Travel Mode Choice Modeling Using Fuzzy Extension of the Analytic Network Process: A Case Study of Palinda Nuwara

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Abstract

This paper presents the Fuzzy extension of the Analytic Network Process (FANP) to model the travel mode choice for shopping trips. Palinda Nuwara division in Sri Lanka was selected for the case study. Six mode choices and eight attributes were included in the decision network structure. A home based questionnaire survey with face to face interviews was conducted. Based on the field survey data analysis, the fuzzy membership functions for travel time and travel expenses were developed and applied to the FANP model. The results of the study showed that the FANP model replicated about 60% of the observed choices. It was concluded that the proposed FANP model establishes fairly good performance in mode choice prediction.

Keyword: Travel mode choice models, Fuzzy extension of the Analytic Network Process, Uncertainty in decision making

1. Introduction

Clear understanding of individual evaluations of attributes and alternatives is necessary for mode choice modeling. Large number of researches has been focused on the relationships between factors and personal travel patterns. Most common influential trip characteristics that effect mode choice are listed below.

Travel time and travel cost are cited as mode choice influential factors [1]. Adverse weather conditions have an impact on the level of service an operator provides [2]. Reliability and safety issues are considered when selecting the travel mode [3]. The comfort and convenience is valued differently by people. The value of personal environment control, flexibility of scheduling, punctuality, and ease of carrying things affect overall convenience. In the United States private automobile was found as the safest mode of transportation, while bus was described as least safe [4].

Travel mode choice methods model a travel choice among a set of mutually exclusive alternative modes of travel. For several decades, numerous technical analyses such as discrete choice models have been focused on solving mode choice problem. The Analytic Network Process (ANP) is a widely accepted decision making method which facilitates to solve the mode choice problem [5]. The uncertainly associated with the traveler's vagueness expression can be addressed using the fuzzy mathematics. However, any attempt to validate the fuzzy extension of the ANP for mode choice modeling has not been observed in the literature.

2. Methodology

The FANP method applied in this study is described step by step below.

Step 1 Formulation of mode choice decision problem

The formulation of the mode choice decision network structure was based on the travelers' characteristics and the identified existing mode choices of the study area. The

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structure was formulated based on previous studies related to mode choice.

Step 2 Making pairwise comparisons

Once the FANP network was constructed then pairwise comparisons between elements with respect to a controlling element were made. The intensity of preference was assigned to the comparisons process using the Saaty's nine-point scale and degree of fuzziness (δ) (Table 1, Figure 1) based on [6].

Step 3 Checking of the consistency judgments

In the large scale decision making problem, while doing pairwise comparisons, the respondents might misevaluate unconsciously. Hence, it was required to control the consistency of judgments using the Consistency Index (CI) as presented in Equation 1.

$$CI = \frac{\lambda_{\max} - 1}{n - 1} \quad (1)$$

Where $\lambda_{\max}$ is the largest eigenvalue of the pairwise

Table 1 Nine-point scale for pairwise comparative judgments $\tilde{a}_{ij} \geq 1$

Numerical scale	Linguistic scale $I(element\ i) \geq I(element\ j)^*$	Fuzzy scale $(l_{ij}\ m_{ij}\ u_{ij})$
1	Just equal importance	$(\max(L_B,1-\delta)\ ,1\ ,\min(U_B,1+\delta))$
2	Equal to weak importance	$(\max(L_B,2-\delta)\ ,2\ ,\min(U_B,2+\delta))$
3	Weak importance	$(\max(L_B,3-\delta)\ ,3\ ,\min(U_B,3+\delta))$
4	Weak to strong importance	$(\max(L_B,4-\delta)\ ,4\ ,\min(U_B,4+\delta))$
5	Strong importance	$(\max(L_B,5-\delta)\ ,5\ ,\min(U_B,5+\delta))$
6	Strong to very strong importance	$(\max(L_B,6-\delta)\ ,6\ ,\min(U_B,6+\delta))$
7	Very strong importance	$(\max(L_B,7-\delta)\ ,7\ ,\min(U_B,7+\delta))$
8	Very strong to absolute importance	$(\max(L_B,8-\delta)\ ,8\ ,\min(U_B,8+\delta))$
9	Absolute importance	$(\max(L_B,9-\delta)\ ,9\ ,\min(U_B,9+\delta))$

