ประสิทธิภาพของการประมาณค่าพารามิเตอร์เอนเอียงบางตัว สำหรับตัวประมาณ Liu ในตัวแบบการถดถอยเชิงเส้น ที่ตัวแปรอิสระมีความสัมพันธ์เชิงเส้นพหุคูณ

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บทคัดย่อ

ในการประมาณค่าสัมประสิทธิ์การถดถอยเชิงเส้นพหุ เมื่อตัวแปรอิสระมีความสัมพันธ์เชิงเส้นพหุ (Multicollinearity) จะส่งผลให้การประมาณค่าตัวแปรตามที่ได้ไม่เหมาะสม อีกทั้งมีผลทำให้ค่าประมาณสัมประสิทธิ์การถดถอยเชิงเส้นพหุด้วยวิธีกำลังสองน้อยที่สุดมีความเอนเอียงและค่าคลาดเคลื่อนกำลังสองเฉลี่ยมีค่าสูง เพื่อแก้ไขปัญหาดังกล่าวข้างต้นจึงได้มีผู้เสนอวิธีการ และพัฒนาวิธีการประมาณค่าสัมประสิทธิ์การถดถอยเชิงเส้นพหุเมื่อตัวแปรอิสระมีพหุสัมพันธ์กัน Liu Keijian (1993) ได้เสนอตัวประมาณใหม่ เรียกว่าตัวประมาณค่า Liu โดยมีค่าพารามิเตอร์เอนเอียง (d) เป็นพารามิเตอร์ที่สำคัญในการสร้างตัวประมาณ Liu ในงานวิจัยนี้ผู้วิจัยจึงศึกษาประสิทธิภาพของวิธีการประมาณค่าพารามิเตอร์เอนเอียง (d) โดยเกณฑ์ที่ใช้ในการเปรียบเทียบประสิทธิภาพของวิธีการประมาณค่าคือค่าคลาดเคลื่อนกำลังสองเฉลี่ย (MSE) ผลจากการจำลองข้อมูลและข้อมูลจริง พบว่า ตัวประมาณค่า Liu ที่ประมาณค่าพารามิเตอร์เอนเอียงด้วย D_8, D_9, D_{13} and D_{15} มีประสิทธิภาพสูงที่สุด

คำสำคัญ: การถดถอยเชิงเส้น, ตัวแปรอิสระมีความสัมพันธ์เชิงเส้นพหุคูณ, ค่าคลาดเคลื่อนกำลังสองเฉลี่ย, ตัวประมาณ Liu, พารามิเตอร์เอนเอียง

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Performance of Some Biasing Parameter Estimates for Liu Estimator in Linear Regression Model with Multicollinearity

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Abstract

In multiple linear regression models, the multicollinearity occurs when the explanatory variables in a regression model are correlated. The multicollinearity effects on the ordinary least squares estimation method since the estimated regression coefficients become unstable and difficult to interpret in the presence of multicollinearity. In order to mitigate the problem of multicollinearity, Liu regression is widely used as a biased method of estimation with biasing parameter d. The purpose of this research is to investigate the performance of some biasing parameter estimates in Liu regression in the presence of moderate to high correlation among the explanatory variables. The simulation study and application using real data have been performed to evaluate the performance of these biasing parameter estimation methods due to the mean squared errors (MSE). Based on the results from the simulation and application using real data, it can be concluded that the estimators D_8, D_9, D_{13} and D_{15} have a better performance for Liu regression.

Keywords: linear regression, multicollinearity, mean square error, Liu estimator, biasing parameter

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1. Introduction

Multiple Linear regression is the statistical procedure to predict the values of a response variable from a collection of predictor explanatory variables values. The multiple linear regression model is known as

$$Y = X\beta + \varepsilon \tag{1}$$

where Y is an $n \times 1$ vector of the response variable, X is a known $n \times p$ full rank matrix of explanatory variables, β is an $p \times 1$ vector of unknown regression parameters, ε is an $n \times 1$ vector of errors such that $E(\varepsilon)=0$, and $V(\varepsilon)=\sigma^2 I_n$, I_n is an $n \times n$ identity matrix. The commonly used method of estimation in multiple linear regression models is the method of ordinary least squares (OLS). The ordinary least squares estimator of β in equation (1) is written as

$$\hat{\beta}_{OLS} = (X'X)^{-1} X'Y \tag{2}$$

As known, using the OLS method for estimating the parameter in the regression when the assumption of linear independence among the regressors is violated which caused multicollinearity is problematic. The results obtained through this method might be misleading when the problem of multicollinearity is present among the explanatory variables (Damodar N., 2009). Many methods exist in literature to combat the multicollinearity problem. Biased estimators with one biasing parameter include the ridge regression (RR) estimator by Hoerl and Kennard (Hoerl et al., 1970) and the Liu estimator by Liu Kejian (Liu, 1993), among others. In addition, since the RR estimator is not linear in terms of this tuning parameter, tuning parameter is difficult to choose. Therefore, Liu estimator (LE) as an alternative to RR estimator which also has a tuning parameter affecting the performance of the model. LE can deal with multicollinearity and provides easier and solid ways on the selection of tuning parameters. LE is usually preferred over RR, because it is the linear function of its biasing parameter d (Qasim et al., 2020). The LE of β in equation (1) is written as

$$\hat{\beta}_{LE} = (X'X + I)^{-1} (X'Y + d\hat{\beta}) \quad ; \quad 0 < d < 1 \tag{3}$$

The advantage of the LE, compared with the traditional RR method proposed by Hoerl and Kennard [4], is that the estimated coefficients are a linear function of the biasing parameter d (Liu, 1993) instead of a non-linear function as in the case of RR. This leads to a more stable biasing of the vector of estimated coefficients. The optimal value of biasing parameter d in Liu regression plays an important role in minimizing the variance. Many researchers have suggested several Liu regression estimators for estimating d . Different studies proposed and investigated the biasing parameter estimators for the LE such as Liu (Liu, 1993), Qasim (Qasim et al., 2020), Khalaf (Khalaf G. et al., 2005), Shukur (Shukur et al., 2015), and Muhammad Suhail (Suhail et al., 2021).

