Numerical Simulation of Concrete Beams subjected to Bi-Axial Loading

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Abstract

This paper presents the numerical simulation of concrete beams subjected to bi-axial loading by performing nonlinear finite element analysis which adopts isotropic three-dimensional brick elements. For the constitutive model of reinforced concrete under complicated state of stress, the present analysis utilizes the proposed model by Hauke and Maekawa [2] which accounts for the anisotropy of tension softening and stiffening of concrete cracks. The numerical results are compared with the experimental results of the true-scale concrete beams under bi-axial loading in terms of load-deflection relationship. In the present test, there are four rectangular beams of 150 X 350 mm. cross-section and 1600 mm. lengths, and two parameters, i.e. the stirrup steel and the ratio of two lateral loads are considered. The comparisons indicate that the proposed model by Hauke and Maekawa [2] can reasonably simulate the nonlinear behavior of all four tested beams. Hence, this model could be a numerical tool to study the nonlinear behavior of concrete beams under complicated loads associated with the true-scale test which is more expensive and time-consuming.

1. Introduction

At present, due to advanced technology in civil engineering, the shape of concrete structure becomes complex, and it may be subjected to the complicated loads, such as unexpected loads or accidental loads. For the reinforced concrete beam, the direction of loads may be arbitrary with respect to principal axes of beam cross section. This type of loading condition is hereinafter called bi-axial loading. With the advent of digital computer and nonlinear finite element analysis, understanding of behavior reinforced concrete beam up to failure state can be achieved. This study attempts to numerically simulate the test results of reinforced concrete beams under bi-axial loading. The three-dimensional nonlinear finite element analysis is performed by using the program developed by Concrete Laboratory, The University of Tokyo, called as COM3 [1].

In the finite element mesh of beams, isotropic 20-node brick element is adopted. By using the smeared crack model proposed by Hauke and Maekawa [2], an anisotropy behavior related to arbitrary crack direction in the three- dimensional space can be taken into account. In order to trace nonlinear behavior of the beams, numerical simulation is performed incrementally. Iterative numerical procedure is used for each computation step.

2. Nonlinear Finite Element Analysis

2.1 Numerical Procedure

Following the normal procedure for nonlinear finite element formulation [3], the incremental stiffness equation is expressed by

$$[K_t]\{\hat{u}\} = \{R_t\} - \{F_t\} \quad (1)$$

where

$[K_t]$ is tangent stiffness matrix
$\{\hat{u}\}$ is nodal displacement vector
$\{R_t\}$ is external force vector
$\{F_t\}$ is internal force vector

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In order to establish nonlinear load-displacement curve, numerical simulation is performed incrementally and unbalanced forces correction is performed by iterative procedure. Typically, the numerical procedure is performed as follows

1. Set up incremental stiffness equation in the global system based on the current geometry and material properties.
2. Solve the incremental stiffness equation Eq. (2), for load increment and update to the current deformed structure.

$$[K_t]_n^m \{\hat{u}\}_n^m = \{R\}_n - \{F\}_n^m = \{\Delta R\}_n^m \qquad (2)$$

where

$[K_t]_n^m$ is incremental or tangent stiffness matrix at iteration m and incremental load step n.

$\{\hat{u}\}_n^m$ is nodal displacement vector at iteration m and incremental load step n.

$\{\Delta R\}_n^m$ is unbalanced forced vector at iteration m and incremental load step n.

$\{R\}_n$ is external force vector at incremental load step n.

$\{F\}_n^m$ is internal force vector at iteration m and incremental load step n.

m and n are iteration and incremental load step, respectively.

3. Compute strains from the updated displacement.
4. Using material models to compute stresses and tangent stiffness.
5. Form internal force and tangent stiffness matrix. Comparing the internal force with the external force which is assumed constant during iteration step, the unbalanced forces are determined.
6. Using the unbalanced forces and the current tangent stiffness matrix, the incremental equation is resolved.

The above steps (1) to (6) are repeated until the unbalanced forces are within allowable tolerances. For the next incremental load step, the steps (1) to (6) are restarted until failure occurs.