L_B - Lower bound U_B - Upper bound ; δ - degree of fuzziness

- pairwise verbal comparisons, Importance of $element\ i$ over $element\ i$ may be interpreted as importance, preference, likelihood or influence
- Triangular fuzzy number is denoted by (l_{ij}, m_{ij}, u_{ij})

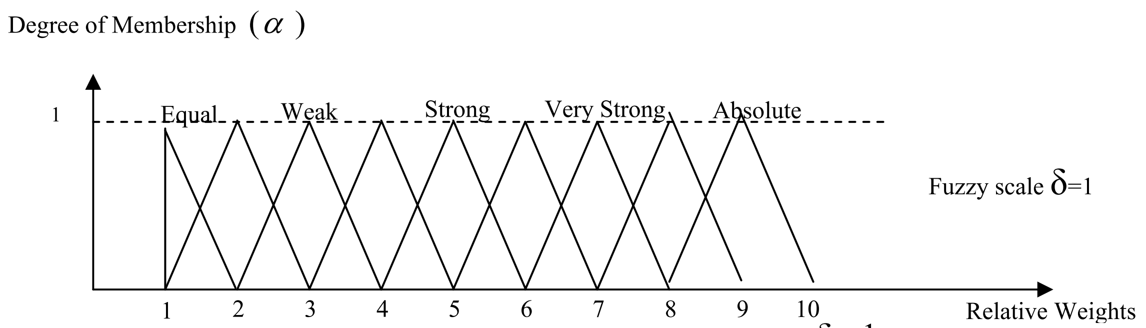

Figure 1 Fuzzy membership function at $\delta = 1$

comparison matrix. The CI of the randomly generated reciprocal matrices using nine point scale were given by Saaty [7] as shown in Table 2. Comparison matrices were checked if it had an acceptable level of inconsistency according to Saaty's Consistency Ratio (CR) [7] as presented in Equation 2.

$$CR = \frac{CI}{RI} \quad (2)$$

To calculate CR, the same order of Random consistency (RI) is used. Normally if the CR is less than 0.1, the comparison matrix was considered as acceptable and satisfactory.

Step 4 Aggregating the judgment matrices

The aggregations of individual judgments were done by using geometric mean method. Let $A_1, A_2, \ldots, A_s$ be judgment matrices for the same decision problem, then the weighted geometric mean complex judgment matrix $\overline{A}$ is;

$$\overline{A} = A_1^{\lambda_1} \circ A_2^{\lambda_2} \circ \ldots \circ A_s^{\lambda_s} \quad (3)$$

$$\sum_{k=1}^{s} \lambda_k = 1, \quad \lambda_k > 0 \; (k = 1, 2, \ldots s) \quad (4)$$

The Hadamard product of matrices is denoted by $\circ$. If R is the set of real numbers, we denote $A^{\lambda} = \left(a_{ij}^{\lambda}\right)$, where $\lambda \in R$.

Step 5 Generating the fuzzy comparison matrices

By using triangular fuzzy number in fuzzy scales, via pairwise comparison, the fuzzy judgment matrix $\widetilde{A}$ is an $n \times n$ fuzzy matrix containing fuzzy numbers $\widetilde{a}_{ij}$ as presented in Figure 2. The arithmetic operations on these fuzzy numbers were based on the fuzzy arithmetic for all levels of decision.

$$\widetilde{A} = \begin{bmatrix} 1 & \widetilde{a}_{12} & \cdots & \widetilde{a}_{1n} \\ \widetilde{a}_{21} & 1 & \cdots & \widetilde{a}_{2n} \\ \vdots & \vdots & & \vdots \\ \widetilde{a}_{n1} & \widetilde{a}_{n2} & \cdots & 1 \end{bmatrix}$$

where $\widetilde{a}_{ji} = \widetilde{a}_{ij}^{\,-1}$

Figure 2 Fuzzy comparison matrix

Step 6 Calculation of the fuzzy eigenvalues

At this step the fuzzy eigenvector was calculated while identifying the maximum fuzzy eigenvalue, $\widetilde{\lambda}$.