Therefore, we investigate and examine different methods for estimating the biasing parameter d of the LE in the linear regression model. The sections of this paper are arranged as follows: the statistical methodology that includes the model estimation and existing LEs are discussed in Section 2. The design of the experiment is described in Section 3. Simulation Results in Section 4. Real data are analyzed in Section 5 . At last, a brief summary and conclusions in Section 6.

2. Some existing Liu estimators

The popular canonical form of the model in equation (1) is

$$Y = Z\alpha + \varepsilon, \tag{4}$$

where $Z = XD$ and $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_p)' = D'\beta,$ D is an orthogonal matrix such that $D'D = I$ and $Z'Z = D'X'XD = \Lambda,$ $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_p)$ consists of the eigenvalues of the $X'X$ matrix. The OLS estimator can be defined in canonical form as follows:

$$\hat{\alpha}_{OLS} = \Lambda^{-1} Z'y, \tag{5}$$

and the mean square error matrix (MSEM) is

$$MSEM(\hat{\alpha}) = \sigma^2 \Lambda^{-1}. \tag{6}$$

The RR estimator of α (Hoerl et al., 1970) is

$$\hat{\alpha}_{RR} = W\Lambda\hat{\alpha}_{OLS}, \tag{7}$$

where $W = (\Lambda + kI)^{-1}$ and the mean square error matrix (MSEM) is

$$MSEM(\hat{\alpha}_{RR}) = \sigma^2 W\Lambda W' + (W\Lambda - I)\alpha\alpha'(W\Lambda - I)'. \tag{8}$$

The LE is defined as

$$\hat{\alpha}_{LE} = F\hat{\alpha}_{OLS}, \tag{9}$$

where $F = (\Lambda + I)^{-1}(\Lambda + dI)$, and the mean square error matrix (MSEM) is

$$MSEM(\hat{\alpha}_{LE}) = \sigma^2 F\Lambda^{-1}F' + (1-d)^2 (\Lambda + I)^{-1}\alpha\alpha'(\Lambda + I)^{-1}. \tag{10}$$

The first estimator for d was suggested by Liu (1993) and is given below:

$$d_j = \frac{\alpha_j^2 - \sigma^2}{\frac{1}{\lambda_j} + \alpha_j^2} \tag{11}$$

where λ_j is the j th eigenvalue of the matrix of cross-products $X'X$ and σ^2 is the true residual variance. In order to estimate the optimal value of d in equation (1), several methods will be proposed in this paper. The idea behind these proposed estimators is taken from the work by Hoerl and Kennard (Hoerl et al., 1970), where several different methods of estimating the biasing parameter for ridge regression were proposed. As in those papers, d_j will be estimated by a single value d. The first estimator applied is:

$$D_1 = \max\left(0, \frac{\alpha_{\max}^2 - \sigma^2}{\frac{1}{\lambda_{\max}} + \alpha_{\max}^2}\right) \tag{12}$$

where we define $\hat{\alpha}_{\max}^2$ is a maximum element of $\hat{\alpha}_j^2$, where $\hat{\lambda}_{\max}$ is the maximum eigenvalue of λ_j, and $\hat{\sigma}^2$ is the sample residual variance. Replacing the values of the unknown parameters with the maximum value of the unbiased estimators is an idea taken from Hoerl and Kennard (Hoerl et al., 1970).

Shukur et al., (Shukur et al., 2015) considered the idea of Khalaf G. et al. (2005) and Kibria (2003) and suggested the following six estimators:

$$D_2 = \max\left(0, median\left(\frac{\hat{\alpha}_j^2 - \hat{\sigma}^2}{\frac{1}{\hat{\lambda}_j} + \hat{\alpha}_j^2}\right)\right) \tag{13}$$

$$D_3 = \max\left(0, \frac{1}{p}\sum_{j=1}^{p}\left(\frac{\hat{\alpha}_j^2 - \hat{\sigma}^2}{\left(1/\lambda_j\right) + \hat{\alpha}_j^2}\right)\right) \tag{14}$$

$$D_4 = \max\left(0, \max\left(\frac{\hat{\alpha}_j^2 - \hat{\sigma}^2}{\left(1/\lambda_j\right) + \hat{\alpha}_j^2}\right)\right) \tag{15}$$

$$D_5 = \max\left(0, median\left(\hat{q}_j\right)\right) \tag{16}$$

$$D_6 = \max\left(0, \frac{1}{p}\sum_{j=1}^{p}\left(\hat{q}_j\right)\right) \tag{17}$$

$$D_7 = \max\left(0, \max\left(\hat{q}_j\right)\right) \tag{18}$$

where $$\hat{q}_j = \frac{\hat{\alpha}_j^2 - 1}{\max\left(1/\lambda_j\right) + \hat{\alpha}_j^2} \tag{19}$$

Suhail et al. (Suhail et al., 2021) propose the biasing parameter estimator depends on the quantile probability whose value is selected according to the degrees of multicollinearity, also suggested the following estimators:

$$D_8 = \max\left(0, \frac{\left(\hat{\alpha}_{(j)}\right)_{0.00} - \hat{\sigma}^2}{\left(\hat{\sigma}^2 / \left(\lambda_{(j)}\right)_{0.00}\right) + \left(\hat{\alpha}_{(j)}^2\right)_{0.00}}\right) \tag{20}$$

$$D_9 = \max\left(0, \frac{\left(\hat{\alpha}_{(j)}\right)_{0.25} - \hat{\sigma}^2}{\left(\hat{\sigma}^2 / \left(\lambda_{(j)}\right)_{0.25}\right) + \left(\hat{\alpha}_{(j)}^2\right)_{0.25}}\right) \tag{21}$$

