Bend Beam Method for Determining Fracture Behavior of Concrete: II. Performance of the Fracture Models

Pusit Lertwattanaruk
Faculty of Architecture and Planning, Thammasat University,
Khlong Luang, Pathumthani, 12121, Thailand
Tel: (+662) 986-9605, Fax: (+662) 986-8067
E-mail: Lertwatt@tu.ac.th

Abstract

In this paper, test results and analytical data of other researchers were compared with those predicted by the fracture mechanics model based on bend beam method presented in the previous publication [1]. Although a large number of bend beam tests of concrete have been reported, the load vs. deflection responses and load vs. crack-mouth-opening displacement (CMOD) responses were rarely published in details. As investigated by researchers, error in load-line deflection measurement greatly affects the fracture energy (G_F) and the fracture parameters such as the critical crack length (a_c), the critical stress intensity factor (K_{IC}) and the critical energy release rate (G_C). Test data referred were selected to highlight the importance of the accurate measurement of the load-line deflections and also to verify the validity of the proposed fracture model. The analytical results concluded that a number of concrete fracture tests were carried out by using the erroneous measurements of beam deflections which often included the crushing at supports. By applying the reasonably assumed values of S_1 for the true deflections and the empirically determined values of S_2 from the literature, the proper evaluation of the fracture behavior and the fracture properties of concrete can be obtained by using the load-CMOD response based on the proposed model.

1. Introduction

The Two-Parameter Fracture Model (TPFM) developed by Jenq and Shah [2,3] was derived under the special non-linear fracture models without using the complete stress-strain and stress-deformation softening relationship. It is based on the pre-peak nonlinear behavior of concrete. The linear elastic fracture mechanics (LEFM) principles are modified to approximately reflect the fracture behavior of concrete. Using the TPFM, Jenq and Shah [3] and Shah [4] showed that along with the increase in critical stress intensity factor (K_{IC}) and compressive strength (f'_c) there was a considerable decrease in the pre-peak nonlinearity or the critical effective crack length (a_c). The TPFM results generally imply that the toughness, which is measured by K_{IC}, increases with f'_c.

Based on the load-CMOD curve from the three-point bend test on notched beams and available LEFM relationship [5], one can determine the fracture parameters of interest as proposed by the RILEM Technical Committee 89-FMT [6]. The brief procedures of TPFM calculation are presented as follows:

The Young's Modulus (E) is determined from the initial slope C_i by using an empirical equation:

$$E = \frac{6Sa_0V(\alpha)}{C_iD^2B} \tag{1}$$

where S = Specimen loading span; B = width of the beam; D = depth of the beam

$\alpha = \frac{(a_0 + h)}{(D + h)}$; a_0 = initial notch depth; h = thickness of holder of clip gauge

$$V(\alpha) = 0.76 - 2.28\alpha + 3.78\alpha^2 - 2.04\alpha^3 + \frac{0.66}{(1-\alpha)^2}$$

C_i = the initial slope determined from the load-CMOD curve (experimentally determined)

The critical effective crack length (a_c) is calculated by using E from Equation 1 and by knowing the unloading compliance C_u. The a_c is found when the following equation is satisfied:

$$E = \frac{6Sa_c V(\alpha)}{C_u D^2 B} \quad (2)$$

where a_c = the critical effective crack length
C_u = the unloading slope at 95% of peak load (experimentally determined)

The critical stress intensity factor (K_{IC}), after which a_c is known, is calculated by using the following relationship.

$$K_{IC} = \frac{3(P_{max} + 0.5W)S\sqrt{\pi a_c}F(\alpha)}{2D^2 B} \quad (3)$$

where

$$F(\alpha) = \frac{1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.7\alpha^2)}{\sqrt{\pi}(1+2\alpha)(1-\alpha)^{3/2}}$$

$\alpha = \frac{a_c}{D}$; P_{max} = the measured maximum load; $W = W_0 \frac{S}{L}$; W_0 = self-weight of the beam and L = length of beam

The maximum load (peak load) and the slope of the unloading-reloading portion of the load-CMOD curve (used to calculate C_u) are experimentally determined. With known specimen geometry and Young's Modulus, the critical effective crack length (a_c) can be determined using LEFM formulae. Once a_c is calculated, K_{IC} can be obtained.

In addition, based on the linear elastic fracture mechanics [2,7], the critical strain energy release rate (G_C) can be related to the critical stress intensity factor (K_{IC}) as

$$G_C = \frac{K_{IC}^2}{E'} \quad (4)$$

where $E' = E$ for plane stress condition; $E' = \frac{E}{(1-\upsilon^2)}$ for plain strain condition;
E = Young's modulus; υ = Poisson's ratio.

In this study, since G_C is closely related to K_{IC}, G_C can therefore be used as a measure of fracture toughness in the same way that the critical stress intensity factor (K_{IC}) is for the linear fracture mechanics.

2. The Proposed Fracture Mechanics Model

The indirect method for obtaining fracture energy, G_F, suggested by RILEM [8] requires the complete load vs. load-line deflection curve from the three-point bend beam test. To obtain an accurate deflection, the test setup with a reference frame was used (see Fig. 1). In Fig. 2(a), a typical load vs. displacement (deflection or CMOD) response was shown. Fig. 2(b) shows a typical relationship between deflection and CMOD which is bilinear in shape. The initial slope S_1 is valid in the linear elastic portion of the load vs. deflection responses. Near the peak load, the slope S_1 then gradually changes to S_2 during the formation of the fracture process zone, which is the nonlinear zone in the vicinity of the crack tip. At the peak load, the fracture process zone is fully developed, and produces traction-free cracked surface after which the specimen exhibits a linear relationship between deflection and CMOD with a constant slope S_2.

Common test data needed to implement the proposed fracture model are the load vs. deflection curves and load vs. CMOD curves of the notched beam specimens. Details of the theoretical development were described in the previous publication [1]. The procedure to apply the proposed model for studying the behavior of material during fracture process can be briefly described as follows:

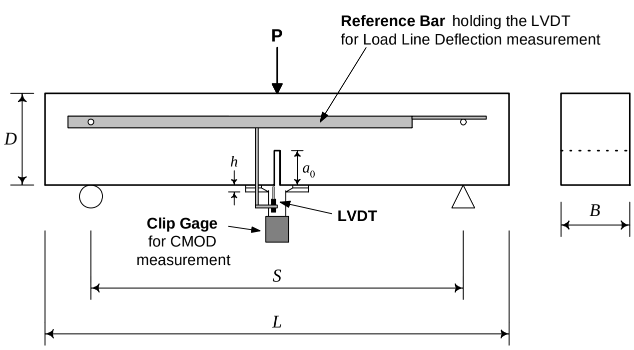


Fig. 1 Three-point-bend beam test setup

L is the specimen length (400 mm); *S* is the span length (300 mm); *D* is the beam depth (75 mm); *B* is the width (75 mm), and a_0 is the initial notch depth (25 mm).