$$\widetilde{A}\,\widetilde{w} = \widetilde{\lambda}\,\widetilde{w} \quad (5)$$

$\widetilde{w}$ is a positive $1 \times n$ fuzzy vector containing fuzzy numbers $\widetilde{w}_i$. To perform fuzzy addition $(\oplus)$ and multiplication $(\otimes)$, Equation 5 was arranged as Equation 6.

$$\left(\widetilde{a}_{i1} \otimes \widetilde{w}_1\right) \oplus \ldots\ldots\ldots \left(\widetilde{a}_{in} \otimes \widetilde{w}_n\right) = \widetilde{\lambda} \otimes \widetilde{w}_i \quad \forall\, 1 \le i \le n \quad (6)$$

Where $\widetilde{A} = \widetilde{a}_{ij}$, $\widetilde{w}^t = \left(\widetilde{w}_1 \cdots\cdots \widetilde{w}_n\right)$ and $\widetilde{\lambda}$ is fuzzy eigenvalue.

Step 7 Transformation of fuzzy matrices to crisp matrices

Degree of optimism was presented using index of optimism index β [5]. The use of optimism index allows the decision maker's attitude towards the fuzziness of judgment. For example, an 'optimistic' decision maker ($\beta = 1$) was to assign higher values of the crisp interval at a given level of confidence (α) while a 'pessimistic' decision maker ($\beta = 0$) tended to favor the lower values of the said interval. The optimism index (β) was defined as the linear convex combination as presented in Equation 7.

Table 2 Random consistency index for difference sizes of matrix (n)

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	.52	.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

$$\tilde{a}_{ij}^{\alpha} = \beta a_{iju}^{\alpha} + (1-\beta) a_{ijl}^{\alpha} \quad \forall \beta \in [0,1] \tag{7}$$

Where (u, l) are the interval values of fuzzy number a at confidence level α. After α and β are set, the following crisp pairwise comparison matrix was obtained (Figure 3).

$$\hat{A} = \begin{bmatrix} 1 & \hat{a}_{12} & \cdots & \hat{a}_{1n} \\ \hat{a}_{21} & 1 & \cdots & \hat{a}_{2n} \\ \vdots & \vdots & & \vdots \\ \hat{a}_{n1} & \hat{a}_{n2} & \cdots & 1 \end{bmatrix}$$

Figure 3 Crisp comparison matrix

Step 8 Calculating the crisp eigenvalues

From the matrix $\hat{A}$, the local priorities were calculated using the normalized principal right eigenvector for all α-cut and β values. The principal right eigenvector was approximated from Equation 8 [5].

$$\lim_{k \to \infty} \frac{\hat{A}^k e}{e^T \hat{A}^k e} = c\hat{w} \tag{8}$$

where e^T is a unit row vector; c is a constant; and $\hat{w}$ is the eigenvector corresponding to the principal eigenvalue of the matrix $\hat{A}$.

Step 9 Populating the supermatrix

The initial supermatrix can be populated using selected local priorities. The initial supermatrix is adjusted to be column stochastic so that the sum of each column is equal to one using the cluster weights. The adjusted matrix is now called the weighted supermatrix. Then the meaningful overall priorities can be derived using Equation 9.

$$\lim_{p \to \infty} \frac{\hat{S}^p e}{e^T \hat{S}^p e} = q\hat{v} \tag{9}$$

Where q is a constant and $\hat{v}$ is the eigenvector corresponding to the principal eigenvalue of the primitive supermatrix $\hat{S}$.

Step 10 Aggregating the solutions

The calculations were repeated from steps 3 to 5 in order to derive the set of desired priorities for all α-cuts at different β values.

Step 11 Validating the decision model

The FANP model performance was evaluated by comparing the simulated choice with actual behavior by each respondent in the validating process.