$$D_{10} = \max\left(0, \frac{\left(\hat{\alpha}_{(j)}\right)_{0.50} - \hat{\sigma}^2}{\left(\hat{\sigma}^2 / \left(\lambda_{(j)}\right)_{0.50}\right) + \left(\hat{\alpha}_{(j)}^2\right)_{0.50}}\right) \tag{22}$$

$$D_{11} = \max\left(0, \frac{\left(\hat{\alpha}_{(j)}\right)_{0.75} - \hat{\sigma}^2}{\left(\hat{\sigma}^2 / \left(\lambda_{(j)}\right)_{0.75}\right) + \left(\hat{\alpha}_{(j)}^2\right)_{0.75}}\right) \tag{23}$$

$$D_{12} = \max\left(0, \frac{\left(\hat{\alpha}_{(j)}\right)_{1.00} - \hat{\sigma}^2}{\left(\hat{\sigma}^2 / \left(\lambda_{(j)}\right)_{1.00}\right) + \left(\hat{\alpha}_{(j)}^2\right)_{1.00}}\right) \tag{24}$$

Babar et al. (Babar, 2021) present the role of quantile probability, choose some specific values for quantile probability as: 0 (minimum), 0.25 (first quartile) and 0.50 (median), is given below:

$$D_{13} = \max\left(0, \frac{\left(\hat{\alpha}_j\right)_{0.00} - \hat{\sigma}^2}{\max\left(\hat{\sigma}^2 / \lambda_j\right) + \max\left(\hat{\alpha}_j^2\right)}\right) \tag{25}$$

$$D_{14} = \max\left(0, \frac{\left(\hat{\alpha}_j\right)_{0.25} - \hat{\sigma}^2}{\max\left(\hat{\sigma}^2 / \lambda_j\right) + \max\left(\hat{\alpha}_j^2\right)}\right) \tag{26}$$

$$D_{15} = \max\left(0, \frac{\left(\hat{\alpha}_j\right)_{0.50} - \hat{\sigma}^2}{\max\left(\hat{\sigma}^2 / \lambda_j\right) + \max\left(\hat{\alpha}_j^2\right)}\right) \tag{27}$$

3. The Design of an Experiment

This section covers the Monte Carlo simulation experiment and performance criterion measures.

3.1 The Monte Carlo Simulation

The performance of LE is compared in this section through the simulation study. Following (McDonald et al., 2021) the explanatory variables are generated as

$$x_{ij} = \left(1-\rho^2\right)^{1/2} z_{ij} + \rho z_{i,p+1}, \quad i=1,2,\ldots,n \;, j=1,2,\ldots,p \tag{28}$$

where ρ is the degree or level of multicollinearity between the explanatory variables (p), which are given as 0.80, 0.90, and 0.99. z_{ij} are the random numbers obtained from the standard normal distribution. Considering $p=4$ and $p=8$ in simulation where the variables are standardized. The response variable y_i are defined as:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip} + \varepsilon_i, \quad i=1,2,\ldots,n \tag{29}$$

where $\varepsilon_i \sim N\left(0,\sigma^2\right)$, σ^2 is the error variance are given as 1, 5 and 10. β_0 is considered to be identically zero. Sample size (n) are given as 10, 30, 50, 100, 150 and 200. Following Kibria (2003), the eigenvector corresponding to maximum eigenvalue of the $X'X$ matrix is taken as the vector of regression coefficients.

3.2. Performance Evaluation Criteria

The estimated MSE of estimators are measured in each replicate and computed as:

$$MSE = \frac{\sum_{i=1}^{M}\left(\hat{\beta}_d - \beta\right)'\left(\hat{\beta}_d - \beta\right)}{1,000} \tag{30}$$

where $\hat{\beta}_d$ is the LE estimated value of β. The experiment is replicated 1000 times. The MSE simulation results for p = 4 and 8, ρ = 0.80, 0.90, and 0.99, σ^2 = 1, 5 and 10 are presented in Tables 1–6 for n = 10, 30, 50, 100, 150 and 200, respectively. The results are discussed in the following section.

4. Simulation Results

In this section present the results of our Monte Carlo experiment concerning the MSEs of the different estimation methods can be found in Table 1-6. The factors influencing the estimated MSE are the degree of correlation (ρ), error variance $\left(\sigma^2\right)$, sample size (n), and the explanatory variables (p). A general remark from the literature is that these factors have the significant effect on the simulation design. The effect of each factor on MSE of estimators is discussed below:

The effect of n increase leads to a lower MSE of all the estimators. The estimator D_{13} exhibit lowest MSE in $n=$ 10. When $n=$ 30, 100, 150 and 200, the estimators D_3 D_8, D_9 and D_{15} are the closest competitor to D_{13}

The effect of p increasing while holding n, ρ, and σ^2 fixed is most commonly to increase the estimated MSE. It is evident from these tables that the LE is always superior to the OLS estimator. LE with biasing parameter D_{13} exhibits the lowest MSE.

The MSE of the estimators increases with the increase in the value of σ^2. However, the performance of LE is better than OLS estimator.

The increase of ρ gives an increase of the estimated MSE values. LE with biasing parameter D_{13} performs better than the other estimators.

The concluded remarks from Tables 1–6 are that the estimators D_{13} have shown better performance than the other existing LEs in terms of smaller MSE. However, when $n=$ 30, 100, 150 and 200, the estimators D_3, D_8, D_9, D_{13} and D_{15} are almost equivalent. Therefore, based on MSE criterion, we can infer that the estimator D_{13} is more efficient and is the best choice for the practitioners in the presence of high and severe multicollinearity. The least robust option among the different methods of estimating the biasing parameter are the D_1 and D_{12}.