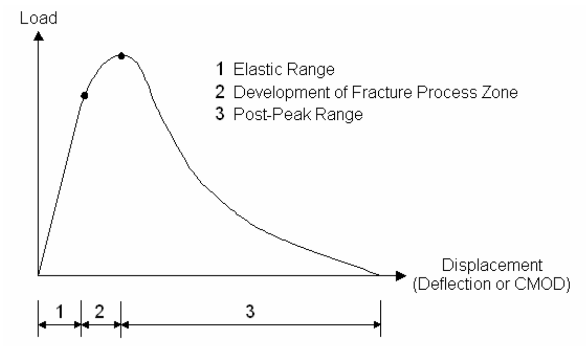


Fig. 2(a) Typical load-displacement response

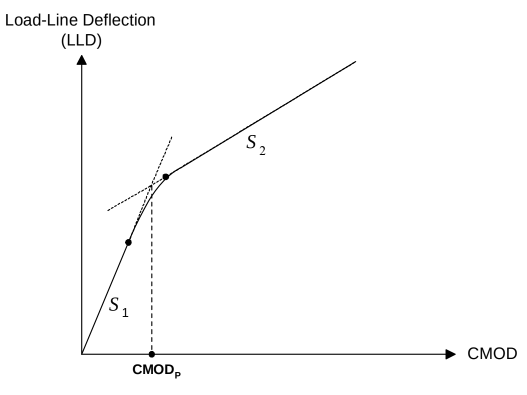


Fig. 2(b) Relationship between load-line deflection (LLD) and CMOD

1. *Determination of crack growth due to applied load*

The crack growth (Δa) at any instant of time Δt, under the applied load *P*, can be determined by using the following expression:

$$\Delta a = \frac{1}{G_F B}\left(S_1 \int_0^{CMOD_P} PdCMOD + S_2 \int_{CMOD_P}^{CMOD} PdCMOD \right) \quad (5)$$

where S_1 is the slope of deflection-CMOD curve in the linear-elastic region; S_2 is the slope of deflection-CMOD curve in the post-peak region. The region between the linear-elastic and post-peak response is approximated by extrapolating both the slopes S_1 and S_2 till they intersect as shown in Fig. 2(b). The intersection of S_1 and S_2 is represented by $CMOD_P$ in Eq. 5, B = the width of the beam, and G_F = the fracture energy (a material property), which can be calculated from:

$$G_F B(D-a_0) = S_1 \int_0^{CMOD_P} PdCMOD + S_2 \int_{CMOD_P}^{\infty} PdCMOD \quad (6)$$

where *D* is the depth of the beam and a_0 is the pre-notched or initial crack length.

2. *Determination of energy release rate*

The energy release rate (G_R) at any instant of time (R curve) during the fracture process can be determined by the following expression:

$$G_R = \frac{1}{B\Delta a}\left[S_1 \int_0^{CMOD_P} PdCMOD + S_2 \int_{CMOD_P}^{CMOD} PdCMOD \right] - \frac{S_1}{B\Delta a}\left[\frac{P^2}{2K_i^{CMOD}} \right]_{P(\delta=0)}^{P(\delta=\delta)} \quad (7)$$

where K_i^{CMOD} is the initial stiffness of the beam determined from the slope of the load-CMOD curve. By knowing Δa, *P*, S_1, S_2 and the area under *P-CMOD* curve at any instant of time, G_R at that instant can be determined.

3. Experimental Program

3.1 Details of Concrete Mix and Materials

A series of specimens for the control concrete (CC) and silica fume concrete (SF) were tested. Details of the mix proportions of the concrete are presented in Table 1. In this study, Portland cement Type I conforming to ASTM C 150 [9] were used. Silica fume with particle size less than 1 μm were used as an additive. Local siliceous sand, passing through sieve No. 4 (opening size 4.75 mm) conforming to ASTM C 33, were used as fine aggregates. Coarse aggregates were crushed limestone of 3/8 inch (9.5 mm) maximum size conforming to ASTM C 33 [10].

Table 1 Mix proportions of concrete by weight

Materials	Control Concrete (CC)	Silica Fume Concrete (SF)
Cement	1	0.9
Sand	2	2
Coarse Aggregate	3	3
Water	0.5	0.5
Silica Fume	0	0.1

3.2 Three-Point Bend Test on Notched Beam

The three-point bend tests on notched beams were conducted to obtain the test data required to implement the proposed fracture model, comprising of the deflection, crack-mouth-opening displacement (CMOD) and applied loads.

The 75 x 75 x 400 mm beams were cast and rested in the normal room environment. After 24 hours they were demolded, and transferred into a 100% humidity room for curing until one day before testing. Prior to testing the beams were notched using a circular diamond saw. All beams were tested at the age of 56 days. Fig. 1 shows the diagram of the beam test setup and the dimension of the test specimens.

All beam tests were performed under the CMOD control in an MTS closed-loop testing system at a displacement rate of 5×10^{-4} mm/second to produce a controlled failure, allowing all parameters of interest to be measured. The deflection was measured using a transducer (LVDT) fixed to a reference frame (see Fig. 1) in order to eliminate the effects of support crushing. The measurements of CMOD were done by an MTS clip-on gage. In order to compare with the TPFM Model [2], the applied load was unloaded when passed the maximum load at about 95%, and reloaded when the applied load approached zero. It took approximately 45 minutes to an hour to complete the entire test.

4. Test Results and Discussions

4.1 Results of Bend Beam Test on the Proposed Fracture Model and TPFM Model

Typical load vs. deflection, load vs. crack-mouth-opening displacement (CMOD), and deflection vs. CMOD curves are shown in Fig. 2. Details of the test results were presented in the previous publication [1]. Table 2 summarizes the test results obtained from the bend beam tests. CC represents the control concrete, and SF denotes silica fume concrete.

4.1.1 Critical Crack Length

Based on the TPFM model, the critical crack length (a_c) and the stress intensity factor (K_{IC}) are two material properties defined according to the elastic behavior of the material response without any consideration of the fracture process zone. From Table 2, the results from the TPFM show as suggested by Shah [4] that there was a decrease in the critical crack length (a_c) when the compressive strength (f'_c) increases, and the smaller of a_c means a more brittle material. With modern technology, the modification of the cement matrix-aggregate interfaces can possibly produce high strength concretes, which are also less brittle. Obviously, without considering the non-linear behavior of the fracture process zone, which is quite sensitive to the changes of cement matrix-aggregate interfaces, the TPFM model may not be suitable for studying the fracture behavior of high performance concrete.

