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# Dissipative Heat and Mass Transfer Effects on MHD Flow of a Continuously Moving Plate Through a Porous Medium With Constant Heat and Mass Flux

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**Abstract:** The dissipative effects on heat and mass transfer of natural convection about a continuously moving vertical plate bounding fluid saturated porous medium on one side in presence of magnetic field with constant heat and mass flux has been studied. The plate is subjected to a constant suction with heat and mass flux. The governing equations are solved by perturbation method and the pertinent findings are represented through graphs. It is interesting to note that the magnetic parameter has a retarding effect on velocity profiles, temperature profiles and skin friction coefficient in the presence of porous matrix. The effect of increasing the permeability of the porous medium enhances velocity, temperature and skin friction. Further, it is observed that the effect of increasing dissipative heat parameter is to accelerate the temperature and skin friction coefficient..

**Keywords:** Moving plate, Heat and mass transfer, Dissipation, Porous medium, Magnetic field, heat and mass flux.

## 1 Introduction

The problem of fluid flow in an electromagnetic field has been studied for its importance in geophysics, metallurgy and aerodynamic extrusion of plastic sheets and

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other engineering processes such as in petroleum engineering, chemical engineering, composite or ceramic engineering and heat exchangers. Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle reentry. In the last five years, many investigations dealing with heat flow and mass transfer over a vertical porous plate with variable suction, heat absorption/generation or Hall current have been reported.

The study of natural convection heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. Processes involving heat and mass transfer in porous media are often encountered in the chemical industry, in reservoir engineering in connection with thermal recovery process and in the study of dynamics of hot and salty springs of a sea. Underground spreading of chemical wastes and other pollutants, grain storage, evaporation, cooling and solidification are the few other application areas where the combined thermo-solute natural convection in porous media is observed.

Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. Studies pertaining to coupled heat and mass transfer due to free convection has got wide applications in different realms, such as, mechanical, geothermal, chemical sciences etc. In nature, there exist flows which are caused not only by the temperature differences but also by concentration differences. These mass transfer differences do affect the rate of heat transfer. In industries, many transport process exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect of thermal and mass diffusion. The phenomenon of heat and mass transfer frequently exists in chemically processed industries such as food processing and polymer production. Magetohydrodynamics is attracting the attention of the many authors due to its applications in geophysics; it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering in MHD pumps, MHD bearings etc. at high temperatures attained in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic and alter heat transfer and friction characteristic.

Despite all these important investigations, there are few investigations in porous medium taking dissipation aspects into account. It is worth mentioning that viscous dissipation in porous regime is to be reckoning with in many situations simply because due to its qualitative effects it may alter the thermal regime. Viscous heating serves as a heat source since it is the local production of heat energy due to shear stresses. Viscous dissipation aspects in porous media warrant careful attention simply because the porous matrix walls obstruct the fluid traversal inside it hence the shear within fluid itself and due to porous medium has to be taken care of while dealing with dissipation aspects. However, it is worth to note that though the dissipation is not that much dominant quantitatively as compared to its other counterpart effects but certainly there are areas such as tribology, instrumentation etc where its qualitative effects are earnestly observed. Extensive analysis can be found in the literature about the expressions envisaging dissipation in porous medium. Nield [1] have presented an excellent relevant document. In this communication following Al-Hadhrami et al. [2] the dissipation $\varphi$ in the porous medium is taken as

$$\varphi = \mu \left( \frac{du}{dy} \right)^2 + \frac{\mu}{k} u^2 \tag{1}$$

Where $\mu$: Viscosity of the fluid, $u$: Velocity of the space variable, $k$: The permeability of the porous medium

Abdel Khalek [3] examined MHD free convection with mass transfer from a moving permeable vertical surface and produced a perturbation solution. Israel-Cookey et al.[4] discussed Influence of viscous dissipation on unsteady MHD free convection flow past an infinite heated vertical plate in porous medium with time-dependent suction. Senapati et.al [5] have studied magnetic effect on mass and heat transfer of a hydrodynamic flow past a vertical oscillating plate in presence of chemical reaction. Senapati et al. [6] also discussed the chemical effects on mass and heat transfer on MHD free convection flow of fluids in vertical plates and in between parallel plates in poiseuille flow. Vyas et al.[7] have been discussed the dissipative heat and mass transfer in porous medium due to continuously moving plate. The heat and mass transfer on MHD flow through a porous medium over a stretching surface with heat source has been studied by Barik et al. [8]. The mass transfer and radiation effect on MHD flow past an impulsively started exponentially accelerated inclined porous plate with variable temperature in the presence of heat source and chemical reaction has also investigated by Barik [9].

Kumar [10] studied the chemical reaction effects on MHD flow of continuously moving vertical surface with heat and mass flux through porous medium. The heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity have been studied by Singh and Singh [11]. Reddy et al.[12] have investigated the heat and mass transfer effects on MHD flow continuously moving vertical surface with uniform heat and mass flux. Ravikumar et al.[13] have studied the heat and mass transfer effects on MHD flow of viscous fluid through non-homogeneous porous medium in the presence of temperature dependent heat source. Muthucumaraswamy and Ganesan [14] focused the first-order chemical reaction on flow past an impulsively started vertical plate with heat and mass flux. The effect of viscous dissipative heat and uniform magnetic field on the free convective flow through a porous medium with heat generation / absorption was studied by Amakiri and Ogulu [15]. Recently, Barik et al. [16] studied the MHD flow and heat transfer over a stretching porous sheet subject to power law heat flux in the presence of chemical reaction and viscous dissipation. Girish Kumar [17] has studied the chemical reaction effects on MHD flow of continuously moving vertical surface with heat and mass flux through porous medium. Cramer and Pai [18] have studied the magneto flow Dynamics for engineers and applied physicists. The boundary layer theory has investigated by Schlichting and Gersten [19]. Lucian et al. [20] studied An MHD study of the behavior of an electrolyte solution having 3D numerical solution and experimental results.

The objective of the present paper is to study the effects of dissipative heat and mass transfer in porous medium due to continuously moving plate in presence of magnetic field with constant heat and mass flux. In the present study we have incorporated the magnetic field on the flow and heat transfer phenomena with heat and mass flux in the work of Vyas et al. [7] whereas in the literature of earlier work, the authors have restricted to non-conducting fluid with heat flux.