5. Applications Using Real Data

In this section, we used the Portland cement data that adopted by Woods et al. (Woods H. et al., 1932) is used to examine the Liu estimator performance with the different biasing parameter proposed estimators of d. Then, this data was applied in various studies as Kaciranlar et al. (Kaciranlar et al., 1999), Li et al., 1932 (Li et al., 2012), Lukman et al. (Lukman et al., 2019) and recently Dawouda et al. (Dawouda et al., 2022), among others. The dataset consists of 13 observations. The response variable (y) is to study the heat evolved after 180 days of curing and measured in calories/gram of cement with 40% water at 35°C (95°F). Besides, the response variable (y), the four explanatory variables considered are the clinker compounds (CALCD) defined as

X_1 : Tricalcium aluminate ($3CaO * Al_2 O_3$)

X_2 : Tricalcium silicate ($3CaO * SiO_2$)

X_3 : Tetracalcium aluminoferrite ($4CaO * Al2 O_3 * Fe_2 O_3$)

X_4 : Dicalcium silicate ($2CaO * SiO_2$)

This data regression model is given as

$$y=\beta_0+\beta_1 X_1+\beta_2 X_2+\beta_3 X_3+\beta_4 X_4+\varepsilon \tag{31}$$

Condition number (CN) is used to measure the severity of multicollinearity among predictor. It can be defined as (Kibria, 2017)

$$CN=\sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}}, \tag{32}$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are represent the maximum and minimum eigenvalue of the $X'X$ matrix, respectively, if the value of CN < 10 this means there is no problem of multicollinearity between the explanatory variables and if it is 10< CN <30 then there is a problem of moderate multicollinearity between the explanatory variables and if the value of CN > 30 this means that there is a strong multicollinearity problem between the explanatory variables (Inan et al., 2013). The eigenvalues of $X'X$ matrix are 2.2357, 1.5761, 0.1866 and 0.0016. Then, the condition number is 37.38064. Therefore, multicollinearity exists among the predictors. The Shapiro-Wilk (W) normality test is used to test the normality of response variable. We obtain the value test statistic W = 0.96967 and p-value = 0.8903, which shows that the response variable is normal at 5% level of significance.

MSE of OLS and Liu estimators from Liu (1993) can be written as

$$MSE\left(\hat{\alpha}\right)=\sigma^2 \sum_{j=1}^{p} \frac{1}{\lambda_j}, \tag{33}$$

$$MSE\left(\hat{\alpha}\right)=\sigma^2 \sum_{j=1}^{p} \frac{\left(\lambda_j+d\right)^2}{\lambda_j\left(\lambda_j+1\right)^2}+\left(d-1\right)^2 \sum_{j=1}^{p} \frac{\hat{\alpha}_j^2}{\lambda_j\left(\lambda_j+1\right)^2}. \tag{34}$$

The estimated values for d , coefficients and MSE of estimators are presented in Table 7. This table shows that the LE is better than the OLS estimator. However, the Liu estimator, estimators D_{10} and D_{15} outperform others; therefore, they are highly efficient among others. Estimators D_5, D_6, D_7, D_8, D_9, D_{13} and D_{14} were the close competitors to D_{10} and D_{15}.

6. Conclusions

In this paper, we investigate the performance of some biasing parameter estimates in Liu regression. Within different biasing parameter methods, D_1 to D_{15}, we focus on determining the ones that outperform better by a simulation study and a real data application. The simulations were carried out by varying the values of correlation, error variance, sample sizes, and different numbers of the explanatory variables. Results from the simulation results demonstrate that biasing parameter estimator D_{13} produces the lowest MSE in all cases and performs the best among the considered estimators. However, the estimators D_3 D_8, D_9, D_{13} and D_{15} achieve comparable MSE results. For the real data application, among the estimators, D_{10} and D_{15} perform better than the other considered estimators, while the estimators D_5, D_6, D_7, D_8, D_9, D_{13} and D_{14} were the close competitors to D_{10} and D_{15}. Finally, from the simulation study and the real data results, the LE is shown to be consistently more robust than the OLS when the multicollinearity is present. It can also be concluded that D_8, D_9, D_{13} and D_{15} were the ones to be recommended for practitioners when the LE is used.

Table 1 MSE with n=10

$p=4$									
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	3.672	8.550	90.813	97.794	199.829	2128.749	371.728	793.124	9107.695
D_1	2.615	6.985	88.074	74.148	166.793	2072.666	277.907	664.922	8883.769
D_2	0.529	0.696	2.244	13.255	15.929	294.819	50.358	60.339	1861.549
D_3	0.537	0.717	2.579	12.904	14.027	22.425	49.258	53.101	75.933
D_4	1.059	1.873	13.228	62.459	130.315	1499.157	264.045	615.054	8021.994
D_5	0.490	0.551	0.376	18.382	20.463	55.205	108.471	150.136	929.859
D_6	0.535	0.657	0.841	25.347	33.754	164.162	128.619	196.949	1372.055
D_7	0.809	1.320	5.899	74.921	143.833	1387.537	338.587	708.152	7917.600
D_8	0.489	0.550	0.348	**12.773**	13.538	**8.908**	**49.235**	**52.826**	**34.887**
D_9	0.510	0.602	0.730	12.826	13.892	15.796	49.253	53.285	66.397
D_{10}	0.621	0.798	3.384	13.640	16.017	77.492	50.993	60.924	304.930
D_{11}	1.719	4.767	80.764	41.412	102.542	1883.321	150.156	410.664	8111.919
D_{12}	2.652	7.016	88.095	70.715	161.673	2061.186	264.113	644.023	8835.881
D_{13}	**0.489**	**0.549**	**0.347**	**12.773**	**13.536**	**8.908**	**49.235**	**52.826**	**34.887**
D_{14}	1.059	1.873	13.228	62.459	130.315	1499.157	264.045	615.054	8021.994
D_{15}	0.496	0.558	0.393	12.819	13.634	9.780	49.349	53.135	38.525
$p=8$									
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	184.645	374.730	4057.833	4443.360	9219.762	99238.358	17043.967	35672.742	383526.203
D_1	182.096	371.950	4054.942	4387.546	9159.852	99175.579	16817.239	35408.446	383263.949
D_2	1.790	3.736	35.829	275.440	739.854	21610.765	1676.151	4815.823	162591.215
D_3	3.167	6.773	74.457	118.084	208.731	3187.724	535.778	715.969	9657.095
D_4	47.667	95.620	1063.896	3282.043	6885.701	78306.929	15274.631	32178.062	355557.742
D_5	1.147	1.196	0.453	31.520	31.521	20.719	157.314	188.973	509.481
D_6	1.452	1.706	4.902	118.184	197.647	1583.108	772.997	1423.282	12510.913
D_7	14.190	25.383	257.761	3025.144	6147.465	65341.738	14884.652	30961.493	329486.295
D_8	1.148	1.197	0.462	29.716	29.330	11.012	115.609	120.143	44.851
D_9	3.257	8.609	37.613	60.716	79.614	676.286	306.171	286.075	2254.298
D_{10}	13.852	29.660	304.381	239.995	487.483	6934.724	1066.274	1797.644	24953.107
D_{11}	64.451	146.164	1726.570	1608.845	3453.314	43321.319	5886.617	12898.477	155012.147
D_{12}	182.116	371.955	4054.944	4380.467	9152.628	99168.127	16788.230	35375.726	383231.564
D_{13}	**1.146**	**1.195**	**0.449**	**29.713**	**29.327**	**11.010**	**115.594**	**120.130**	**44.850**
D_{14}	47.667	95.620	1063.896	3282.043	6885.701	78306.929	15274.631	32178.062	355557.742
D_{15}	1.150	1.200	0.462	29.811	29.457	11.325	115.873	120.506	46.318

