come out of the crankshaft and another 20% are caused by evaporation in the carburetor. Nitrogen oxides (nitric oxide, NO), nitrogen dioxide (NO_2) and almost all nitrous oxide (N_2O) are also come out of the exhaust pipe which directly causes toxic to humans. Besides, lead in gasoline also increases the amount of lead in the air too. In [1], Pollution Travelling Salesman Problem (PTSP) is proposed as a modified combination of Travelling Salesman Problem (TSP) and Pollution Routing Problem (PRP). The purpose of PTSP is to reduce the amount of pollution that released from vehicles. Since there are several related factors causing air pollution such as vehicle load, vehicle speed, road conditions and distance, it is more efficient to design the route considering those factors rather than those in general TSP. There are very few research on PTSP, thus, this study aims to introduce PTSP to find optimal route in order to minimize the pollution problem released from vehicle. This helps to reduce a direct impact on daily life which can cause human health problems.

2. Literature Review

Traveling salesman problem (TSP), known as TSP, originated in the 1880s by Irish mathematician Sir William Rowan Hamilton and British mathematician Thomas Penyngton Kirkman shown in [2]. TSP is a fundamental problem that focuses on finding the best value (optimization), such as the results of the shortest total distance, the lowest cost and use the least amount of time traveling. Based on 34 literature reviews, it was found that many types of research use their algorithms to solve most TSP problems. Most researchers used the exact algorithm to obtain the best solution. For example, [3] used an exact algorithm with adaptive parallelizing to solve TSP which can solve small size of problem. In [4], an exact algorithm was used to solve Equitable TSP for a small and medium size instances. However, the disadvantage of the exact algorithm is it can only solve small problems. For medium to-large size problem, the exact method is not be able to find the global optimal within suitable time. Therefore, another research approach applies the approximation algorithm to find near-best solutions such as a research in [5] which applied a Differential Evolution (DE) to solve TSP and outperforms other algorithms. Some of researches are using Hybrid algorithm to solve TSP such as a research in [6] that using Hybrid Differential Evolution Algorithm for TSP and compete with other algorithms which can outperforms a single algorithm and some hybrid algorithm. There are only one research on PTSP which was proposed by Cacchiani et al. [7] in early 2018. In their study, new parameters were introduced to the classical TSP. Then, they used an Iterated Local Search to solve the problem. The experimental results showed that the proposed algorithm can solve the problem up to 50 nodes.

3. Mathematical Model

This research used a mathematical model of PTSP proposed by Cacchiani et al [7]. The PTSP is defined on a complete directed graph $G = (N, A)$ where $N = \{0, \dots, n\}$ is the set of nodes, 0 is a depot and A is the set of arcs. The distance from node i to node j is denoted by d_{ij}. The set $N_0 = N \setminus \{0\}$ is the customer set. Each customer $i \in N_0$ has a non-negative demand q_i, and a service time t_i. According to Cacchiani et al. [7], the recommended value for all parameters in the PTSP model is showed in Table 1.

Table 1 Parameters used in the PTSP model [7]

Notation	Description	Typical value	Unit
W	Curb-weight	6350	Kg
ε	Air–fuel ratio	1	AFR
k	Engine friction factor	0.2	KJ/rev/L
N	Engine speed	33	Rev/s
V	Engine displacement	5	L
g	Gravitational constant	9.81	m/s^2
C_d	Coefficient of aerodynamic drag	0.7	-
ρ	Air density	1.2041	Kg/m^3
A	Frontal surface area	3.912	m^2
C_r	Coefficient of rolling resistance	0.01	-
η_{tf}	Vehicle drive train efficiency	0.4	-
η	Efficiency parameter for diesel engines	0.9	-
f_d	Driver wages per	0.011	Baht/s
κ	Heating value of a typical diesel fuel	44	KJ/g
ϕ	Conversion factor	737	g/s to L/s
V^l	Lower speed limit	5.5 (or 20 km/h)	m/s
V^u	Upper speed limit (m/s)	25 (or 90 km/h)	m/s