3. Case Study

Palinda Nuwara, a division western province of Sri Lanka (Figure 4), was selected as the survey site primarily because it is a typical town with various transportation modes available. Sunday fair (weekend market) is an attractive market in the region. Bus owners provide special short distance bus services in order to achieve the passenger demand. Additionally, Sunday van services and three wheelers (motorized) play major roles. People who have their own vehicles tend to use those for shopping trips. The total population in the study area is 32,790, which includes 8,247 households. The questionnaires were developed in accordance with the carefully configured decision network structure which all clusters compared were linked to each other, as shown in Figure 5. If there is only one-way connection between two clusters, only one-way dependencies exist and such a situation is represented with directed arrow. If there is a two-way dependence between two clusters, bi-directed arrows are used. In order to develop the FANP models, the respondents were asked to rank the paired options (Figure 6). Simple random sampling was used to collect the data of sample size of approximately 497 around 5 km distance from the market. The surveyors went to different villages such that all locations are covered in May 2007. People, who make shopping trips were given questionnaire forms and asked to complete it. To validate the proposed model each traveler was given 30

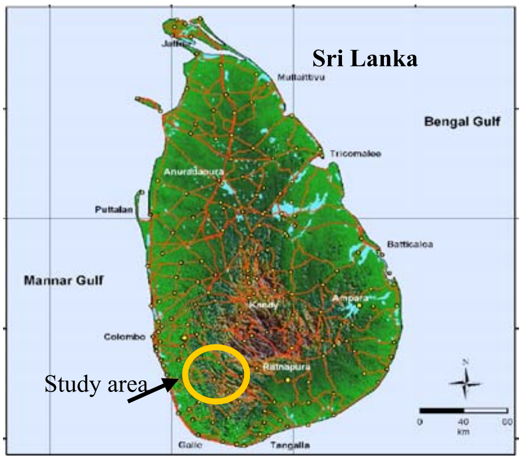

Figure 4 Location of study area

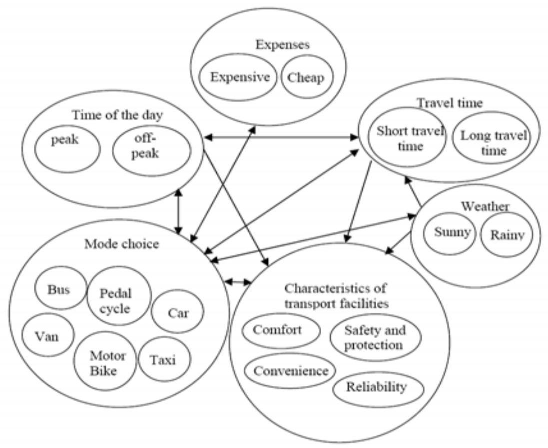

Figure 5 Decision network structure

different scenarios considering travel time, expenses, weather condition and time of the day with six alternatives (Figure 7).

4. Results

After developing the mode choice decision model, relative importance of the related factors were examined.

According to the synthesis weights in Figure 8, transportation characteristics (38.13%) and travel time (14.85%) can be regarded as more important than other related factors.

The synthesis weights show that travel time, traffic congestion (peak- off peak) and travel cost, convenience, safety, reliability, are also important measurement indicators for the travel mode choice in the study area. It is seen that most often respondents assign the highest weight to travel time and the lowest weight to comfort. According to cluster normalized priorities, the most preferable condition regarding travel time is short travel time (65.79%). Among expenses related factors, the most preferable one is cheap travel cost (67.96%). Sunny weather (54.32%) is regarded as the most preferable weather condition. When time of the day is taken into consideration, the most preferable factor is off peak period (64.84%). Regarding the characteristics of transport facilities, safety has the most significant value (27.84%) followed by convenience (27.56%), reliability (25.24%), and comfort (19.37%). In travel choice studies, it is common to apply the model at the level of aggregation. As in Figure 8, the most preferred mode choice is car (25%), followed by three wheeler (21%), motorbike (17%), and bus (14%). Van (11%) and pedalcycle (11%) are the least preferred modes among other alternatives. These global preferences represent the mode share of shopping trips in most general case. The cumulative frequency of respondents was plotted against the travel time in order to determine the fuzzy membership

Suppose it is sunny day. Which mode has grater preference to use shopping trips and how strongly?

	Sunny day																		
Q1	car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	bus
Q2	car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	motorbike
Q3	car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	three wheeler

Figure 6 An example of pairwise comparison questions

Suppose it is sunny day. Your trip is in peak time.