Bold values indicate the minimum MSE.

Table 2 MSE with n=30

	$p = 4$								
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	0.490	0.941	9.431	12.356	24.564	240.990	49.667	98.514	1001.038
$\mathbf{D}_{1}$	0.365	0.603	7.308	8.959	15.757	194.470	35.921	63.668	817.666
$\mathbf{D}_{2}$	0.343	0.493	0.683	**8.661**	**12.900**	29.759	**34.680**	**50.722**	157.710
$\mathbf{D}_{3}$	0.344	0.495	0.726	8.661	**12.900**	**14.696**	**34.680**	**50.722**	**60.274**
$\mathbf{D}_{4}$	0.358	0.536	1.214	8.910	15.096	118.311	35.859	62.782	692.892
$\mathbf{D}_{5}$	**0.343**	0.492	0.594	9.065	14.348	34.120	39.426	64.631	284.156
$\mathbf{D}_{6}$	**0.343**	0.492	0.632	9.273	14.999	41.087	39.820	66.406	287.693
$\mathbf{D}_{7}$	0.346	0.504	0.815	10.663	19.186	115.678	46.801	89.397	764.452
$\mathbf{D}_{8}$	**0.343**	**0.491**	0.574	**8.661**	**12.900**	**14.696**	**34.680**	**50.722**	**60.274**
$\mathbf{D}_{9}$	**0.343**	**0.491**	0.583	**8.661**	**12.900**	14.970	**34.680**	**50.722**	61.102
$\mathbf{D}_{10}$	0.344	0.493	0.659	**8.661**	12.901	16.862	**34.680**	50.730	69.781
$\mathbf{D}_{11}$	0.349	0.532	4.935	8.664	13.204	125.870	34.684	52.150	531.838
$\mathbf{D}_{12}$	0.366	0.605	7.314	8.917	15.456	187.667	35.729	62.320	789.016
$\mathbf{D}_{13}$	**0.343**	**0.491**	**0.574**	**8.661**	**12.900**	**14.696**	**34.680**	**50.722**	**60.274**
$\mathbf{D}_{14}$	0.358	0.536	1.214	8.910	15.096	118.311	35.859	62.782	692.892
$\mathbf{D}_{15}$	0.343	0.492	0.596	**8.661**	12.901	15.270	**34.680**	50.725	62.746
	$p = 8$								
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	1.476	2.973	31.791	36.469	72.290	806.864	154.439	326.399	3161.653
$\mathbf{D}_{1}$	1.004	2.031	28.822	25.042	50.134	738.627	107.584	234.782	2888.276
$\mathbf{D}_{2}$	**0.735**	0.987	1.463	**18.267**	**24.933**	57.438	**75.247**	**103.984**	324.107
$\mathbf{D}_{3}$	**0.735**	0.988	1.594	**18.267**	**24.933**	**35.301**	**75.247**	**103.984**	**139.176**
$\mathbf{D}_{4}$	0.817	1.183	3.479	23.189	41.305	432.627	104.756	219.676	2379.909
$\mathbf{D}_{5}$	**0.735**	**0.987**	1.380	18.609	25.849	43.985	83.708	124.771	298.409
$\mathbf{D}_{6}$	0.735	0.989	1.426	19.693	28.263	63.495	90.184	140.230	426.891
$\mathbf{D}_{7}$	0.764	1.058	1.896	28.002	48.155	344.769	139.011	279.320	2247.218
$\mathbf{D}_{8}$	**0.735**	**0.987**	**1.375**	**18.267**	**24.933**	**35.301**	**75.247**	**103.984**	**139.176**
$\mathbf{D}_{9}$	**0.735**	**0.987**	1.382	**18.267**	**24.933**	35.372	**75.247**	**103.984**	139.439
$\mathbf{D}_{10}$	**0.735**	0.987	1.463	**18.267**	**24.933**	37.065	**75.247**	103.985	146.655
$\mathbf{D}_{11}$	0.738	1.004	2.207	18.277	25.048	57.170	**75.247**	104.263	216.946
$\mathbf{D}_{12}$	1.007	2.035	28.831	24.645	49.150	732.066	105.662	230.080	2861.305
$\mathbf{D}_{13}$	**0.735**	**0.987**	**1.375**	**18.267**	**24.933**	**35.301**	**75.247**	**103.984**	**139.176**
$\mathbf{D}_{14}$	0.817	1.183	3.479	23.189	41.305	432.627	104.756	219.676	2379.909
$\mathbf{D}_{15}$	**0.735**	0.987	1.381	**18.267**	**24.933**	35.432	**75.247**	103.984	139.725

Bold values indicate the minimum MSE.