# 2 Mathematical Formulation

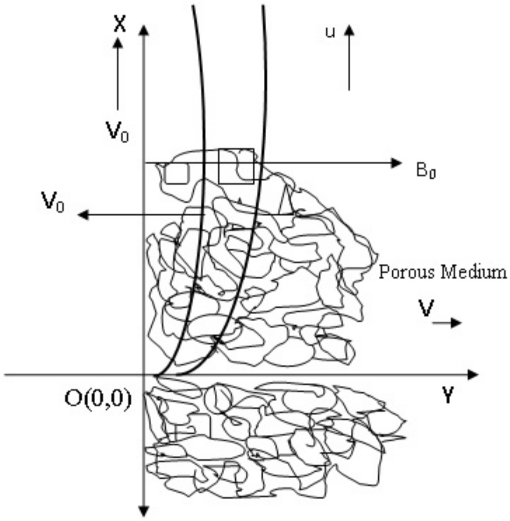


Figure 1: Sketch of the problem

We consider steady laminar flow and mass transfer of viscous incompressible fluid filled in a homogeneous porous medium in presence of magnetic field. The porous medium is bounded on one side by a vertical plate moving upwards with a uniform velocity $V_0$ and subjected to uniform heat and mass flux vis a vis a constant suction $V_0$. A Cartesian coordinate system is used where X-axis is taken along the plate in the moving direction of the plate and a magnetic field of uniform strength $\beta_0$ is applied normal to the plate along $Y$-axis. $T'_\infty$ is the temperature and $C'_\infty$ is the mass concentration of fluid. Applying Boussinesq approximation and taking viscous dissipation in porous regime into account, the governing equations are as under:

$$\frac{\partial V}{\partial y'} = 0 \Rightarrow V = -V_0 \tag{2}$$

$$V\frac{\partial u'}{\partial y'} = g\beta(T - T_\infty) + g\beta_c(C - C_\infty) + \overline{\nu}\frac{\partial^2 u'}{\partial {y'}^2} - \frac{\nu}{k_p}u' - \frac{\sigma {B_0}^2}{\rho}u' \tag{3}$$

$$V\frac{\partial T'}{\partial y'} = \frac{\overline{k}}{\rho c_p}\frac{\partial^2 T'}{\partial {y'}^2} + \frac{\overline{\nu}}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{\nu}{c_p k_p}{u'}^2 \tag{4}$$

$$V\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial {y'}^2} \tag{5}$$

with the following boundary conditions

$$\left.\begin{array}{l} u' = V_0, \quad \frac{\partial T'}{\partial y'} = -\frac{q}{k}, \quad \frac{\partial C'}{\partial y'} = -\frac{m}{D}, \quad \text{at} \quad y' = 0 \\ u'_0 \to 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty, \quad \text{at} \quad y' \to \infty \end{array}\right\} \tag{6}$$

where (u.V ) are velocity components in x and y directions, g is acceleration due to gravity, T' is the temperature of the fluid, C' is the species concentration in the regime , $\beta$ is the coefficient of thermal expansion, $\beta_c$ is the volumetric expansion coefficient, $\overline{\nu}$ is the effective kinematic viscosity in the porous medium and $\nu$ is the kinematic viscosity of the fluid, $\overline{k}$ is effective thermal conductivity, $\rho$ is the density of the fluid, $k_p$ is the permeability, $c_p$ is the specific heat at constant pressure, $D$ is the diffusion coefficient, $q$ is the heat flux per unit area and $m$ is the mass flux per unit area.

Let us introduce the following non dimensional quantities

$$\left.\begin{array}{c} y = \dfrac{V_0 y'}{\nu}, \quad u = \dfrac{u'}{V_0}, \quad \theta = \dfrac{(T' - T'_\infty)V_0 k}{q\nu}, \quad P_r = \dfrac{\rho v c_p}{k}, \\ C = \dfrac{(C' - C'_\infty)V_0 D}{mv}, \quad K = \dfrac{{V_0}^2 k_p}{v^2}, \quad M = \dfrac{\sigma {B_0}^2 v}{\rho {V_0}^2}, \quad G_r = \dfrac{g\beta q v^2}{{V_0}^4 k} \\ G_m = \dfrac{g\beta_c m v^2}{{V_0}^4 D}, \quad E_c = \dfrac{k{V_0}^3}{v c_p q}, \quad S_1 = \dfrac{\overline{\nu}}{\nu}, \quad S_2 = \dfrac{\overline{k}}{k}, \quad S_c = \dfrac{v}{D} \end{array}\right\} \tag{7}$$

Into equations (3), (4) and (5) with boundary conditions (6) we get

$$-\frac{\partial u}{\partial y} = G_r\theta + G_m C + S_1\frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u \tag{8}$$

$$-\frac{\partial \theta}{\partial y} = S_2\left(\frac{1}{P_r}\right)\frac{\partial^2 \theta}{\partial y^2} + S_1 E_c\left(\frac{\partial u}{\partial y}\right)^2 + \frac{E_c}{K}u^2 \tag{9}$$

$$-S_c\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} \tag{10}$$

with the following boundary conditions

$$\left.\begin{array}{c} u = 1, \quad \frac{\partial \theta}{\partial y} = -1, \quad \frac{\partial C}{\partial y} = -1, \quad \text{at} \quad y = 0 \\ u \to 0, \quad \theta \to 0, \quad C \to 0, \quad \text{at} \quad y \to \infty \end{array}\right\} \tag{11}$$

where $G_r$ is Grashof number, $G_m$ modified Grashof number, $M$ is magnetic number, $P_r$ is prandtl number, $S_c$ is Schmidt number, $K$ is permeability parameter porous medium, ratio parameter of kinematic viscosity $(S_1)$, ratio parameter of conductivity $(S_2)$ and $E_c$ is Eckert number.