Where $\lambda = \varepsilon / \kappa\phi$ and $\gamma = 1/1000\eta_{tf}\ \eta_{tf}$ are constants, $\beta = 0.5C_d\rho A$ is a vehicle specific constant. $Distance_{ij}$ is the distance from location *i* to location *j*, T_i Is the service time it takes at the destination *i*, q_i is customer demand at the destination *i*, *D* is the summation of demand in all destination which is $D = \sum_{i \in N_0} q_i$ and $\alpha_{ij} = \tau + g\sin\theta_{ij} + gC_r\cos\theta_{ij}$ is an arc specific constant depending on the road angle θ_{ij} .

The main purpose of the PTSP is to determine the minimum polluted route that departs from the depot, visits each customer exactly once by serving its demand, and return to the depot, where the pollution is calculated by the sum of the fuel consumption in liter and driver wage.

The following decision variables for PTSP is shown as follows: [7]

$$X_{ij}\begin{cases} 1 \text{ when travel from destination i to j} \\ 0 \text{ otherwise} \end{cases}$$

$$f_{ij} = R \text{ load on vehicle (kg) when travel from destination i to j}$$

$$Z_{ij}^{r}\begin{cases} 1 \text{ when travel from destination i to j with speed level r} \\ 0 \text{ otherwise} \end{cases}$$

The MILP model for PTSP is shown as follows: [7]

Objective function:

Minimize

$$\left(\sum_{(i,j)\in A} \lambda KNVd_{ij} \sum_{r\in R} \frac{z_{ij}^r}{v^r}\right)$$
$$+\sum_{(i,j)\in A} \lambda\beta\gamma d_{ij} \sum_{r\in R} z_{ij}^r \left(v^r\right)^2$$
$$+\sum_{(i,j)\in A} \lambda\gamma\alpha_{ij} d_{ij} f_{ij} +\sum_{(i,j)\in A} \lambda w\gamma\alpha_{ij} d_{ij} x_{ij}$$
$$+f_d\left(\sum_{(i,j)\in A} \sum_{r\in R} \left(\frac{d_{ij}}{v^r}\right) z_{ij}^r + \sum_{i\in N_0} t_i\right) \quad (1)$$

Constraints:

$$\sum_{j\in N_0} f_{0j} = D \quad (2)$$
$$\sum_{j\in N_0} f_{0j} = 0 \quad (3)$$
$$\sum_{j\in N} x_{ij} = 1, \ \forall i \in N \quad (4)$$
$$\sum_{i\in N} x_{ij} = 1, \ \forall j \in N \quad (5)$$
$$\sum_{j\in N} f_{ji} - \sum_{j\in N} f_{ij} = q_i, \ \forall i \in N_0 \quad (6)$$
$$q_j x_{ij} \le f_{ij} \le (D - q_i) x_{ij}, \ \forall (i,j) \in A \quad (7)$$
$$\sum_{r\in R} z_{ij}^r = x_{ij}, \ \forall (i,j) \in A \quad (8)$$
$$x_{ij} \in \{0,1\}, \ \forall (i,j) \in A \quad (9)$$
$$f_{ij} \ge 0, \ \forall (i,j) \in A \quad (10)$$
$$z_{ij}^r \in \{0,1\}, \ \forall (i,j) \in A, \forall r \in R \quad (11)$$