Table 3 MSE with n=50

	$p = 4$								
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	0.283	0.537	5.060	7.218	13.775	122.830	28.886	53.852	511.374
$\mathbf{D}_{1}$	0.234	0.387	3.539	5.879	9.795	87.881	23.570	38.352	372.819
$\mathbf{D}_{2}$	**0.230**	0.366	0.751	**5.840**	**9.290**	20.218	**23.360**	**36.509**	92.403
$\mathbf{D}_{3}$	**0.230**	0.366	0.779	**5.840**	**9.290**	**17.472**	**23.360**	**36.509**	**72.675**
$\mathbf{D}_{4}$	0.233	0.379	0.996	5.873	9.673	53.508	23.559	38.198	314.175
$\mathbf{D}_{5}$	**0.230**	**0.366**	0.720	5.929	9.632	24.750	24.737	40.611	163.725

Bold values indicate the minimum MSE.

Table 3 Continued

$p = 4$									
σ^2	1			5			10		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
D_6	**0.230**	**0.366**	0.737	5.998	9.900	28.684	24.974	41.561	174.832
D_7	0.230	0.368	0.813	6.445	11.318	57.233	27.431	49.413	376.191
D_8	**0.230**	**0.366**	**0.713**	**5.840**	**9.290**	**17.472**	**23.360**	**36.509**	**72.675**
D_9	**0.230**	**0.366**	0.716	**5.840**	**9.290**	17.504	**23.360**	**36.509**	72.867
D_{10}	**0.230**	0.366	0.737	**5.840**	**9.290**	17.873	**23.360**	**36.509**	74.645
D_{11}	0.230	0.371	2.261	5.840	9.305	49.995	**23.360**	36.521	216.820
D_{12}	0.234	0.388	3.545	5.873	9.735	83.940	23.536	38.099	356.564
D_{13}	**0.230**	**0.366**	**0.713**	**5.840**	**9.290**	**17.472**	**23.360**	**36.509**	**72.675**
D_{14}	0.233	0.379	0.996	5.873	9.673	53.508	23.559	38.198	314.175
D_{15}	**0.230**	**0.366**	0.721	**5.840**	**9.290**	17.599	**23.360**	**36.509**	73.293
$p = 8$									
σ^2	1			5			10		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	1.038	2.110	21.380	24.371	52.023	547.807	96.638	215.966	2142.686
D_1	0.724	1.416	18.690	16.781	34.954	485.349	67.265	147.416	1892.963
D_2	**0.624**	**0.883**	1.437	**14.718**	**22.031**	45.271	**58.616**	**89.330**	208.894
D_3	**0.624**	0.883	1.504	**14.718**	**22.031**	**37.222**	**58.616**	**89.330**	**148.191**
D_4	0.654	0.980	2.341	16.054	29.269	239.194	66.275	137.828	1441.023
D_5	**0.624**	**0.883**	1.412	14.824	22.421	41.899	61.558	98.222	225.412
D_6	**0.624**	**0.883**	1.435	15.302	23.794	52.347	64.719	107.588	306.190
D_7	0.631	0.911	1.642	18.730	33.940	194.643	85.693	178.214	1339.611
D_8	**0.624**	**0.883**	**1.410**	**14.718**	**22.031**	**37.222**	**58.616**	**89.330**	**148.191**
D_9	**0.624**	**0.883**	1.412	**14.718**	**22.031**	37.239	**58.616**	**89.330**	148.257
D_{10}	**0.624**	**0.883**	1.431	**14.718**	**22.031**	37.813	**58.616**	**89.330**	150.497
D_{11}	0.624	0.887	1.647	**14.718**	22.038	43.855	**58.616**	89.353	173.656
D_{12}	0.725	1.418	18.694	16.635	34.357	479.740	66.674	144.723	1869.722
D_{13}	**0.624**	**0.883**	**1.410**	**14.718**	**22.031**	**37.222**	**58.616**	**89.330**	**148.191**
D_{14}	0.654	0.980	2.341	16.054	29.269	239.194	66.275	137.828	1441.023
D_{15}	**0.624**	**0.883**	1.412	**14.718**	**22.031**	37.274	**58.616**	**89.330**	148.398

Bold values indicate the minimum MSE.

Table 4 MSE with n=100

$p = 4$									
σ^2	1			5			10		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	0.132	0.266	2.992	3.288	6.851	71.482	13.299	27.556	281.991
D_1	0.119	0.214	1.946	**2.942**	5.465	47.697	**11.906**	21.954	188.767
D_2	**0.118**	**0.210**	0.684	**2.942**	**5.416**	17.229	**11.906**	**21.791**	65.893
D_3	**0.118**	**0.210**	0.695	**2.942**	**5.416**	**16.976**	**11.906**	**21.791**	**64.982**
D_4	0.119	0.213	0.768	**2.942**	5.449	28.815	**11.906**	21.938	149.897
D_5	**0.118**	**0.210**	0.677	2.948	5.456	18.694	12.085	22.539	87.108
D_6	**0.118**	**0.210**	0.682	2.958	5.532	20.719	12.167	22.985	99.363
D_7	0.118	0.211	0.706	3.036	5.877	31.012	12.723	25.330	183.529
D_8	**0.118**	**0.210**	**0.677**	**2.942**	5.416	**16.976**	**11.906**	**21.791**	**64.982**
D_9	**0.118**	**0.210**	0.677	**2.942**	**5.416**	16.979	**11.906**	**21.791**	**64.982**
D_{10}	**0.118**	**0.210**	0.680	**2.942**	**5.416**	17.025	**11.906**	**21.791**	65.096

Bold values indicate the minimum MSE.