Based on the proposed fracture model, the critical crack growth (Δa_c) is derived based on both elastic and inelastic parts of the material response considering effects of the fracture process zone. From Table 2, the results based on the proposed fracture model contradict the suggestion by Shah [4] that there was a decrease in the a_c when the f'_c increases. Silica fume concrete (SF) has a higher a_c than that of control concrete. Taking into consideration that the average size of coarse aggregate used is 3/8 inches (9.50 mm), only the silica fume concrete (SF) yields the average critical crack growth of 9.93 mm, which is more than the average size of coarse aggregate. The control concrete (CC) yields the critical crack growth of 7.69 mm, which are less than the average size of the coarse aggregate (9.50 mm).

For the silica fume concrete, due to the strong bond between cement matrix and coarse aggregate, the cracks tend to penetrate straight through the coarse aggregates rather than deflect around them [11,12]. The straight crack through coarse aggregate, which is a tougher homogenous material, consumes higher energy (G_R) to propagate than the crack that goes through the cement matrix-aggregate interface. At the moment of fracture (at the peak load), the coarse aggregate releases the energy (G_R), absorbed while resisting the crack from propagating at a shorter time period resulting in a larger critical crack length (a_c).

On the other hand, for the control concrete, due to the weaker cement matrix-coarse aggregate interface, the cracks tend to deflect to the path of least resistance, which is along the interface or around the coarse aggregate particles [11,12]. Since the cement matrix-coarse aggregate interface, considered to be non-homogeneous, is weaker than the coarse aggregate, the non-planar crack along the interface gradually consumes G_R, and slowly propagates during fracture, resulting in more stable crack propagation.

4.1.2 Critical Energy Release Rate

Based on the TPFM model, the critical energy release rate (G_C) is defined according to the elastic part of the material response without consideration of fracture process zone. For the proposed fracture model, the G_C was derived considering both the elastic and inelastic parts of the material. The proposed model presents the fracture behavior in the terms of the energy release rate (G_R) as a function of crack length (a) or so-called R curve. The value of G_R at the peak load is defined as the critical energy release rate, G_C. From Table 2, the results from the TPFM and the proposed fracture model compare favorably, and show that G_C increases with increasing compressive strength.

Table 2 Test results from the proposed fracture model and the TPFM model

Specimen Type	Compressive Strength f'_c (MPa)	Relationship between CMOD and Deflection		Fracture Energy G_F (N/m)	Critical Crack Growth		Critial Energy Release Rate (G_C)	
		S_1 (mm/mm)	S_2 (mm/mm)		Proposed CMOD Method (mm)	TPFM Method (mm)	Proposed CMOD Method (N/m)	TPFM Method (N/m)
CC-1	41.42	1.279	0.780	82.88	7.08	10.35	45.18	49.14
CC-2	41.42	1.284	0.829	70.88	7.54	9.96	43.65	43.82
CC-3	41.42	1.247	0.731	76.87	8.47	11.81	44.98	51.62
CC (Average)	**41.42**	**1.270**	**0.780**	**76.88**	**7.69**	**10.71**	**44.60**	**48.19**
SF-1	51.27	1.445	0.903	89.48	9.61	10.36	58.93	54.40
SF-2	51.27	1.446	0.883	86.54	9.41	10.02	50.89	55.71
SF-3	51.27	1.565	0.906	96.29	10.76	10.20	56.73	61.61
SF (Average)	**51.27**	**1.486**	**0.897**	**90.77**	**9.93**	**10.19**	**55.52**	**57.24**

4.2 Comparison with Test Results of Other Researchers

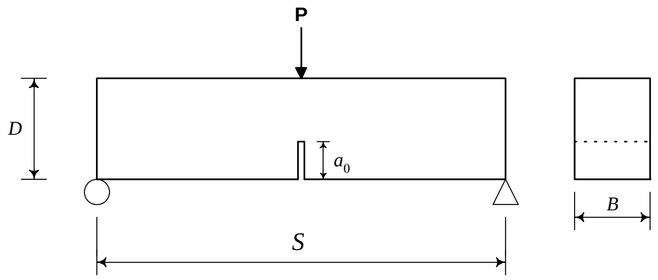
Test data and analytical results of other researchers were compared with those predicted by the proposed fracture mechanics model. Although a large number of the fracture tests of concrete have been reported in the literature, the corresponding load vs. deflection responses and load vs. crack-mouth-opening displacement (CMOD) responses of the notched beams were rarely published in details. Error in deflection measurement greatly affects the fracture energy (G_F) and the fracture parameters such as the critical crack length (a_c), the critical stress intensity factor (K_{IC}) and the critical energy release rate (G_C). Test data were selected to highlight the importance of the accurate measurement of load-line deflections and also to verify the validity of the proposed fracture mechanics model.

The selected test series in this study as shown in Table 3 were from:

1. *Jenq and Shah* [2,3] for normal strength concrete: Two concrete beams, JS1 and JS2, were made with a mix proportion by weight of 1: 2.6: 2.6: 0.65 of cement, sand, coarse aggregate and water. The maximum aggregate size was 3/4 inches (1.9 mm).
2. *Ratanalert and Wecharatana* [13] for medium strength mortar: Two mortar beams, RW1 and RW2 were made with a mix proportion by weight of 1: 2.6: 0.45 of cement, sand and water. The maximum aggregate size was 3/8 inches (9.5 mm).
3. *Gopalaratnam and Ye* [14] for plain concrete model: A plain concrete beam, GY1, was modeled by a numerical model and the finite element methods. Dimensions of the beams are shown in Table 3.

Table 3 Test results of other works compared with the proposed fracture model

Specimen Type	Beam Dimensions S x B x D x a_o (cm)
JS1	60 x 5.6 x 15 x 4.9
JS2	30 x 2.8 x 7.5 x 2.2
RW1	57 x 5 x 15 x 4.8
RW2	20 x 5 x 5 x 2.4
GY1	120 x 10 x 30 x 5



Specimen Type	S_1		G_F		a_c		G_C	
	Refer to Literature (mm/mm)	Proposed Model (mm/mm)	Refer to Literature (N/m)	Proposed Model (N/m)	Refer to Literature (mm)	Proposed Model (mm)	Refer to Literature (N/m)	Proposed Model (N/m)
JS1	1.087	1.087	88.61	88.61*	7.76	7.63	63.40	66.72
JS2	1.324	1.324	66.20	66.20*	4.06	4.13	54.81	53.76
RW1	2.678	1.270	50.79	36.08	N.A.	9.49	19.61	22.24
RW2	3.184	1.270	64.80	49.39	N.A.	2.82	18.16	18.39
GY1	2.441	1.270	52.54	52.54+	10.90	10.63	N.A.	25.22

* The load-deflection curves were not modified by the proposed model, and therefore G_F were not re-calculated.