# 3 Method of Solution

Solving equation (10) using the boundary condition (11), we get

$$C = \frac{e^{-S_c y}}{S_c} \tag{12}$$

The coupled momentum and energy equations are solved by perturbation method. We have now to solve the coupled non-linear equations (8) and (9) under the boundary condition (11). For an approximate solution, we expand the physical variables $u$ and $\theta$ in the power of $E_c$, the Eckert number. For incompressible fluids $E_c < 1$ .Physically, this possible and can be interpreted as the flow due to viscous dissipative heat is superimposed on the main flow. Hence, mathematically we assume

$$u = u_0 + E_c u_1 + o({E_c}^2) \quad and \quad \theta = \theta_0 + E_c\theta_1 + o({E_c}^2) \tag{13}$$

Using the above perturbations in the governing equation (8) - (9) and equating the constant and coefficients of $E_c$, we get
**Zeroth order equations** with boundary conditions are given by

$$S_1 u_0'' + u_0' - \left(M + \frac{1}{K}\right) u_0 = -G_r\theta_0 - \frac{G_m e^{-S_c y}}{S_c} \tag{14}$$

$$S_2\theta_0'' + P_r\theta_0' = 0_0 \tag{15}$$

$$\left.\begin{array}{l} u_0 = 1, \quad \theta_0 = -1, \quad \text{at} \quad y = 0 \\ u_0 \to 0, \quad \theta_0 \to 0, \quad \text{at} \quad y \to \infty \end{array}\right\} \tag{16}$$

**First order equations** with boundary conditions are given by

$$S_1 u_1'' + u_1' - \left(M + \frac{1}{K}\right) u_1 = -G_r\theta_1 \tag{17}$$

$$KS_2\theta_1'' + KP_r\theta_1' = -S_1P_rK{u_0'}^2 - P_r{u_0}^2 \tag{18}$$

$$\left.\begin{array}{l} u_1 = 0, \quad \theta_1 = 0, \quad \text{at} \quad y = 0 \\ u_1 \to 0, \quad \theta_1 \to 0, \quad \text{at} \quad y \to \infty \end{array}\right\} \tag{19}$$

By solving equations (14) and (15) using the boundary conditions (16) ,we get

$$u_0 = A_{11}e^{-A_1 y} + A_{12}e^{-\frac{P_r}{S_2}y} + A_{13}e^{-S_c y} \tag{20}$$

$$\theta_0 = \frac{S_2}{P_r} e^{-\frac{P_r}{S_2} y} \tag{21}$$

$$\begin{aligned} u_1 &= A_{28} e^{-A_1 y} + A_{21} e^{-\frac{P_r}{S_2} y} + A_{22} e^{-2A_1 y} + A_{23} e^{-\frac{2P_r}{S_2} y} + A_{24} e^{-2S_c y} \\ &\quad + A_{25} e^{-\left(A_1 + \frac{P_r}{S_2}\right) y} + A_{26} e^{-\left(S_c + \frac{P_r}{S_2}\right) y} + A_{27} e^{-(S_c + A_1) y} \end{aligned} \tag{22}$$

$$\begin{aligned} \theta_1 &= A_{20} e^{-\frac{P_r}{S_2} y} + A_{14} e^{-2A_1 y} + A_{15} e^{-\frac{2P_r}{S_2} y} + A_{16} e^{-2S_c y} \\ &\quad + A_{17} e^{-\left(A_1 + \frac{P_r}{S_2}\right) y} + A_{18} e^{-\left(S_c + \frac{P_r}{S_2}\right) y} + A_{19} e^{-(S_c + A_1) y} \end{aligned} \tag{23}$$

Substituting equations (20) to (23) in equation (13) then, we get

$$\begin{aligned} u &= A_{11} e^{-A_1 y} + A_{12} e^{-\frac{P_r}{S_2} y} + A_{13} e^{-S_c y} + E_c \Big( A_{28} e^{-A_1 y} + A_{21} e^{-\frac{P_r}{S_2} y} \\ &\quad + A_{22} e^{-2A_1 y} + A_{23} e^{-\frac{2P_r}{S_2} y} + A_{24} e^{-2S_c y} + A_{25} e^{-\left(A_1 + \frac{P_r}{S_2}\right) y} \\ &\quad + A_{26} e^{-\left(S_c + \frac{P_r}{S_2}\right) y} + A_{27} e^{-(S_c + A_1) y} \Big) \end{aligned} \tag{24}$$

$$\begin{aligned} \theta &= \frac{S_2}{P_r} e^{-\frac{P_r}{S_2} y} + E_c \Big( A_{20} e^{-\frac{P_r}{S_2} y} + A_{14} e^{-2A_1 y} + A_{15} e^{-\frac{2P_r}{S_2} y} + A_{15} e^{-\frac{2P_r}{S_2} y} \\ &\quad + A_{16} e^{-2S_c y} + A_{17} e^{-\left(A_1 + \frac{P_r}{S_2}\right) y} + A_{18} e^{-\left(S_c + \frac{P_r}{S_2}\right) y} + A_{19} e^{-(S_c + A_1) y} \Big) \end{aligned} \tag{25}$$

The non-dimensional Shearing stress (skin friction) at the wall,

$$\begin{aligned} \tau_0 &= \frac{\nu}{{V_0}^2} \frac{\partial u'}{\partial y'}|_{y'=0} = \frac{\partial u}{\partial y}|_{y=0} = -\Big( A_{11} A_1 + \frac{P_r A_{12}}{S_2} + S_c A_{13} \\ &\quad + E_c \Big( A_1 A_{28} + 2A_1 A_{22} + 2S_c A_{24} + \frac{P_r A_{21}}{S_2} + \frac{2P_r A_{23}}{S_2} \\ &\quad + A_{25} \Big( A_1 + \frac{P_r}{S_2} \Big) + A_{26} \Big( S_c + \frac{P_r}{S_2} \Big) + A_{27} \Big( S_c + A_1 \Big) \Big) \Big) \end{aligned} \tag{26}$$

The non-dimensional rate of heat transfer, Nusselt Number

$$\begin{aligned} Nu &= -\frac{k}{q} \frac{\partial T'}{\partial y'}|_{y'=0} = -\frac{\partial \theta}{\partial y}|_{y=0} \\ &= 1 + E_c \Big( \frac{P_r A_{20}}{S_2} + 2A_1 A_{14} + \frac{2P_r A_{15}}{S_2} + 2S_c A_{16} \\ &\quad + A_{17} \left( A_1 + \frac{Pr}{S_2} \right) + A_{18} \left( S_c + \frac{P_r}{S_2} \right) + A_{19} \left( S_c + A_1 \right) \Big) \end{aligned} \tag{27}$$