The objective function in equation (1) consists of four main components to be minimized. The first component is the summation of fuel consumption (*L*) from the engine efficiency when traveling from location *i* to *j* with the selected speed level *r*, where the selected speed is an average speed between two cities without considering traffic condition. Second component is the summation of fuel consumption (*L*) calculated from the transmission efficiency and frontal surface area resistance from the wind when traveling from location *i* to *j* at the speed level *r*. Third component is the summation of fuel consumption (*L*) according to vehicle weight, vehicle load, road angle, transmission efficiency when traveling from location *i* to *j*. The last component is the amount of fuel consumption (*L*) calculated from the driver's wage when traveling at the speed level *r.* The model converges the driver's wage into fuel consumption so that the number of working hours of drivers is minimized. Constraints (2) and (3) ensure that the vehicle leaves full and returns empty at the depot. Constraints (4) and (5) guarantee that each node is visited exactly once. Constraints (6) and (7) define the load of the vehicle on each visited arc. Constraint (8) link *x* and *z* variables by imposing that exactly one speed level is chosen for each arc *(i, j)* ∈ *A*. Constraints (9) to (11) define the variable domains.

4. Experimental Results

From the mathematical model of PTSP that have been developed by [3]. This study generates 15 instances with different conditions (e.g. number of cities, vehicle load, speed level, distance between cities, road angle between cities, and services time). The mathematical model is solved by the exact method with LINGO optimization program version 18.0 on PC core i5 4560, 16gb of ram with G-force GTX1060Ti. The experimental results are shown in Table 2.

Table 2 Results from LINGO program

Instance	Number of cities	#Global Obj. (L)	Computational time (hh:mm:ss)
1	5	401.615	0:00:00
2	5	405.870	0:00:00
3	5	351.414	0:00:00
4	5	421.424	0:00:00
5	5	383.000	0:00:00
6	5	303.688	0:00:00

Table 2 Results from LINGO program (continued)

Instance	Number of cities	#Global Obj. (L)	Computational time (hh:mm:ss)
7	5	438.275	0:00:00
8	5	407.128	0:00:00
9	5	403.150	0:00:00
10	5	232.553	0:00:00
11	10	533.652	0:00:18
12	15	438.849	03:28:38
13	15	711.151	08:15:42
14	20	N/A	>10:00:00
15	20	N/A	>10:00:00

According to numerical results from Table 2, the LINGO program can find optimal routing for the problems up to 10 cities. When problem size increases to 15 cities, LINGO takes approximately 3 to 8 hours to obtain the global optimal solution. Then, when the number of cities increases to 20, LINGO cannot find the global optimal solution within an acceptable time of 10 hours. It is noted that although Cacchiani et al. [7] can solve the PTSP up to 50 nodes, the solutions are not optimal. This paper aims to find global optimal solution; however, when the problem is more complex, the optimal solution cannot be found within an unacceptable time. Therefore, the exact algorithm has limitation of solving only small size problems. Figure 1 show a relationship between the number of cities and computing time. It can be seen that the computational time increases in exponential form.

The example of optimal routing obtained from instance 1 can be illustrated in Figure 2. Table 3 shows the data of distance between cities and depot, road angle between cities (α), service time (T_i), and speed level (r).

Table 3 distance between 5 cities and depot

city/city	Depot	2	3	4	5	6
Depot	M	34	52	41	65	98
2	34	M	39	29	77	65
3	52	39	M	37	45	43
4	41	29	37	M	85	77
5	65	77	45	85	M	89
6	98	65	43	77	89	M

q = -20000 4000 4000 4000 4000 4000

D = 20000

t = 0 3600 3600 3600 3600 3600

α = 1

V = 5.5 10 15 20 22 25

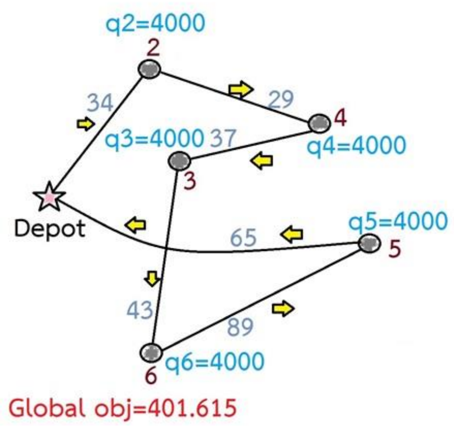

Figure 2 Optimal route for Instance 1

In this model, the main factors affecting the route choices are loads on the vehicle, road angle, and distance between cities. For example, a vehicle will try to deliver to the city with largest demand first, then it will deliver to other remaining cities.