Table 4 Continued

$p=4$									
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
D11	**0.118**	0.211	1.232	**2.942**	**5.416**	27.460	**11.906**	**21.791**	104.507
D12	0.119	0.214	1.947	**2.942**	5.458	45.673	**11.906**	21.931	180.352
D13	**0.118**	**0.210**	**0.677**	**2.942**	**5.416**	**16.976**	**11.906**	**21.791**	**64.982**
D14	0.119	0.213	0.768	**2.942**	5.449	28.815	**11.906**	21.938	149.897
D15	**0.118**	**0.210**	0.677	**2.942**	**5.416**	16.988	**11.906**	**21.791**	65.009
$p=8$									
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	0.438	0.905	8.883	10.824	22.411	219.302	44.017	92.288	858.795
D1	0.355	0.651	6.872	8.756	15.986	172.378	35.425	66.381	673.121
D2	**0.349**	**0.575**	1.535	**8.611**	**14.271**	37.406	**34.751**	**58.007**	148.245
D3	**0.349**	**0.575**	1.554	**8.611**	**14.271**	**37.109**	**34.751**	**58.007**	**146.377**
D4	0.353	0.593	1.798	8.704	15.179	78.634	35.342	64.857	466.725
D5	0.349	**0.575**	1.533	8.618	14.324	38.457	35.124	59.544	167.928
D6	**0.349**	**0.575**	1.539	8.683	14.617	41.648	35.850	62.110	195.670
D7	0.349	0.579	1.594	9.287	16.742	73.191	40.421	78.937	480.256
D8	**0.349**	**0.575**	**1.533**	**8.611**	**14.271**	**37.109**	**34.751**	**58.007**	**146.377**
D9	**0.349**	**0.575**	**1.533**	**8.611**	**14.271**	37.109	**34.751**	**58.007**	146.379
D10	**0.349**	**0.575**	1.534	**8.611**	**14.271**	37.132	**34.751**	**58.007**	146.458
D11	**0.349**	**0.575**	1.565	**8.611**	**14.271**	37.732	**34.751**	**58.007**	148.858
D12	0.355	0.651	6.874	8.744	15.882	169.229	35.368	65.881	660.367
D13	0.349	**0.575**	**1.533**	**8.611**	**14.271**	**37.109**	**34.751**	**58.007**	**146.377**
D14	0.353	0.593	1.798	8.704	15.179	78.634	35.342	64.857	466.725
D15	**0.349**	**0.575**	1.533	**8.611**	**14.271**	37.111	**34.751**	**58.007**	146.386

Bold values indicate the minimum MSE.

Table 5 MSE with n=150

$p=4$									
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	0.097	0.177	1.778	2.226	4.537	45.638	9.615	18.110	179.283
D1	0.090	0.153	1.089	**2.060**	3.873	28.742	**8.901**	15.489	112.558
D2	**0.089**	**0.151**	0.612	**2.060**	**3.866**	15.499	**8.901**	**15.469**	62.237
D3	**0.089**	**0.151**	0.617	**2.060**	**3.866**	**15.486**	**8.901**	**15.469**	**62.122**
D4	0.090	0.153	0.650	**2.060**	3.871	19.968	**8.901**	15.487	94.946
D5	**0.089**	**0.151**	0.611	2.061	3.877	16.268	8.968	15.716	73.243
D6	**0.089**	**0.151**	0.612	2.063	3.902	17.311	9.007	15.904	79.852
D7	**0.089**	0.152	0.621	2.089	4.025	21.910	9.258	16.824	118.776
D8	**0.089**	**0.151**	**0.611**	**2.060**	**3.866**	**15.486**	**8.901**	**15.469**	**62.122**
D9	**0.089**	**0.151**	**0.611**	**2.060**	**3.866**	**15.486**	**8.901**	**15.469**	**62.122**
D10	**0.089**	**0.151**	0.611	**2.060**	**3.866**	15.493	**8.901**	**15.469**	62.153
D11	**0.089**	0.152	0.783	**2.060**	**3.866**	18.221	**8.901**	**15.469**	72.036
D12	0.090	0.153	1.089	**2.060**	3.872	27.663	**8.901**	15.486	108.281
D13	**0.089**	**0.151**	**0.611**	**2.060**	**3.866**	**15.486**	**8.901**	**15.469**	**62.122**
D14	0.090	0.153	0.650	**2.060**	3.871	19.968	**8.901**	15.487	94.946
D15	**0.089**	**0.151**	0.611	**2.060**	**3.866**	15.488	**8.901**	**15.469**	62.130

Bold values indicate the minimum MSE.

Table 5 Continued

$p = 8$									
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	0.269	0.563	5.660	6.814	13.421	143.900	26.454	55.546	597.072
D_1	0.235	0.438	4.082	5.922	10.410	106.530	23.114	42.988	446.800
D_2	**0.234**	**0.422**	1.429	**5.912**	**10.102**	36.603	**23.098**	**41.600**	147.927
D_3	**0.234**	**0.422**	1.438	**5.912**	**10.102**	**36.582**	**23.098**	**41.600**	**147.882**
D_4	0.235	0.428	1.569	5.918	10.263	55.418	23.112	42.715	319.872
D_5	**0.234**	**0.422**	**1.428**	5.913	**10.112**	37.274	23.203	42.085	159.915
D_6	**0.234**	**0.422**	1.430	5.927	10.197	39.084	23.404	42.991	178.107
D_7	**0.234**	0.423	1.456	6.123	10.865	54.071	24.780	48.829	339.702
D_8	**0.234**	**0.422**	**1.428**	**5.912**	**10.102**	**36.582**	**23.098**	**41.600**	**147.882**
D_9	**0.234**	**0.422**	**1.428**	**5.912**	**10.102**	**36.582**	**23.098**	**41.600**	**147.882**
D_{10}	**0.234**	**0.422**	1.428	**5.912**	**10.102**	36.583	**23.098**	**41.600**	147.886
D_{11}	**0.234**	**0.422**	1.435	**5.912**	**10.102**	36.724	**23.098**	**41.600**	148.506
D_{12}	0.235	0.438	4.082	5.921	10.390	104.490	23.112	42.892	438.207
D_{13}	**0.234**	**0.422**	**1.428**	**5.912**	**10.102**	**36.582**	**23.098**	**41.600**	**147.882**
D_{14}	0.235	0.428	1.569	5.918	10.263	55.418	23.112	42.715	319.872
D_{15}	**0.234**	**0.422**	**1.428**	**5.912**	**10.102**	36.582	**23.098**	**41.600**	147.883

Bold values indicate the minimum MSE.