The non-dimensional rate of mass transfer, Sherwood Number

$$Sh = -\frac{D}{m}\frac{\partial C'}{\partial y'}\bigg|_{y'=0} = -\frac{\partial C}{\partial y}\bigg|_{y=0} = 1 \tag{28}$$

where $A_1 = \dfrac{1+\sqrt{1+4S_1\left(M+\frac{1}{K}\right)}}{2S_1}$

$$A_{11} = 1-(A_{12}+A_{13}), \quad A_{12} = \frac{-G_r {S_2}^3}{S_1 {P_r}^3 - S_2 {P_r}^2 - {S_2}^2\left(M+\frac{1}{K}\right)},$$

$$A_{13} = \frac{-G_m}{S_1 {S_c}^3 - {S_c}^2 - S_2\left(M+\frac{1}{K}\right)}, A_{14} = -\left(\frac{S_1 P_r K {A_1}^2 {A_{11}}^2 + {A_{11}}^2 P_r}{4KS_2{A_1}^2 - 2KP_rA_1}\right)$$

$$A_{15} = -\left(\frac{P_r {A_{12}}^2 {S_2}^2 + S_1 {P_r}^3 {A_{12}}^2 K}{4K{P_r}^2 S_2 - 2K{P_r}^2 S_2}\right), A_{16} = -\left(\frac{S_1 P_r K {S_c}^2 {A_{13}}^2 + {A_{13}}^2 P_r}{4KS_2{S_c}^2 - 2KP_rS_c}\right)$$

$$A_{17} = -\left(\frac{2S_1{P_r}^2KA_1A_{11}A_{12} + 2P_rA_{11}A_{12}S_2}{K(A_1S_2+P_r)^2 - KP_r(A_1S_2+P_r)}\right)$$

$$A_{18} = -\left(\frac{2S_1{P_r}^2KS_cA_{13}A_{12} + 2P_rA_{13}A_{12}S_2}{K(S_cS_2+P_r)^2 - KP_r(S_cS_2+P_r)}\right)$$

$$A_{19} = -\left(\frac{2S_1S_cP_rKA_1A_{11}A_{12} + 2P_rA_{11}A_{13}S_2}{KS_2(A_1+S_c)^2 - KP_r(A_1+S_c)}\right)$$

$$A_{20} = -\frac{S_2}{P_r}\left[2A_{14}A_1 + \frac{2A_{15}P_r}{S_2} + 2A_{16}S_c + A_{17}\left(A_1+\frac{P_r}{S_2}\right) + A_{18}\left(S_c + \frac{P_r}{S_2} + A_{19}(S_c+A_1)\right)\right]$$

$$A_{21} = -\left\{\frac{G_rA_{20}}{S_1(\frac{P_r}{S_2})^2 - \frac{P_r}{S_2} - \left(M+\frac{1}{K}\right)}\right\}, \quad A_{22} = -\left\{\frac{G_rA_{14}}{4{A_1}^2S_1 - 2A_1 - \left(M+\frac{1}{K}\right)}\right\}$$

$$A_{23} = -\left\{\frac{G_rA_{15}}{4S_1(\frac{P_r}{S_2})^2 - \frac{P_r}{S_2} - \left(M+\frac{1}{K}\right)}\right\}, \quad A_{24} = -\left\{\frac{G_rA_{16}}{4{S_c}^2S_1 - 2S_c - \left(M+\frac{1}{K}\right)}\right\}$$

$$A_{25} = -\left\{\frac{G_rA_{17}}{S_1(A_1+\frac{P_r}{S_2})^2 - (A_1+\frac{P_r}{S_2}) - \left(M+\frac{1}{K}\right)}\right\}$$

$$A_{26} = -\left\{\frac{G_rA_{18}}{S_1(S_c+\frac{P_r}{S_2})^2 - (S_c+\frac{P_r}{S_2}) - \left(M+\frac{1}{K}\right)}\right\}$$

$$A_{27} = -\left\{\frac{G_rA_{19}}{S_1(A_1+S_c)^2 - (A_1+S_1) - \left(M+\frac{1}{K}\right)}\right\}$$

$$A_{28} = -(A_{21}+A_{22}+A_{23}+A_{24}+A_{25}+A_{26}+A_{27})$$

# 4 Result and Discussion

In this paper we have studied the dissipative heat and mass transfer in porous medium due to continuously moving plate in presence of magnetic field with constant heat and mass flux. The effect of the parameters $G_r, G_m, M, R, E_c, S_1, S_2, P_r$ and $S_c$ on flow characteristics have been studied and shown by means of graphs and tables. In order to have physical correlations, we choose suitable values of flow parameters. Shearing Stress has been calculated for different parameters.

**Velocity Profile:** The velocity profiles are depicted in Figs 2-5. Fig. 2 shows the effect of the parameters $G_r$ and $G_m$ on velocity at any point of the fluid. It is noticed that the velocity increases with the increase of Grashoff number $(G_r)$ and modified Grashoff number $(G_m)$. This implies that the present study is an buoyancy assisting flow with thermal buoyancy $(G_r)$ and mass buoyancy $(G_m)$. Here, it is pertinent to record that C accounts for the convective currents and may take positive or negative values. Physically, $G_r > 0$ signifies heating of the fluid (cooling of the plate). For increasing values of $G_r$ ($G_r > 0$ thermal buoyancy) and $G_m$ ($G_m > 0$ mass buoyancy), the velocity increases. This shows that a buoyant force enhances the momentum transport so that velocity increases.

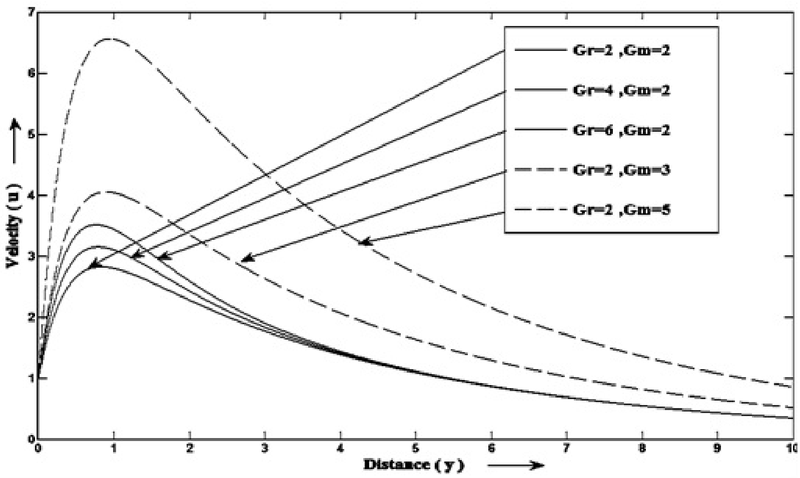


Figure 2: Effect of $G_r$ and $G_m$ on velocity profile, when $S_c = 0.23, P_r = 0.71, M = 2, K = 2, E_c = 0.001, S_1 = 0.8$ *and* $S_2 = 0.8$

Fig. 3 displays the effects of the parameters $M$ and $K$ on velocity. It is observed that the velocity decreases with the increase of Magnetic parameter $(M)$, but increases with the increase of permeability parameter of the porous medium $(K)$.