Next, a vehicle tends to avoid the route with positive incline and choose the negative incline or zero incline condition first. Finally, the distance between cities is another factor that alternates the decision on vehicle routing.

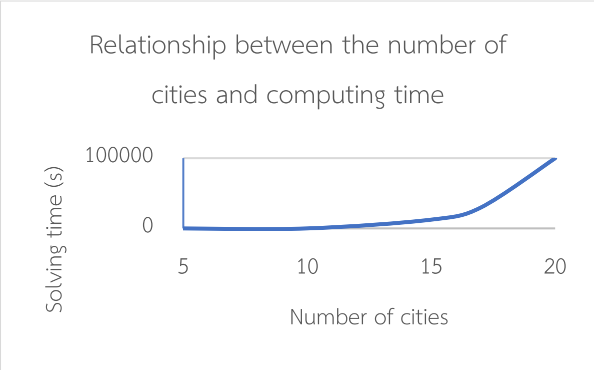

Figure 1 Relationship between the number of cities and computing time

5. Conclusion

This paper presents a mathematical model for Pollution Travelling Salesman Problem (PTSP), a generalization of the TSP, in which fuel consumption is the objective to be minimized. The Mixed Integer Linear Programming (MILP) model is solved by the exact algorithm with LINGO optimization program version 18.0 on PC core i5 4560, 16gb of ram with G-force GTX1060Ti. The experimental results of 15 generated instances shows that LINGO can find optimal solutions for the problems up to 10 cities in fast computing time. When the number of cities increases to 15, LINGO consumes 3 to 8 hours to find the optimal solution, and when the number of cities reaches 20 cities, LINGO cannot find the optimal solution within acceptable computing time of 10 hours.

Since the exact method has limitation of solving large-size problem, the future study is to apply a metaheuristic approach to deal with PTSP for medium and large problems.

Reference

[1] ปริมณภา จงจิตรกลาง, ปวิตรา สายจริงวงศ์, สาวรส จั่นขุนทด, ลักษณา วรศศิกุล, ธวัช ชาวัน, “สาเหตุและผลกระทบของมลพิษทางอากาศ,” [ออนไลน์]. แหล่งที่มา: http://www.rmuti.ac.th/user/thanyaphak/Web%20EMR/Web%20IS%20Environmen%20gr.4/Mola2.html. [วันที่เข้าถึง 11 เมษายน 2562].

[2] W. Cook, “History of the TSP,” [Online]. Available: www.math.uwaterloo.ca/tsp/history/index.html. [Accessed 11 April 2019].

[3] V. V. Burkhovetskiy and B. Y. Steinberg, “Parallelizing an exact algorithm for the traveling salesman problem,” *Procedia Computer Science.*, vol. 119, pp. 97-102, 2017.

[4] J. Kinable, B. Smeulders, E. Delcour, and F. C. R. Spieksma, “Exact algorithms for the equitable traveling salesman problem,” *European Journal of Operational Research.*, vol. 261, pp. 475-485, 2017.

[5] F. J. Ahlers, W. D. Carlo, C. Fleiner, L. Godwin, M. Keenan, R. D. Nath, et al., “Differential Evolution (DE),” [Online]. Available: https://www1.icsi.berkeley.edu/~storn/code.html#matl. [Accessed 11 April 2019]

[6] X. Wang and G. Xu, “Hybrid differential evolution algorithm for traveling salesman problem,” *Procedia Engineering.*, vol. 15, pp. 2716-2720, 2011.

[7] V. Cacchiani, C. C. Bolton, J. Escobar, L. M. E. Falcón, R. Linfati, and P. Toth, “An iterated local search algorithm for the pollution traveling salesman problem, ” *AIRO Springer Series.*, vol. 1, pp. 83-91, 2018.