+ The load-deflection curve was modified by the proposed model, but G_F , which is a given material property for the model in the literature, was not altered.

Table 3 shows the results for the three-point-bend notched beam tests of the other researchers along with the analytical results predicted by the proposed model. For the compatibility of specimen geometry, all of the beams referred here have the ratio between span to depth ratio (S/D) of approximately or equal to 4, which is the same as S/D of the beam tested in the present study.

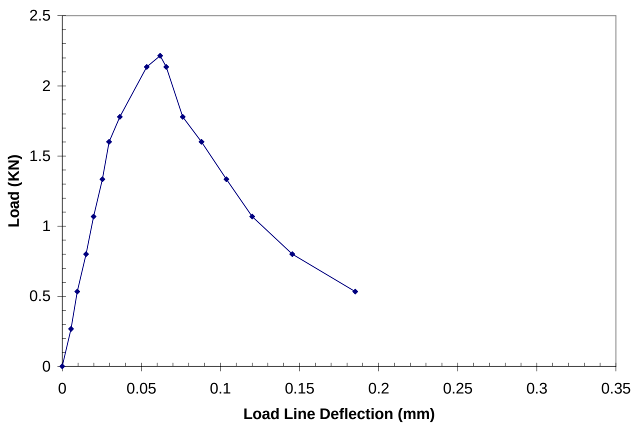


Fig. 3(a) Load-Deflection Relationship of Beam JS1 (Jenq and Shah [2])

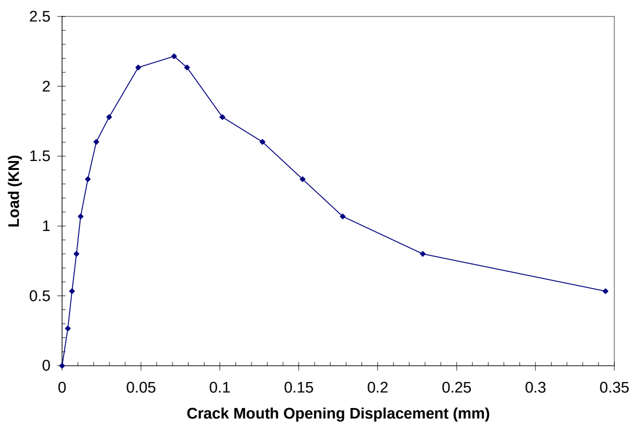


Fig. 3(b) Load-CMOD Relationship of Beam JS1 (Jenq and Shah [2])

Fig. 3 and 4 present test data adapted from Jenq and Shah's study [2,3] for the concrete beam JS1 and JS2 respectively. The fracture parameters, the critical crack length (a_c) and the critical energy release rate (G_C), in the literature were calculated based on the TPFM Model using the relationships from the load vs. CMOD curves. For comparisons, the proposed fracture model was implemented using the referred data to determine the above mentioned fracture parameters.

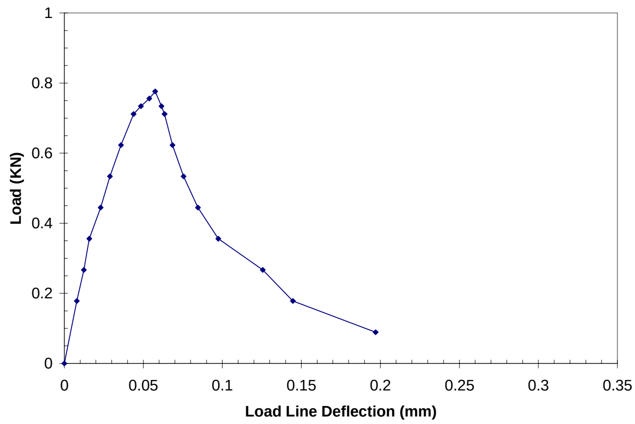


Fig. 4(a) Load-Deflection Relationship of Beam JS2 (Jenq and Shah [3])

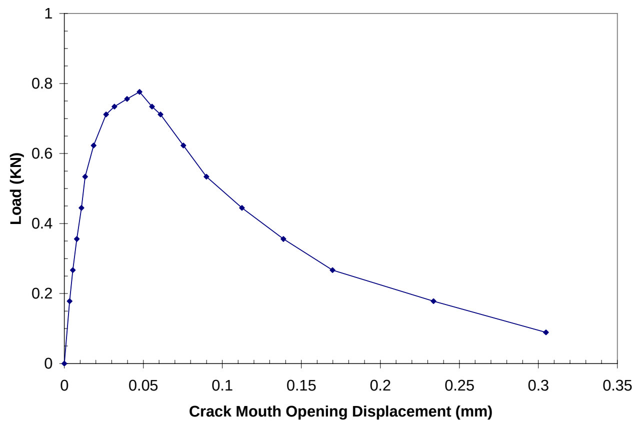


Fig. 4(b) Load-CMOD Relationship of Beam JS2 (Jenq and Shah [3])

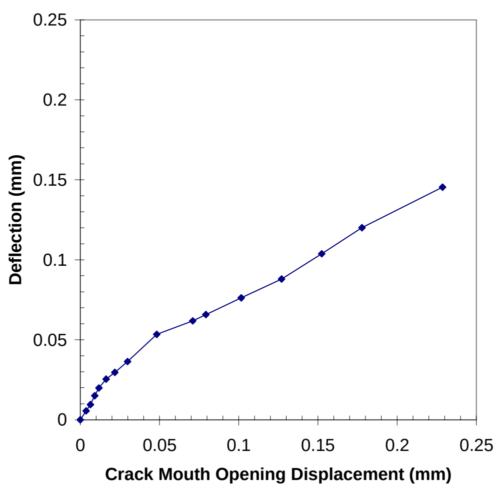


Fig. 5 Deflection-CMOD Relationship of Beam JS1

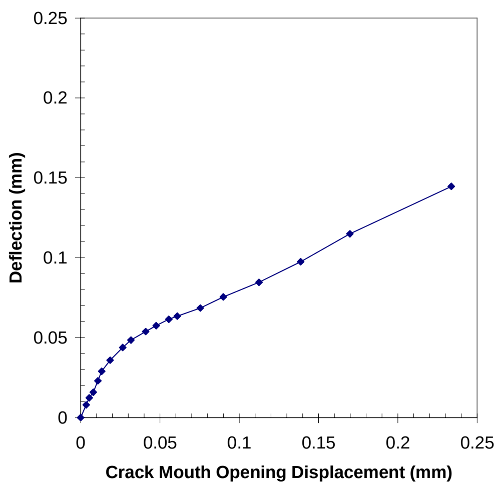


Fig. 6 Deflection-CMOD Relationship of Beam JS2

Fig. 5 and 6 show the deflection-CMOD relationships of the beams JS1 and JS2. From Table 3, the results of S_1 or the initial slope of deflection-CMOD curve for JS1 and JS2 beam tests are 1.087 and 1.324 mm/mm respectively, which are similar to the S_1 of 1.270 mm/mm for control concrete (CC) in the present study. With a bilinear behavior shown, this tends to indicate that the deflection measurements from these tests seem to be accurate.