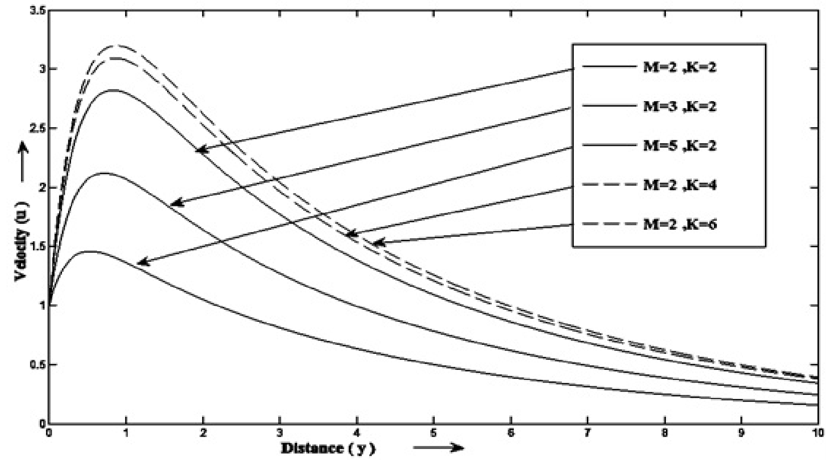


Figure 3: Effect of $M$ and $K$ on velocity profile, when $S_c = 0.23, P_r = 0.71, G_r = 2, G_m = 2, E_c = 0.001, S_1 = 0.8$ *and* $S_2 = 0.8$

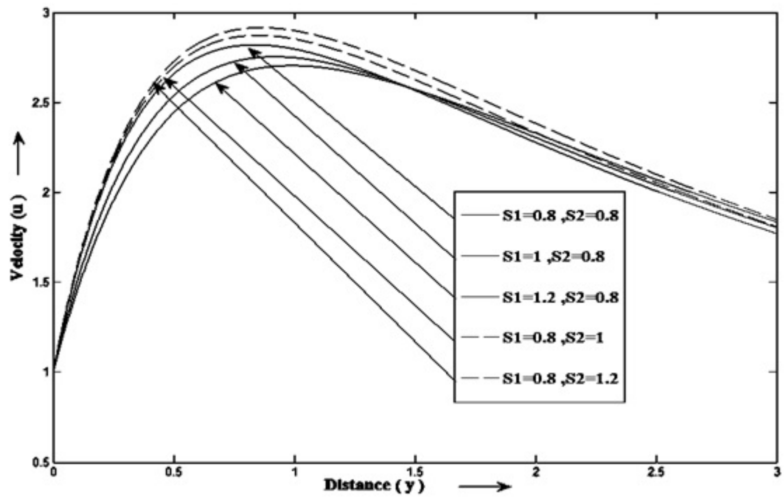


Figure 4: Effect of $S_1$ and $S_2$ on velocity profile, when $S_c = 0.23, P_r = 0.71, M = 2, K = 2, E_c = 0.001, G_r = 0.8$ *and* $G_m = 0.8$

Fig. 4 depicts the effect of the parameters $S_1$ and $S_2$ on velocity in the boundary layer. It is seen that the velocity decreases with the increase of $(S_1)$, but the reverse effects is observed for $(S_2)$.

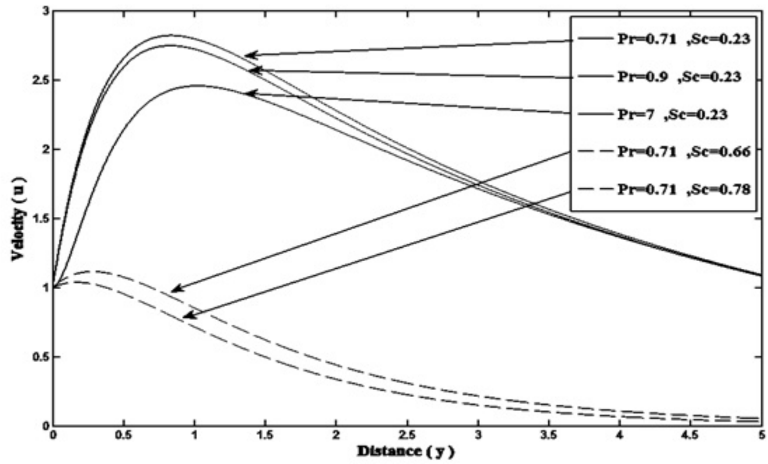


Figure 5: Effect of $S_c$ and $P_r$ on velocity profile, when $M = 2, K = 2, G_r = 2, G_m = 2, E_c = 0.001, S_1 = 0.8$ *and* $S_2 = 0.8$

From fig. 5 It is noticed that the velocity decreases with the increase of Schmidt number $(S_c)$, but increase with the increase of Prandtl number $(P_r)$. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Physically it can be interpreted as; heavier diffusing species, the velocity gets retarded whereas increasing $P_r$ means increasing momentum diffusivity for which velocity increases.

One striking feature of the velocity field is that the velocity decreases under the influence of magnetic field in the presence of porous matrix which is due to electromagnetic resistive forces opposing the motion in the main direction of the flow.

**Temperature Profile:** The temperature profiles are depicted in Figs 6-9. Fig. 6 reveals that the effect of the parameters $(G_r)$ and $(G_m)$ on Temperature profile. It is noticed that the temperature rises in the increase of Grashoff number $(G_r > 0)$ and modified Grashoff number $(G_m > 0)$. Physically it can be interpreted as: $G_r > 0$, cooling of the plate which means that energy flows from the plate to the fluid, so that temperature increases. Similar explanation could be for $G_m > 0$ in case of level of concentration.