2.2 Modeling of Reinforced Concrete

In this study, the reinforced concrete behavior is treated macroscopically as a continuum in a finite region. In this case, the so-called smeared crack modeling which macroscopically deals with cracked concrete and reinforcing bar is introduced by expressing the average stress and strain relationships. The relationships can be described by combining the constitutive laws for concrete and reinforcing bar [1]. With various kinds of the action of stresses, constitutive law of concrete is composed of 1) concrete under tension, 2) concrete under compression and 3) shear transfer along the crack plane. Regarding the constitutive law for reinforcement, models under uni-axial loading in tension and compression are introduced. In addition, the anisotropic cracking model proposed by Hauke and Maekawa [2] is used in the constitutive law of three-dimensional cracked concrete.

(a) Concrete under tension

The model representing concrete subjected to tensile force is shown in Fig.1. From the figure, the softening is expressed by Eq.(3) in which the softening parameter c indicates the effect of bond between reinforcement and concrete surrounded.

$$\sigma_t = f_t\left(\frac{\varepsilon_{tu}}{\varepsilon_t}\right)^c \qquad (3)$$

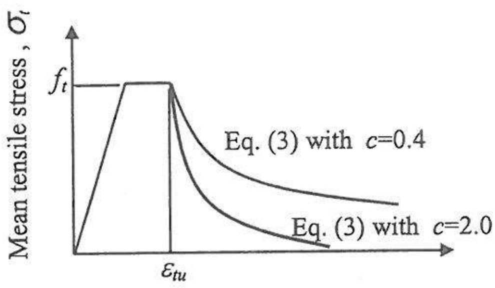


Fig.1 Tension model for concrete [1]

where, σ_t and ε_t are tensile stress and strain respectively. f_t is tensile strength and ε_{tu} is cracking strain. The value of c depends on the crack location and direction relative to reinforcement provided, regardless of reinforcement ratio. In other word, there are two extreme cases defining c value as follow; (1) the crack occurs inside bond effective zone (RC zone-reinforced concrete zone) and direct normal to the reinforcing bar, and (2) the crack occurs outside bond effective zone (PL zone-plain concrete zone) or direct parallel to the reinforcing bar [4]. For the first case, stress can be transferred across the crack with use of bonding between concrete and reinforcing bar. For the other case, stress is transferred across the crack by aggregate interlocking, without bonding effect from reinforcement. Hence, tensile stress reduces gradually for the first case and drastically for the other case in which the c value is respectively defined as 0.4 and 2.0 (Fig.1).

(b) Concrete under compression

Based on the concept of continuum fracturing and plasticity, the relationship between compressive stress and compressive strain in concrete can be obtained, as shown in Eq.(4) and Fig.2.

$$\sigma' = E_0K\left(\varepsilon_c' - \varepsilon_p'\right) \tag{4}$$

where

$$E_0 = \frac{2f_c'}{\varepsilon_{co}'}$$

$$K = exp\left[-0.73\left(\frac{\varepsilon_c'}{\varepsilon_{co}'}\right)\left\{1 - exp\left(-1.25\frac{\varepsilon_c'}{\varepsilon_{co}'}\right)\right\}\right]$$

$$\varepsilon_p' = \varepsilon_c' - \varepsilon_{co}'\left(\frac{20}{7}\right)\left[1 - exp\left\{-0.35\frac{\varepsilon_c'}{\varepsilon_{co}'}\right\}\right]$$

σ', ε_c' are compressive stress and strain, respectively

E_0 is initial tangential elastic constant

K is fracture parameter

ε_p' is plastic compressive strain

f_c' is uniaxial compressive strength

ε_{co}' is compressive strain corresponding to f_c'

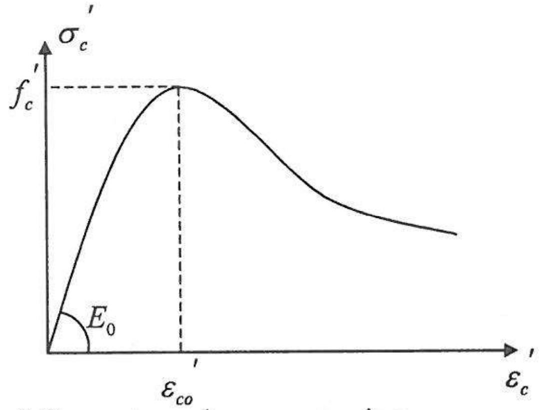


Fig.2 Concrete under compression

(c) Shear transfer along the crack plane

The transfer of shear along a concrete crack plane is expressed by Eq.(5) and Fig.3.