The analytical results of the load vs. crack length relationship of beam JS1 obtained by the proposed model is shown in Fig. 7. From Table 3, the critical crack length (a_c) of 7.63 mm predicted by the proposed model compares favorably with 7.76 mm reported by Jenq and

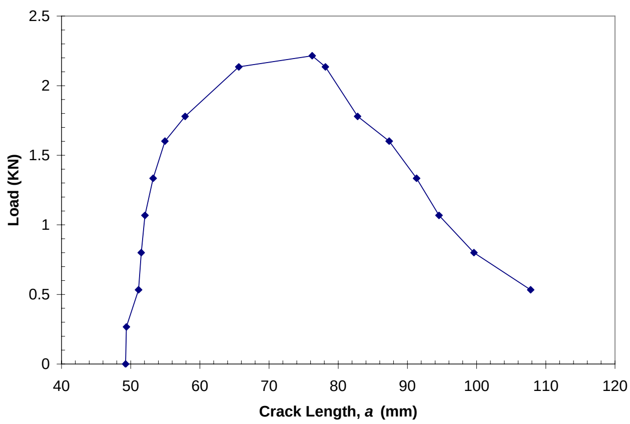


Fig. 7 Load-Crack Length Relationship of Beam JS1

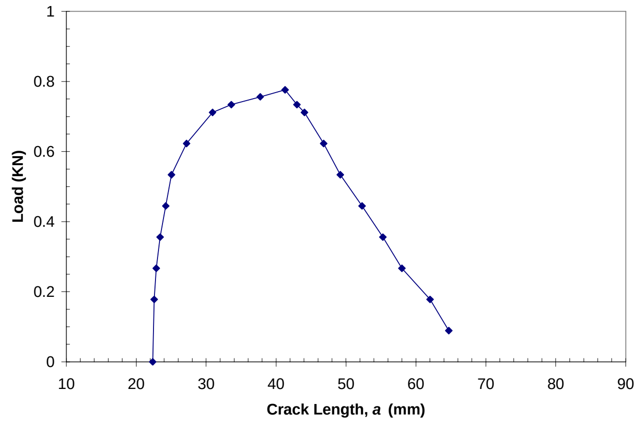


Fig. 8 Load-Crack Length Relationship of Beam JS2

Shah [2], whereas the critical energy release rate (G_C) of 66.72 N/m also agrees well with the 63.4 N/m reported in the literature [2].

For the beam JS2, the parameters predicted by the proposed model, the critical crack length (a_c) of 4.13 mm is closed to 4.06 mm reported [3], and the critical energy release rate (G_C) of 54.81 N/m is in good agreement with 53.76 N/m reported in the literature [3]. Fig. 8 shows the analytical results of load vs. crack length relationship of beam JS2 obtained by the proposed model. From the results discussed, the material behavior and fracture parameters obtained by the proposed model are found to be in good agreement with those from the literature.

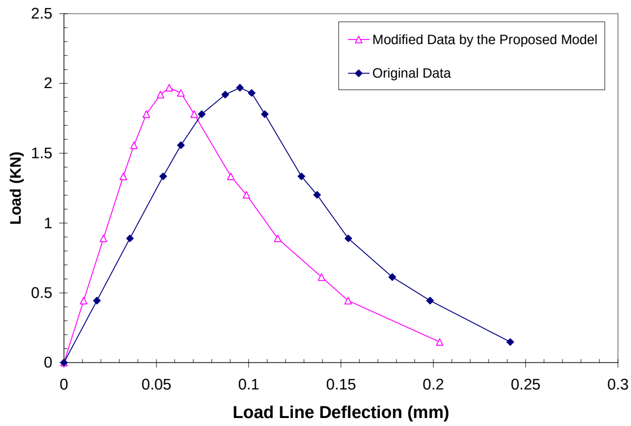


Fig. 9(a) Load-Deflection Relationship of Beam RW1 (Ratanalert and Wecharatana [13])

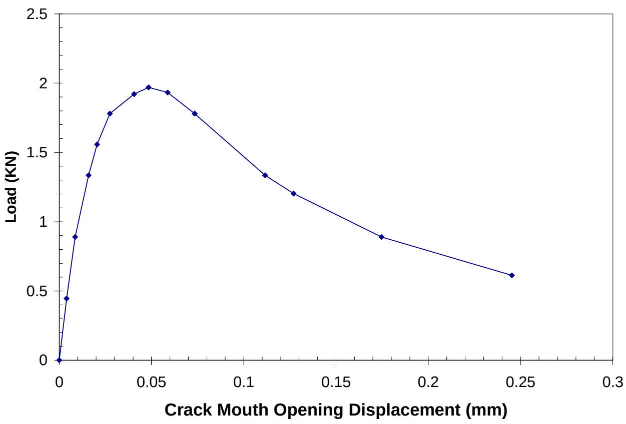


Fig. 9(b) Load-CMOD Relationship of Beam RW1 (Ratanalert and Wecharatana [13])