Transport modes	car	motorbike	bus	three wheeler	pedalcycle	van
Expenses (Rs)	20	60	15	120	non	15
Travel time (min)	20	8	18	8	40	30
I would select						

Figure 7 An example of different scenarios given to traveler

		Travel time		Expenses		Weather		Time of the day		Characteristics of transport facilities				Mode choice						The synthesis weights	Weights (%) by culter	Synthesis weights (%)	Nomalized by culster (%)
		A- short	B- long	C-expensive	D-cheap	E-sunny	F-rainy	G-peak	H-off peak	J-convenience	K-safety	L-comfort	M-reliability	N-car	O-bus	P-motorbike	Q-three wheeler	R-van	S-pedalcycle				
Travel time	A	0	0	0	0	0.246	0.228	0.234	0.218	0	0	0	0	0.156	0.132	0.171	0.168	0.166	0.170	0.0977	14.85	9.77	65.79
	B	0	0	0	0	0.093	0.091	0.099	0.104	0	0	0	0	0.074	0.092	0.074	0.079	0.091	0.084	0.0508		5.08	34.21
Expenses	C	0	0	0	0	0	0	0	0	0	0	0	0	0.058	0.052	0.048	0.056	0.069	0.048	0.0238	8.99	2.88	32.04
	D	0	0	0	0	0	0	0	0	0	0	0	0	0.117	0.158	0.134	0.126	0.116	0.141	0.0611		6.11	67.96
Weather	E	0	0	0	0	0	0	0	0	0	0	0	0	0.085	0.086	0.105	0.088	0.080	0.112	0.0390	7.18	3.90	54.32
	F	0	0	0	0	0	0	0	0	0	0	0	0	0.071	0.083	0.062	0.075	0.079	0.066	0.0328		3.28	45.68
Time of day	G	0.070	0.085	0	0	0	0	0	0	0	0	0	0	0.050	0.085	0.066	0.069	0.068	0.072	0.0359	10.21	3.59	35.16
	H	0.166	0.164	0	0	0	0	0	0	0	0	0	0	0.111	0.148	0.112	0.102	0.108	0.113	0.0662		6.62	64.84
Characteristics of transport facilities	J	0.133	0.112	0	0	0.097	0.096	0.108	0.110	0	0	0	0	0.063	0.049	0.081	0.076	0.059	0.066	0.0582	21.12	5.82	27.56
	K	0.142	0.142	0	0	0.106	0.124	0.116	0.117	0	0	0	0	0.073	0.048	0.052	0.058	0.058	0.052	0.0588		5.88	27.84
	L	0.087	0.098	0	0	0.081	0.080	0.077	0.086	0	0	0	0	0.067	0.029	0.038	0.047	0.051	0.031	0.0409		4.09	19.37
	M	0.125	0.125	0	0	0.101	0.103	0.100	0.113	0	0	0	0	0.075	0.039	0.056	0.054	0.056	0.046	0.0533		5.33	25.24
Mode choice	N	0.043	0.053	0.281	0.071	0.073	0.089	0.045	0.050	0.252	0.294	0.288	0.320	0	0	0	0	0	0	0.0965	38.13	9.65	25.31
	O	0.028	0.047	0.103	0.225	0.038	0.043	0.027	0.038	0.095	0.144	0.098	0.150	0	0	0	0	0	0	0.0543		5.43	14.24
	P	0.071	0.046	0.149	0.200	0.031	0.020	0.061	0.048	0.172	0.108	0.131	0.181	0	0	0	0	0	0	0.0646		6.46	16.94
	Q	0.062	0.053	0.207	0.143	0.057	0.055	0.060	0.050	0.226	0.179	0.215	0.227	0	0	0	0	0	0	0.0785		7.85	20.59
	R	0.044	0.044	0.190	0.106	0.053	0.055	0.037	0.040	0.178	0.192	0.205	0.000	0	0	0	0	0	0	0.0424		4.24	11.12
	S	0.030	0.032	0.069	0.256	0.022	0.016	0.037	0.028	0.078	0.084	0.063	0.121	0	0	0	0	0	0	0.0450		4.50	11.80

Figure 8 Weighted supermatrix (fuzzy scale α=0) and synthesis weights

function of long travel time. The fuzzy membership functions for long travel time were developed for each travel modes (see Figure 9). The degree of membership of the short travel time can be derived by subtracting the degree of membership of long travel time from one. The procedure used to develop the fuzzy membership function for long travel time was applied for development of the fuzzy membership function for expensive travel cost as well (see Figure 10).