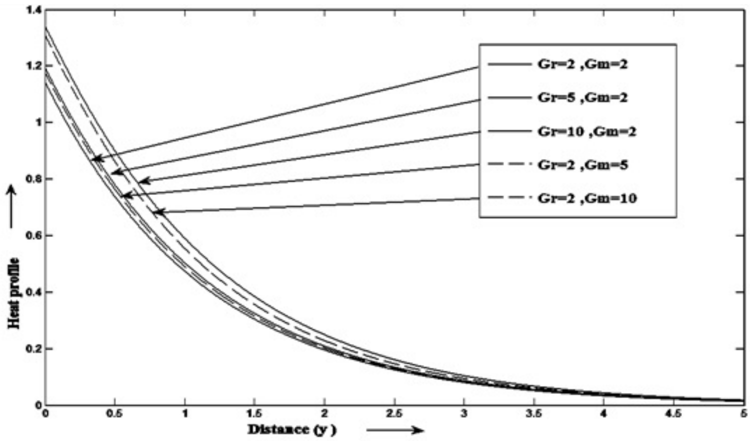


Figure 6: Effect of $G_r$ and $G_m$ on temperature profile, when $S_c = 0.23, P_r = 0.71, M = 2, K = 2, E_c = 0.001, S_1 = 0.8 and S_2 = 0.8$

Fig.7 shows the effect of the parameters $S_1$ and $S_2$ on Temperature profile. It is noticed that the temperature rises with the increase of $S_1$ and $S_2$.

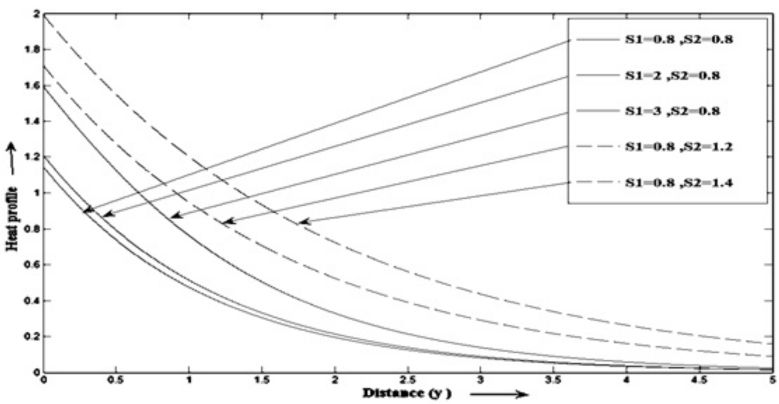


Figure 7: Effect of $S_1$ and $S_2$ on temperature profile, when $S_c = 0.23, P_r = 0.71, G_r = 2, G_m = 2, E_c = 0.001, M = 2 and K = 2$

Fig.8 shows that the temperature falls in the increase of Schmidt number ($S_c$) and Prandtl number ($P_r$). Physically interpretation runs as: higher $S_c$ means heavier diffusing species which absorbs heat more consequently temperature falls. In case of $P_r$, higher $P_r$ means low thermal conductivity, hence temperature falls in the thermal boundary layer.

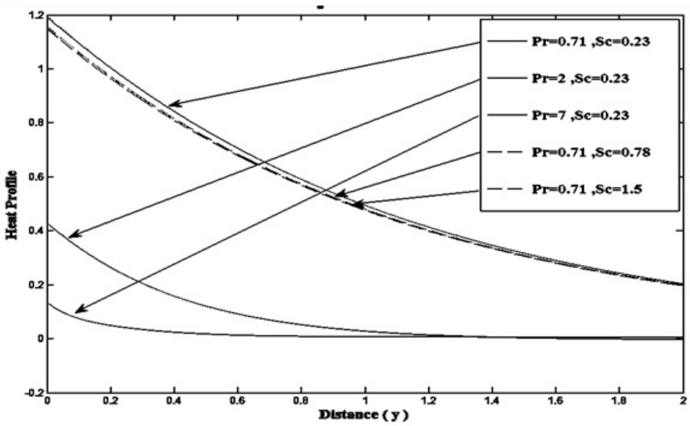


Figure 8: Effect of $P_r$ and $S_c$ on temperature profile, when $S_1$ and $S_2$ on velocity profile, when $G_r = 2, G_m = 2, M = 2, K = 2, E_c = 0.001, S_1 = 0.8 \ \ and \ \ S_2 = 0.8$

Fig.9 shows the effect of the parameters $M$, $K$ and $E_c$ on temperature profile at any point of the fluid. It is remarked that the temperature falls with an increase of magnetic parameter ($M$), whereas rises with the increase of permeability parameter porous medium ($K$) and Eckert number ($E_c$). Higher Eckert number $E_c$ increases the temperature is interpreted as rise in temperature occurs due to rise in stored heat energy in the fluid for frictional heating (viscous dissipation)

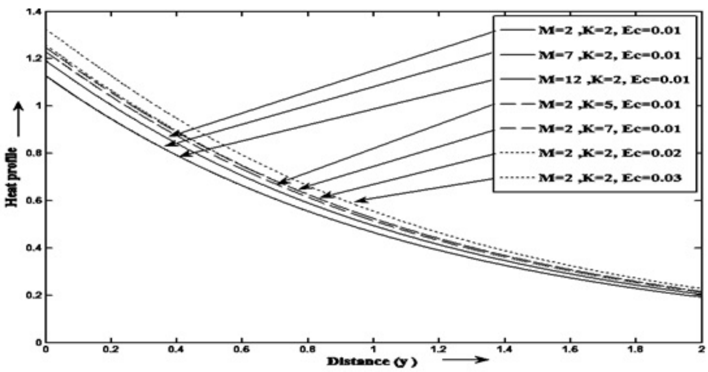


Figure 9: Effect of $M$ $K$ and $E_c$ on temperature profile, when $S_c = 0.23, P_r = 0.71, G_r = 2, G_m = 2, S_1 = 0.8 \ \ and \ \ S_2 = 0.8$

It is concluded that the effect of increasing Grashof number or modified Grashof number or permeability of the porous medium or ratio parameter of conductivity or ratio parameter of kinematic viscosity or Eckert number enhance the temperature field in the flow domain whereas the effect of increasing Schmidt number or Prandtl number or magnetic parameter reduce the temperature field in the flow domain. One striking feature of the temperature field is that the temperature decreases under the influence of magnetic field in the presence of porous matrix.

**Skin friction:** Surface criteria such as skin friction , Nusselt number etc are very much important as because those affects the momentum transport and thermal energy transport from the surface to the flow domain.

The Table-1 depicts the effect of important parameters on the skin friction. It is observed that both the buoyant forces due to thermal buoyancy and solutal buoyancy, porosity of the saturated porous medium and Eckert number enhance the skin friction.