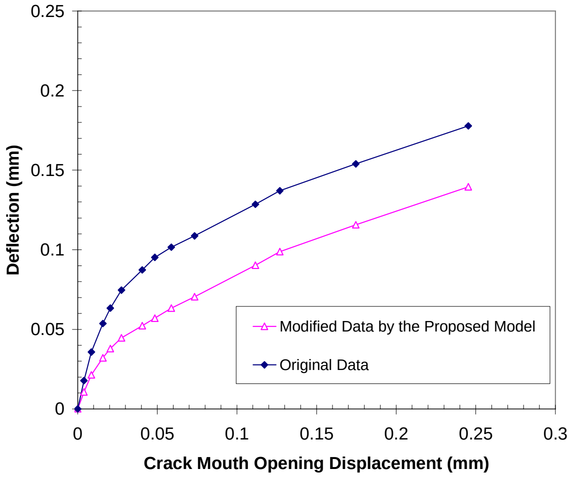


Fig. 10 Deflection-CMOD Relationship of Beam RW1

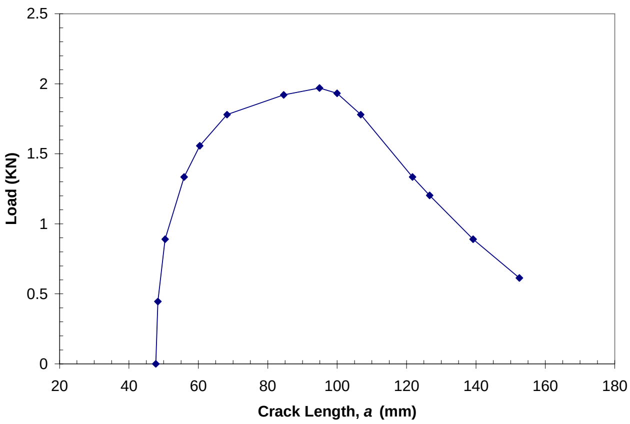


Fig. 11 Load-Crack Length Relationship of Beam RW1

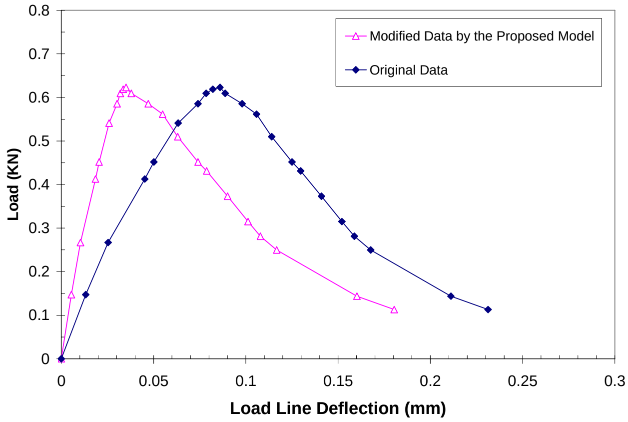


Fig. 12(a) Load-Deflection Relationship of Beam RW2 (Ratanalert and Wecharatana [13])

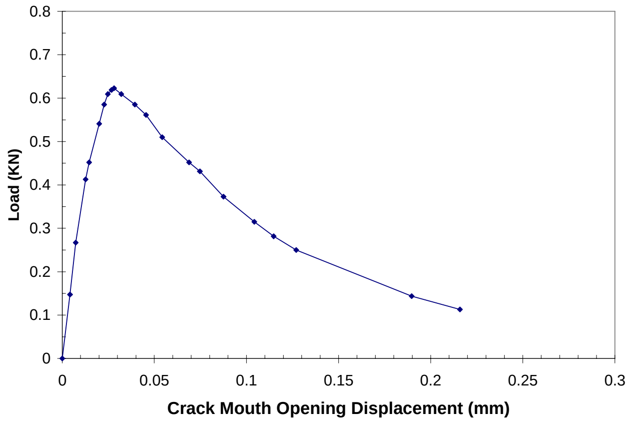


Fig. 12(b) Load-CMOD Relationship of Beam RW2 (Ratanalert and Wecharatana [13])

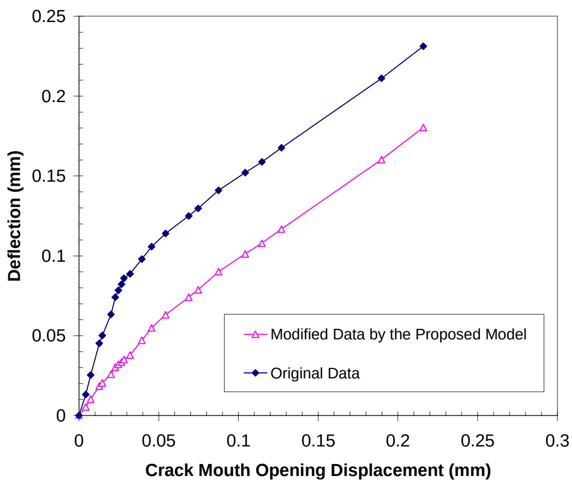


Fig. 13 Deflection-CMOD Relationship of Beam RW2

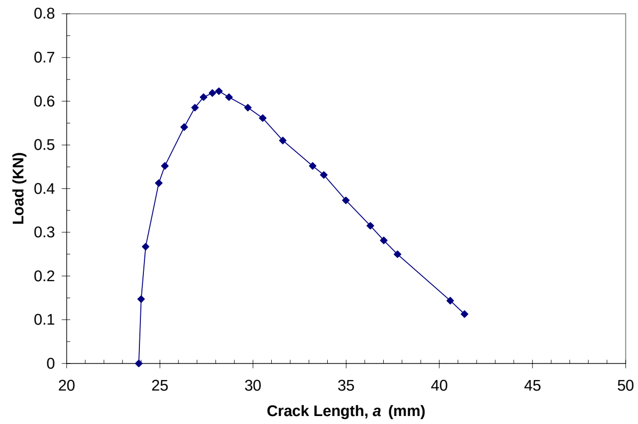


Fig. 14 Load-Crack Length Relationship of Beam RW2

In Fig. 9 and 12, test data adapted from the study of Ratanalert and Wecharatana [13] are presented. In their study, the critical energy release rate (G_C) was also calculated by applying the TPFM model. For the mortar beam RW1, by using the original load-deflection curve and the load-CMOD curve from the literature as shown in Fig. 9, the G_C calculated by the proposed model is 48.67 N/m, which is noticeably higher than the G_C of 19.61 N/m found in the literature. With reference to the proposed model, the G_C can be calculated from the load vs. CMOD curve when the value of S_1 is known. From Table 3, the value of S_1 for the beam RW1 calculated from the original data is 2.678 mm/mm, which is higher than S_1 of 1.270 mm/mm for normal concrete (CC) obtained in this study. This indicates that the deflection could be inaccurately measured. Therefore, the original load vs. deflection curve was modified to have S_1 equal to 1.270 mm/mm in order to investigate the effect of the deflection measurement on determination of the fracture behavior. Fig. 10 shows the deflection-CMOD relationships (S_1 and S_2) of the beam RW1 for the original data and the data modified by the proposed model.

By using the data from the modified load-deflection curve and the original load-CMOD curve for the beam RW1 as shown in Fig. 9, the value of G_C obtained by the proposed model was 22.24 N/m, which compares favorably with the G_C of 19.61 N/m in the literature. Furthermore, from Table 3, the fracture energy (G_F) reported in their study was 50.79 N/m, which is about 40% higher than that calculated by the modified load-deflection curve (36.08 N/m). Fig. 11 shows the analytical results of the load vs. crack length relationship of beam RW1 obtained by the proposed model. It is interesting to note that, from Fig. 12(a), the difference between the measured peak deflection in the literature (0.095 mm) and that of the modified curve (0.057 mm) is more than 65%. The above results indicate that the deflection measurements reported in the literature possibly included the erroneous deflections caused by concrete crushing at the supports and/or the method of measurement.