Table 6 MSE with n=200

$p = 4$									
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	0.075	0.140	1.476	1.799	4.076	38.312	7.270	15.348	147.345
D_1	0.071	0.125	0.878	**1.700**	**3.609**	23.816	**6.868**	**13.584**	90.203
D_2	**0.071**	**0.125**	0.587	**1.700**	**3.609**	15.231	**6.868**	**13.584**	59.009
D_3	**0.071**	**0.125**	0.592	**1.700**	**3.609**	**15.229**	**6.868**	**13.584**	**58.969**
D_4	0.071	0.125	0.616	**1.700**	**3.609**	17.783	**6.868**	**13.584**	77.889
D_5	**0.071**	**0.125**	0.586	1.700	3.615	15.747	6.898	13.742	66.262
D_6	**0.071**	**0.125**	0.587	1.701	3.630	16.427	6.917	13.857	70.879
D_7	0.071	**0.125**	0.591	1.714	3.703	19.311	7.043	14.431	96.174
D_8	**0.071**	**0.125**	**0.586**	**1.700**	**3.609**	**15.229**	**6.868**	**13.584**	**58.969**
D_9	**0.071**	**0.125**	**0.586**	**1.700**	**3.609**	**15.229**	**6.868**	**13.584**	**58.969**
D_{10}	**0.071**	**0.125**	0.586	**1.700**	**3.609**	15.230	**6.868**	**13.584**	58.975
D_{11}	**0.071**	**0.125**	0.685	**1.700**	**3.609**	16.586	**6.868**	**13.584**	63.529
D_{12}	0.071	0.125	0.879	**1.700**	**3.609**	23.065	**6.868**	**13.584**	87.304
D_{13}	**0.071**	**0.125**	**0.586**	**1.700**	**3.609**	**15.229**	**6.868**	**13.584**	**58.969**
D_{14}	0.071	0.125	0.616	**1.700**	**3.609**	17.783	**6.868**	**13.584**	77.889
D_{15}	**0.071**	**0.125**	0.586	**1.700**	**3.609**	15.229	**6.868**	**13.584**	58.971
$p = 8$									
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
OLS	0.164	0.359	3.557	4.184	9.037	87.655	17.338	33.813	353.269
D_1	0.151	0.300	2.443	**3.839**	7.486	61.018	**15.848**	28.142	247.274
D_2	**0.150**	0.295	**1.196**	**3.839**	**7.421**	**29.557**	**15.848**	**27.994**	**119.962**
D_3	**0.150**	0.295	1.199	**3.839**	**7.421**	**29.557**	**15.848**	**27.994**	**119.962**
D_4	0.151	0.297	1.267	**3.839**	7.456	36.621	**15.848**	28.109	187.637

Bold values indicate the minimum MSE.

Table 6 Continued

$p = 8$									
σ^2	**1**			**5**			**10**		
ρ	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**	**0.8**	**0.9**	**0.99**
D_5	**0.150**	**0.295**	**1.196**	3.839	7.423	29.826	15.870	28.125	125.353
D_6	**0.150**	**0.295**	**1.196**	3.840	7.454	30.754	15.950	28.460	134.617
D_7	0.151	0.295	1.209	3.896	7.761	37.742	16.552	30.674	209.583
D_8	**0.150**	**0.295**	**1.196**	**3.839**	**7.421**	**29.557**	**15.848**	**27.994**	**119.962**
D_9	**0.150**	**0.295**	**1.196**	**3.839**	**7.421**	**29.557**	**15.848**	**27.994**	**119.962**
D_{10}	**0.150**	**0.295**	**1.196**	**3.839**	**7.421**	**29.557**	**15.848**	**27.994**	**119.962**
D_{11}	**0.150**	**0.295**	1.197	**3.839**	**7.421**	29.570	**15.848**	**27.994**	120.024
D_{12}	0.151	0.300	2.444	**3.839**	7.481	59.908	**15.848**	28.127	242.814
D_{13}	**0.150**	**0.295**	**1.196**	**3.839**	**7.421**	**29.557**	**15.848**	**27.994**	**119.962**
D_{14}	0.151	0.297	1.267	**3.839**	7.456	36.621	**15.848**	28.109	187.637
D_{15}	**0.150**	**0.295**	**1.196**	**3.839**	**7.421**	**29.557**	**15.848**	**27.994**	**119.962**

Bold values indicate the minimum MSE.

Table 7 Estimated coefficients and mean squared error (MSE) values of the estimators.

Estimators	d	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	**MSE**
OLS	-	-0.657	-0.008	0.303	0.388	14.624
D_1	0.464	-0.644	-0.008	0.253	0.184	3.260
D_2	0.006	-0.634	-0.008	0.210	0.010	0.268
D_3	0.110	-0.636	-0.008	0.220	0.049	0.403
D_4	0.464	-0.644	-0.008	0.253	0.184	3.260
D_5	0.000	-0.633	-0.008	0.209	0.007	0.270
D_6	0.000	-0.633	-0.008	0.209	0.007	0.270
D_7	0.000	-0.633	-0.008	0.209	0.007	0.270
D_8	0.000	-0.633	-0.008	0.209	0.007	0.270
D_9	0.003	-0.633	-0.008	0.210	0.009	0.269
D_{10}	0.007	-0.634	-0.008	0.210	0.010	**0.268**
D_{11}	0.842	-0.653	-0.008	0.288	0.328	10.390
D_{12}	0.923	-0.655	-0.008	0.296	0.359	12.470
D_{13}	0.000	-0.633	-0.008	0.209	0.007	0.270
D_{14}	0.003	-0.633	-0.008	0.210	0.009	0.269
D_{15}	0.007	-0.634	-0.008	0.210	0.010	**0.268**

Bold values indicate the minimum MSE.

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