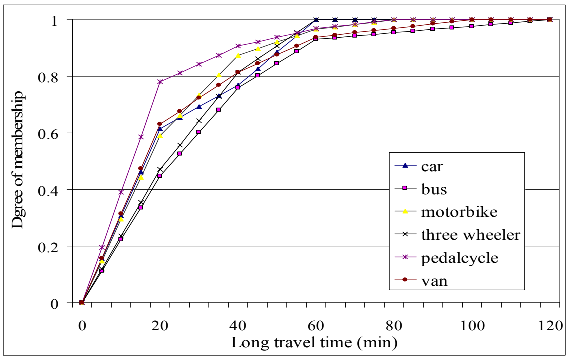

Figure 9 Fuzzy membership functions for long travel time according to the modes for target population for 5 km distance

5. Model Validation

To validate the FANP model, the stated preference method was applied. 30 different scenarios based on travel time, expenses, weather condition and time of the day were set up. The preference score for each alternative in each scenario was calculated according to Equation 10.

$$P_k = \sum_i \alpha_{ik} \times w_{ik} \qquad (10)$$

Where; P_k - Preference score of alternative k

α_{ik} - Degree of fuzziness value of attribute i for alternative k

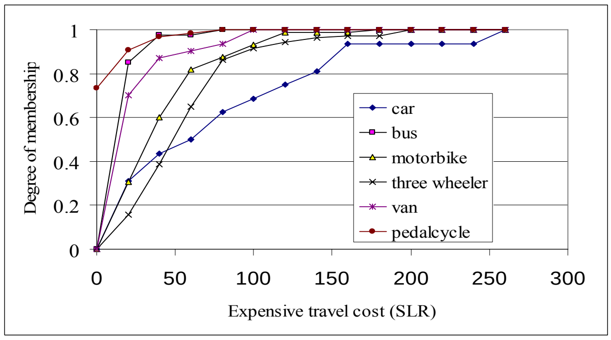

Figure 10 Fuzzy membership functions for expensive travel cost according to the modes for target population for 5 km distance

w_{ik} - FANP global weight of attribute i for alternative k

In the decision structure, the travel time and travel expenses were included as fuzzy variables. The degree of fuzziness of these attributes was determined according to the fuzzy membership function derived. Other variables were treated as crisps variables (weather, time of the day). In the mode choice problem, the travelers made their choices with measures or conditions which are already given. The mode preference scores were calculated against the measures from known conditions and the highest scored alternative was determined as the most preferable alternative. If the most preferable alternative is the same as the observed mode choice, it is considered as a replication of travel behavior. The numbers of replications based on the answers of each individual's 30 scenarios are shown in Figure 11. It is possible to develop the cross tabulation, which rows represent the number of choices made by those subject for each alternative, while the columns represent the number of times an alternative was predicted. Table 3 presents the cross tabulation for the FANP model (fuzzy scale $\delta = 0$). The diagonal elements of the cross tabulation represent the number of the times the choice model correctly predicted the choice of alternative as observed. Thus overall correct prediction was about 60 percent of total number of cases.

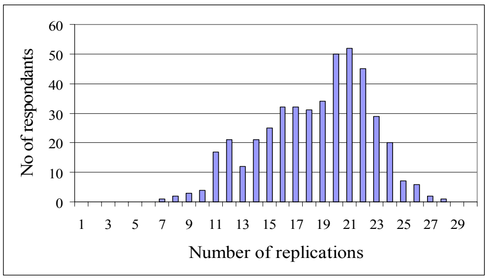

Figure 11 Number of replication for each individual choice observed and predicted out of 30 scenarios