For the other mortar beam, RW2, by using the original load-deflection curve and load-CMOD curve from the literature as shown in Fig. 12, the G_C calculated by the proposed model is 30.21 N/m, which is noticeably higher than the G_C of 18.16 N/m found in the literature. However, it is noted that from Table 3, the value of S_1 for the mortar beam RW2 calculated from the original is 3.184 mm/mm, which is noticeably higher than S_1 of 1.270 mm/mm for normal concrete (CC) obtained in the present study. This again indicates that the deflection of the beam RW2 could be inaccurately measured as well. Therefore in the present study, the original load-deflection curve was modified so S_1 equal to 1.270 mm/mm. The modified responses were then used to study the effect of the deflection measurement on fracture behavior of material as well as to evaluate the performance of the proposed model. Fig. 13 shows the deflection-CMOD relationship of beam RW2 from the original data and that from the data modified by the proposed model.

By using the modified load-deflection curve and the original load-CMOD curve of beam RW2 as shown in Fig. 12, the G_C

obtained by the proposed model is found to be 18.39 N/m, which agrees well with the G_C of 18.16 N/m reported in the literature. Furthermore, from Table 3, the fracture energy (G_F) reported in their study 64.80 N/m, which is about 30% higher than that calculated by the modified load-deflection curve (49.39 N/m). Fig. 14 shows the analytical results of the load vs. crack length relationship of beam RW2 obtained by the proposed model. It is interesting to note that, from Fig. 12(a), the measured peak deflection in the literature (0.086 mm) is much higher than that of the modified curve (0.035 mm). The above results again reflect the possible erroneous deflection measurements reported in the literature. When the load-deflection curve was modified to correct the erroneous deflection response, the fracture parameters obtained by the proposed model are found to be in good agreement with the referred literature.

Fig. 15 presents the test data adapted from Gopalaratnam and Ye's study [14] for the modeled concrete beam GY1. In their study, the critical crack length (a_c) was calculated by applying a numerical model and the finite element method. The fracture behavior of the beam was modeled as functions of the crack-tip-opening displacement and the tensile strength based on the fictitious crack model [15]. Referring to the proposed model, the parameters a_c and G_C can be calculated from the load vs. CMOD curve when the value of S_1 is known. For beam GY1, by using the original load-deflection curve and load-CMOD curve (Fig.15), the a_c calculated by the proposed model is 157.4 mm (see Fig.17), which is noticeably higher than the value 109.0 mm found in the literature.

Fig. 15(a) Load-Deflection Relationship of Beam GY1 (Gopalaratnam and Ye [14])

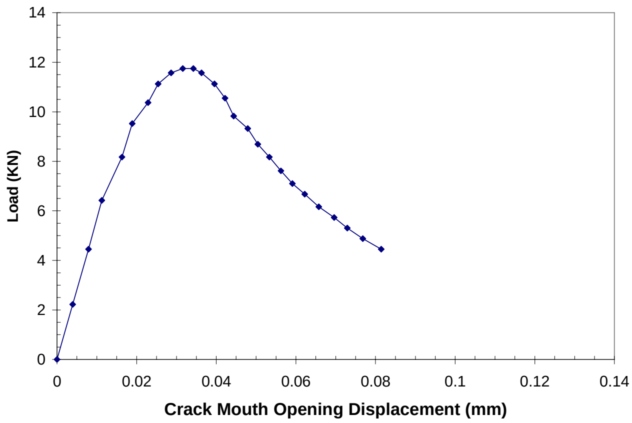


Fig. 15(b) Load-CMOD Relationship of Beam GY1 (Gopalaratnam and Ye [14])

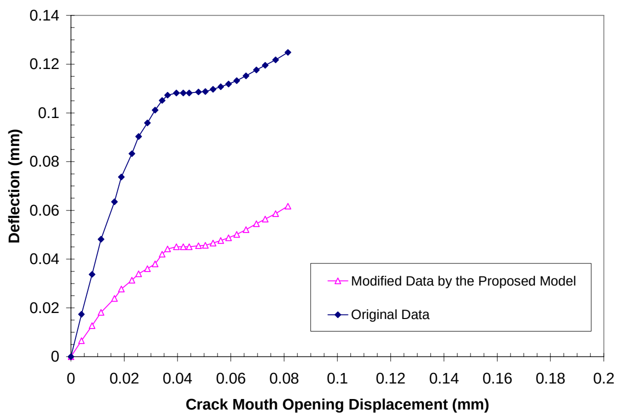
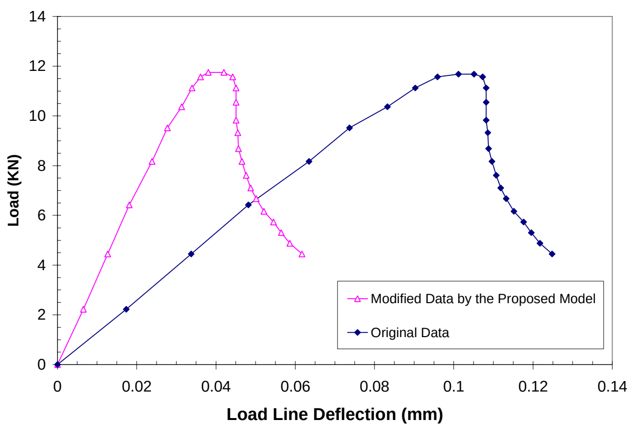


Fig. 16 Deflection-CMOD Relationship of Beam GY1



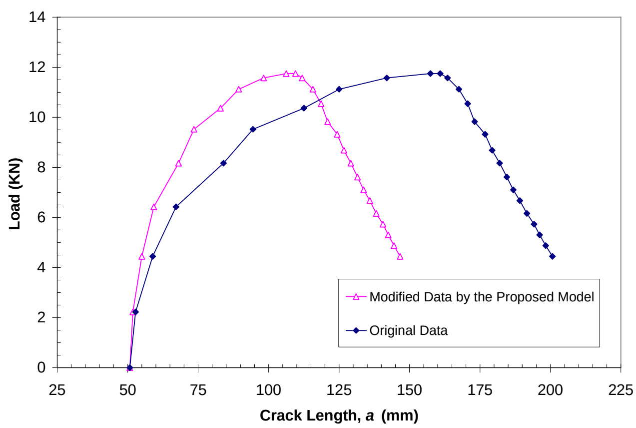


Fig. 17 Load-Crack Length Relationship of Beam GY1

From the discrepancy observed between the analytical results from the literature and the proposed model, it is interesting to investigate the relationship between deflection and CMOD generated by the finite element model. Fig. 16 shows the deflection-CMOD relationship of the modeled beam GY1. The deflection-CMOD curve from the original data does not show the bi-linear relationship (S_1 and S_2). On the other hand, the curve begins rising with the constant slope, and becomes flat after the peak load. Then, it starts to rise again with the slope less than that at the initial stage. From Table 3, the value of S_1 for the beam GY1 calculated from the original data is 2.441, which is higher than S_1 of 1.270 for control concrete (CC) obtained in the present study. This indicates that the deflection responding to the applied load could be modeled in such a way that it does not represent the true deflection behavior of the material.