The buoyant forces enhance the momentum transport in the fluid, consequently, momentum transport decreases at the bounding surface reducing the velocity at the bounding surface. In other words, more shearing stress is experienced, resulting higher skin friction. Similar explanation can be attributed to Eckert number which is measure thermal energy dissipation due to viscosity of the fluid, the resisting inherent property of the fluid; through work done. Hence, the fluid temperature is escalated in the flow domain resulting slow down the energy transport at the surface producing higher friction at the surface. On the other hand, an increase in Prandtl number ($P_r$) and Magnetic parameter ($M$) reduce the skin friction.

An increase in Prandtl number ($P_r$), leads to low rate of thermal diffusion of the flow domain and hence decrease in temperature in the flow domain but enhances the thermal energy transport at the surface, accelerating the fluid flow and reduce the skin friction.

Similar explanation can be attributed to magnetic parameter (a resistive force) which reduces the velocity in the main direction of flow; there by enhances the fluid velocity at the surface, leading the less skin friction.

# 5 Conclusion

Inclusion of viscous dissipation ($E_c \neq 0$) in a flow through porous media is for gaining higher temperature in the flow and experiencing greater shearing stress at the plate.. The effect of increasing Grashof number and modified Grashof number accelerate the velocity profiles, temperature profiles and shearing stress. An increase in Schmidt number decreases the velocity as well as temperature profiles. The effect of increasing of Prandtl number has opposite effect on velocity to that of temperature profiles. Magnetic parameter has a retarding effect on velocity, temperature and skin friction coefficient in the presence of porous matrix which is desirable. The effect of increasing the permeability of the porous medium enhances velocity, temperature and skin friction.

| when $K = 2, E_c = 0.01$, $S_1 = 0.8, S_2 = 0.8$ and $S_c = 0.23$ | | | | shearing stress $\tau$ | when $P_r = 0.71, G_r = 2$, $G_m = 2$, and $S_c = 0.23$ | | | | shearing $\tau_0$ |
|---|---|---|---|---|---|---|---|---|---|
| $P_r$ | $G_r$ | $G_m$ | $M$ | | $K$ | $S_1$ | $S_2$ | $E_c$ | |
| 0.71 | 2 | 2 | 2 | 6.575 | 2 | 0.8 | 0.8 | 0.01 | 6.575 |
| 1.2 | | | | 6.213 | 4 | | | | 7.160 |
| 7 | | | | 0.749 | 6 | | | | 7.364 |
| 0.71 | 4 | | | 7.848 | 0.8 | 1 | | | 5.670 |
| | 6 | | | 9.054 | | 1.2 | | | 5.047 |
| | 2 | 4 | | 14.528 | | 0.8 | 1 | | 6.644 |
| | | 6 | | 22.608 | | | 1.2 | | 6.704 |
| | | 2 | 4 | 3.332 | | | 0.8 | 0.02 | 6.608 |
| | | | 6 | 1.663 | | | | 0.03 | 6.641 |

Table 1: Effect of different parameters on shearing stress

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# 6 Appendix

$$y = \frac{V_0 y'}{\nu}, \quad u = \frac{u'}{V_0}, \quad \theta = \frac{(T' - T'_\infty)V_0 k}{q\nu}, \quad P_r = \frac{\rho v c_p}{k},$$
$$C = \frac{(C' - C'_\infty)V_0 D}{mv}, \quad K = \frac{{V_0}^2 k_p}{v^2}, \quad M = \frac{\sigma {B_0}^2 v}{\rho {V_0}^2}, \quad G_r = \frac{g\beta q v^2}{{V_0}^4 k},$$
$$G_m = \frac{g\beta_c m v^2}{{V_0}^4 D}, \quad E_c = \frac{k{V_0}^3}{v c_p q}, \quad S_1 = \frac{\overline{\nu}}{\nu}, \quad S_2 = \frac{\overline{k}}{k}, \quad S_c = \frac{v}{D}$$

$$V\frac{\partial u'}{\partial y'} = g\beta(T - T_\infty) + g\beta_c(C - C_\infty) + \overline{\nu}\frac{\partial^2 u'}{\partial {y'}^2} - \frac{\nu}{k_p}u' - \frac{\sigma {B_0}^2}{\rho}u'$$
$$\Rightarrow V\frac{\partial(uV_0)}{\partial(\frac{y\nu}{V_0})} = g\beta\frac{\theta q\nu}{V_0 k} + g\beta_c\frac{Cm\nu}{V_0 D} + \overline{\nu}\frac{\partial^2(uV_0)}{\partial(\frac{y\nu}{V_0})^2} - \frac{\nu}{k_p}uV_0 - \frac{\sigma {B_0}^2}{\rho}uV_0$$
$$\Rightarrow -V_0\frac{{V_0}^2}{\nu}\frac{\partial u}{\partial y} = g\beta\frac{\theta q\nu}{V_0 k} + g\beta_c\frac{Cm\nu}{V_0 D} + \overline{\nu}\frac{{V_0}^3}{\nu^2}\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k_p}uV_0 - \frac{\sigma {B_0}^2}{\rho}uV_0$$

Now,on simplification we get

$$-\frac{\partial u}{\partial y} = g\beta\frac{q\nu^2}{{V_0}^4 k}\theta + g\beta_c\frac{m\nu^2}{V_0^4 D}C + \frac{\overline{\nu}}{\nu}\frac{\partial^2 u}{\partial y^2} - \frac{\nu^2}{{V_0}^2 k_p}u - \frac{\sigma {B_0}^2\nu}{\rho {V_0}^2}u$$
$$-\frac{\partial u}{\partial y} = G_r\theta + G_m C + S_1\frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u$$
$$V\frac{\partial T'}{\partial y'} = \frac{\overline{k}}{\rho c_p}\frac{\partial^2 T'}{\partial {y'}^2} + \frac{\overline{\nu}}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{\nu}{c_p k_p}{u'}^2$$

$$\Rightarrow V\frac{\partial\left(\frac{\theta q\nu}{V_0 k} + T'\infty\right)}{\partial\left(\frac{y\nu}{V_0}\right)} = \frac{\overline{k}}{\rho c_p}\frac{\partial^2\left(\frac{\theta q\nu}{V_0 k} + T'\infty\right)}{\partial\left(\frac{y\nu}{V_0}\right)^2} + \frac{\overline{\nu}}{c_p}\left(\frac{\partial\left(uV_0\right)}{\partial\left(\frac{y\nu}{V_0}\right)}\right)^2 + \frac{\nu}{c_p k_p}(uV_0)^2$$
$$\Rightarrow -V_0\frac{V_0}{\nu}\frac{q\nu}{V_0 k}\frac{\partial\theta}{\partial y} = \frac{\overline{k}}{\rho c_p}\frac{{V_0}^2}{\nu^2}\frac{q\nu}{V_0 k}\frac{\partial^2\theta}{\partial y^2} + \frac{\overline{\nu}}{c_p}\frac{{V_0}^4}{\nu^2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\nu}{c_p k_p}{V_0}^2 u^2$$