6. Sensitivity Analysis

Figure 12 shows how the degree of fuzziness could affect the rank order and overall priorities of alternatives in the pooled model at fuzzy scale $\delta = 1$. At β =1, the decision model provided a robust rank order for car, three wheeler, motorbike and pedalcycle (car>three wheeler> motorbike >pedalcycle). Additionally for all α and β values, the model provided a robust rank order for car and three wheeler. However, a rank reversal phenomenon was also observed between some alternatives. At more uncertain conditions, the degree of preference of car and motorbike increase while the degree of preference of three wheeler decreases. These results indicated how the degree of fuzziness could influence the degree of dominance of an alternative over another at different alpha-cuts. It was found that the model was quite robust for rank order for the fuzzy scales less than 3.

7. Discussion

This study is an important start to explore the ability of applying fuzzy extension of analytic network process modeling for use in mode choice modeling. The result of the study showed that about 60% correct replications were given by the model. Hence, the proposed FANP model establishes fairly good performance in prediction.

From this study, it was found that the major disadvantage of the proposed method is the exhaustive establishment of weights for different combinations of criteria, which requires consistency and is time consuming. Hence, in the future research the possibility to split the questionnaire to sub sections and inquiring a traveler part of the questionnaire should be examined.

In the proposed model, the travel time and travel expenses were modeled as fuzzy variables. In future study, the development of other fuzzy variables such as traffic congestion and transportation facilities should be considered. When developing the fuzzy membership functions the biases due to travelers' perception on his previous trip, mode, trip

Table 3 Cross tabulation for FANP model (fuzzy scale $\delta = 0$)

Observed choice \ Predicted choice	Car	Bus	Motorbike	Three wheerler	Van	Pedalcycle	Total
Car	2050	2	34	315	201	21	2658
Bus	333	24	88	367	253	29	1094
Motorbike	110	0	1056	168	244	37	1615
Three wheerler	136	2	65	4490	228	29	4950
Van	144	4	164	189	1809	48	2358
Pedalcycle	127	1	42	81	83	401	735
Total	2935	33	1449	5610	2818	565	13410

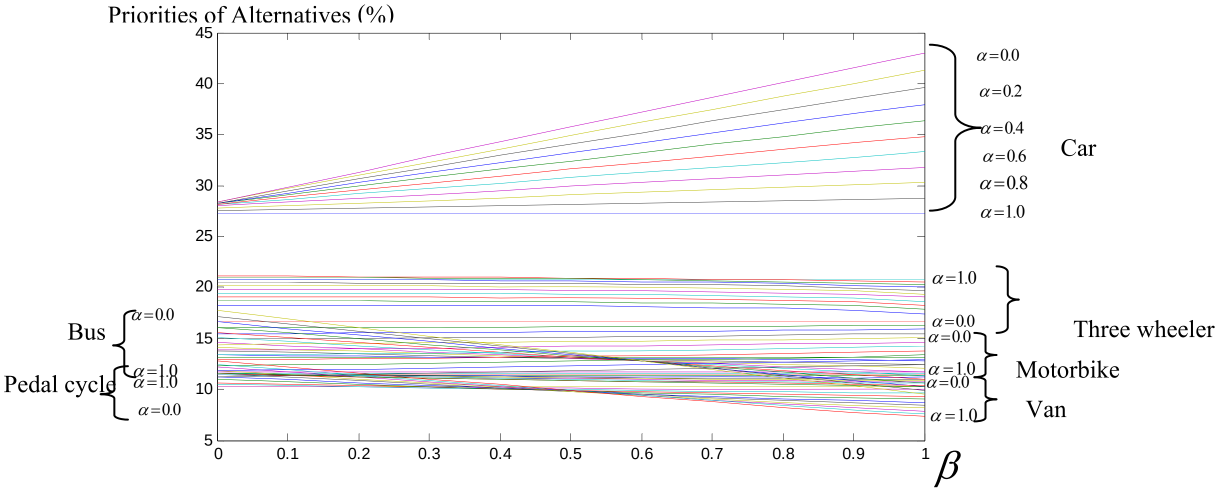

Figure 12 The overall priorities of alternatives at different α and β values, fuzzy scale $\delta = 1$

purpose and other related factors should be further investigated.

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