Referring to the proposed model, the true or reasonably assumed values of fracture energy (G_F) and S_1 of the material are required for evaluating the fracture behavior and determining the fracture parameters. Therefore, in the present study for evaluating the performance of the proposed model, the original load-deflection curve was modified to obtain the S_1 equal to 1.270 as assumed for normal strength concrete. Fig. 16 shows the deflection-CMOD relationship (S_1 and S_2) of the beam GY1 for the original data and the data modified by the proposed model. It is noted that the fracture energy (G_F) of 52.54 N/m from the original data was also used for the modified load-deflection curve to calculate the a_c by the proposed model. This is because the value of G_F of 52.54 N/m was given as a concrete property for the model in their study and not calculated from the area under the load vs. deflection curve.

By using the data (Fig. 15) from the modified load-deflection curve and the original load-CMOD curve for the beam GY1, the critical crack length (a_c) obtained by the proposed model becomes 10.63 mm (see Fig. 17), which compares favorably with that of 10.90 mm found in the literature (see Table3). The results show that S_1 is a material property and important for the proposed model in utilizing the load-CMOD curve to evaluate the fracture behavior of cement-based material. Furthermore, it should be noted that S_1 from the finite element model used by Gopalaratnam and Ye [12] is also sensitive to the modeling of the response of beam deflection. Fig. 17 shows the analytical results of load vs. crack length relationship of the beam GY1 obtained by the proposed model for both the original data in the literature and the modified one. Based on the results, the proposed fracture mechanics model for studying the fracture behavior of cementitious material is found to be in good agreement with the finite element model from the literature.

With reference to the performance of the proposed model using test results from other researchers (Ratanalert and Wecharatana [13], and Gopalaratnam and Ye [14]), in order to reduce the effect of erroneous deflection during fracture process, only the S_1 value was modified, while S_2 remained unchanged. This is due to the fact that the fracture parameters discussed here were determined based on the critical values at the peak load, therefore the S_1, which represents the pre-peak behavior of material, is properly related to the interested fracture parameters rather than the S_2, which covers the post-peak behavior. Eventually, regardless of the S_2, the analytical results for the fracture parameters obtained by the fracture models were found to be in good agreement with those reported experimentally by other researchers. These results confirm that erroneous deflection measurement due to support crushing strongly affects the pre-peak behavior rather than the post-peak behavior of the material during the fracture process.

In the finite element model reported by Gopalaratnam and Ye [14], the fictitious crack concept [15] was used in the numerical scheme to simulate the fracture process zone or the inelastic zone ahead of the traction-free crack into a discrete fictitious crack capable of supporting some traction. Crack growth along crack path was controlled by incrementally releasing one node at a time when the tensile stress at that node reaches the tensile strength of the material. This ensured post-peak stability similar to a crack mouth opening controlled experiment performed in the present study. In their model, the deflection response was not involved in the numerical formulations of the finite element model for the fracture behavior of concrete.

In the proposed fracture model, the crack growth during fracture can be determined from the inelastic energy absorbed in the fracture process, which is calculated by applying the area under the load-CMOD curve, S_1, S_2 and the fracture energy (G_F) [1]. By means of three-point bend tests on notched beams, traditional methods of measuring load-line deflection in the notched beams, which was commonly measured with respect to the base of the testing machine, contain extraneous measurements that affect the values of S_1, S_2 and the fracture energy (G_F). To eliminate these extraneous deformations, the deflections must be measured with reference to its neutral axis using a reference frame attached to the beam.

Based on results of the fracture parameters previously discussed, S_1 and G_F, which are considered material properties, are sensitive to the method of measuring deflection, while S_2 is not. However, from the results throughout the present study, S_1 of cementitious materials can be reasonably predicted if there is sufficient database to relate S_1 with the type of material or the mechanical property of material such as compressive strength. Regardless of the recommended method for measuring accurate deflection, when the S_1 is properly assumed and the S_2 is empirically determined, the fracture behavior of the material during fracture process and other fracture parameters can be reliably obtained by applying the load-CMOD curve using the proposed fracture mechanics model.

5. Conclusions

Based on the analytical and experimental investigations, the main contributions of this work can be concluded as follows:

1. For the notched beam fracture test, by measuring the deflections with reference to its neutral axis using a reference frame attached to the beam, the extraneous deflection measurements as a result of support settlements can be eliminated. With an accurate test setup to measure the deflection, the proposed fracture mechanics model was developed as an alternative means to study the fracture behavior and to determine the fracture parameters of concrete based on the load vs. crack-mouth-opening displacement (CMOD) response.
2. When proper measurements of beam deflection are performed along with CMOD measurements, the bilinear relationships between deflection and CMOD in the pre-peak and post-peak regions defined as the S_1 and S_2 respectively are found to exist. The S_1 is more likely than S_2 to be affected by the erroneous measurements of the beam deflection. For the proposed model, S_1 and S_2 served as the important factors to relate CMOD to the fracture parameters of concrete.
3. In implementing the proposed fracture model, the fracture behavior of concrete such as the load vs. crack growth response and the energy release rate vs. crack growth response (R curve) can be determined without applying the finite element model. The conventional fracture parameters (e.g. a_c, G_C and K_{IC}) can be obtained as well by the proposed model without performing the complicated stable unloading-reloading during testing as required by the TPFM model.
4. The analytical results of the fracture behavior and the fracture parameters of concrete obtained by the proposed model

are found to be in good agreement with the test data and the fracture models of other researchers.

5. Survey of literature on concrete fracture tests found that a number of tests were carried out by using the erroneous measurements of beam deflections which often included the crushing of concrete at supports. By applying the reasonably assumed values of S_1 for the true deflections and the empirically determined values of S_2, the proper evaluation of the fracture behavior and the fracture properties of concrete can be obtained by using the load-CMOD response based on the proposed model.
6. For the three-point bend tests on notched beams, the use of the proposed fracture model with the load-CMOD relationship, which is unaffected by support settlements, could lead to a more reliable testing procedure for determining the fracture properties of cementitious materials.

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