On simplification we get

$$-\frac{\partial\theta}{\partial y} = \frac{\overline{k}}{k}\frac{k}{\rho\nu c_p}\frac{\partial^2\theta}{\partial y^2} + \frac{\overline{\nu}}{\nu}\frac{k{V_0}^3}{\nu c_p q}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{k{V_0}^4}{V_0 c_p q\nu}\frac{\nu^2}{{V_0}^2 k_p}u^2$$
$$-\frac{\partial\theta}{\partial y} = S_2\left(\frac{1}{P_r}\right)\frac{\partial^2\theta}{\partial y^2} + S_1 E_c\left(\frac{\partial u}{\partial y}\right)^2 + \frac{E_c}{K}u^2$$

$$V\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial {y'}^2}$$

$$\Rightarrow -V_0\frac{\partial\left(\frac{Cm\nu}{V_0 D}+C_0'\right)}{\partial\left(\frac{y\nu}{V_0}\right)} = D\frac{\partial^2\left(\frac{Cm\nu}{V_0 D}+C_0'\right)}{\partial\left(\frac{y\nu}{V_0}\right)^2}$$

$$\Rightarrow -V_0\frac{V_0}{\nu}\frac{m\nu}{V_0 D}\frac{\partial C}{\partial y} = D\frac{{V_0}^2}{\nu^2}$$

$$\frac{m\nu}{V_0 D}\frac{\partial^2 C}{\partial y^2}$$

On simplification we get

$$-\frac{\nu}{D}\frac{\partial C}{\partial y} = \frac{V_0 m}{\nu}\frac{\nu}{mV_0}\frac{\partial^2 C}{\partial y^2}$$

$$-S_c\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2}$$

**Shearing stress (skin friction)** at the plate surface is given by

$$-\left(\frac{\partial u'}{\partial y'}\right)_{y=0} = -\frac{\partial\left(u\nu_0\right)}{\partial\left(\frac{y\nu}{\nu_0}\right)} = \frac{{\nu_0}^2}{\nu}\frac{\partial u}{\partial y}\bigg|_{y=0}$$

$$\tau_0 = \frac{\nu}{{V_0}^2}\frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\partial\left(u_0+E_c u_1\right)}{\partial y} = \frac{\partial u_0}{\partial y} + E_c\frac{\partial u_0}{\partial y}$$

Substituting the value of $u$ and evaluating at $y=0$ we get

$$\tau_0 = \frac{\nu}{{V_0}^2}\frac{\partial u'}{\partial y'}\bigg|_{y'=0} = \frac{\partial u}{\partial y}\bigg|_{y=0} = -\Bigg(A_{11}A_1 + \frac{P_r A_{12}}{S_2} + S_c A_{13}$$
$$+ E_c\Bigg(A_1 A_{28} + 2A_1 A_{22} + 2S_c A_{24} + \frac{P_r A_{21}}{S_2} + \frac{2P_r A_{23}}{S_2}$$
$$+ A_{25}\left(A_1 + \frac{P_r}{S_2}\right) + A_{26}\left(S_c + \frac{P_r}{S_2}\right) + A_{27}\left(S_c + A_1\right)\Bigg)\Bigg)$$

**Rate of heat transfer(Nusselt Number):** at the surface is given by

$$-\frac{\partial T'}{\partial y'}|_{y=0} = -\frac{\left(\frac{\theta q\nu}{V_0 k}+T_\infty'\right)}{\left(\frac{y\nu}{V_0}\right)}|_{y=0} = -\frac{V_0}{\nu}\frac{q\nu}{V_0 k}|_{y=0} = -\frac{q}{k}\frac{\partial\theta}{\partial y}|_{y=0}$$

$$Nu = -\frac{k}{q}\frac{\partial T'}{\partial y'}\bigg|_{y'=0} = -\frac{\partial \theta}{\partial y}\bigg|_{y=0}$$
$$\frac{\partial \theta}{\partial y} = \frac{\partial \theta_0}{\partial y} + E_c\frac{\partial \theta_1}{\partial y}$$

Substituting the value of $\theta$ and evaluating at $y = 0$ we get

$$\begin{aligned} Nu &= -\frac{k}{q}\frac{\partial T'}{\partial y'}\bigg|_{y'=0} = -\frac{\partial \theta}{\partial y}\bigg|_{y=0} \\ &= 1 + E_c\Bigg(\frac{P_r A_{20}}{S_2} + 2A_1A_{14} + \frac{2P_rA_{15}}{S_2} + 2S_cA_{16} \\ &\quad + A_{17}\left(A_1 + \frac{Pr}{S_2}\right) + A_{18}\left(S_c + \frac{P_r}{S_2}\right) + A_{19}\left(S_c + A_1\right)\Bigg) \end{aligned}$$

$$-\frac{\partial C'}{\partial y'}\bigg|_{y=0} = -\frac{\left(\frac{Cm\nu}{V_0 D} + C'_\infty\right)}{\left(\frac{y\nu}{V_0}\right)}\Bigg|_{y=0} = -\frac{V_0}{\nu}\frac{m\nu}{V_0 D}\bigg|_{y=0} = -\frac{m}{D}\frac{\partial \theta}{\partial y}\bigg|_{y=0}$$

$$Sh = -\frac{D}{m}\frac{\partial C'}{\partial y'}\bigg|_{y'=0} = -\frac{\partial C}{\partial y}\bigg|_{y=0}$$
$$\frac{\partial C}{\partial y} = \frac{1}{S_c}(-S_c)e^{-S_c y}$$
$$\frac{\partial C}{\partial y}\bigg|_{y=0} = -1$$
$$Sh = -\frac{D}{m}\frac{\partial C'}{\partial y'}\bigg|_{y'=0} = -\frac{\partial C}{\partial y}\bigg|_{y=0} = -1$$

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