$$\tau = G\gamma \tag{5}$$

where

$$G = \frac{1}{1/G_{st} + 1/G_c}$$

$$G_{st} = \frac{\tau_{st}}{\gamma} = f_{st}\frac{\beta}{1+\beta^2}$$

$\beta = 2\frac{\gamma}{\varepsilon_t}$ (2 for reinforced concrete)

$f_{st} = 18\left(f_c'\right)^{1/3}$ (unit: kgf./cm^2)

τ, γ are mean shear stress and strain

G is shear secant modulus

G_c is shear modulus of uncracked concrete
G_{st} is shear modulus due to cracks
ε_t is tensile strain normal to crack plane
f_{st} is intrinsic shear strength

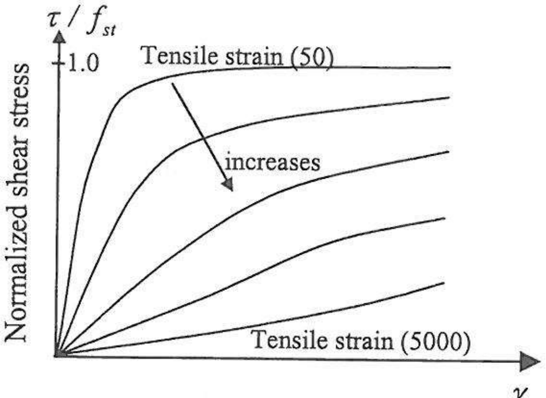


Fig.3 Shear model for concrete

(d) Reinforcement

With cracks, the stresses acting on a bar take the maximum values at crack planes. Hence, the mean yield strength is lower than that of bare bar. Below elastic limit, stress-strain relationship of reinforcement is set equal to that for the bare bars. The model for reinforcement is shown in Fig.4. In case of compressive loading, the stress-strain relationship of bare bar is adopted.

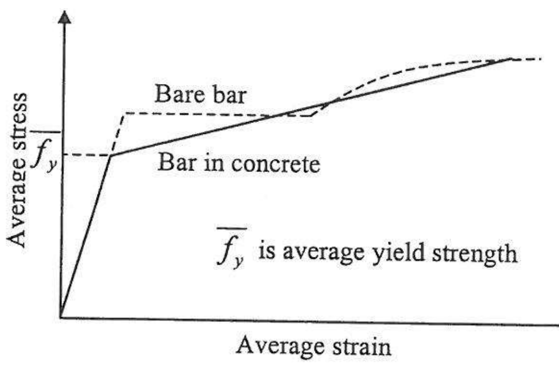


Fig.4 Reinforcement model

(e) Anisotropic cracking

Regardless of arrangement of reinforcement, however, direction of crack in concrete volume is generated (n-direction). Hence, neither parallel (2-direction) nor normal (1-direction) of the direction of crack relative to reinforcement, concrete must shows mixed performance between the two extreme cases. The behavior of cracking regardless of reinforcement brings concrete to anisotropic softening. Fig.5 shows the treatment of mixed formulation of post-cracking behavior [2].

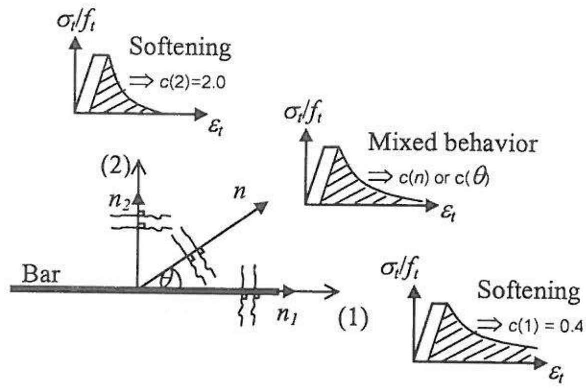
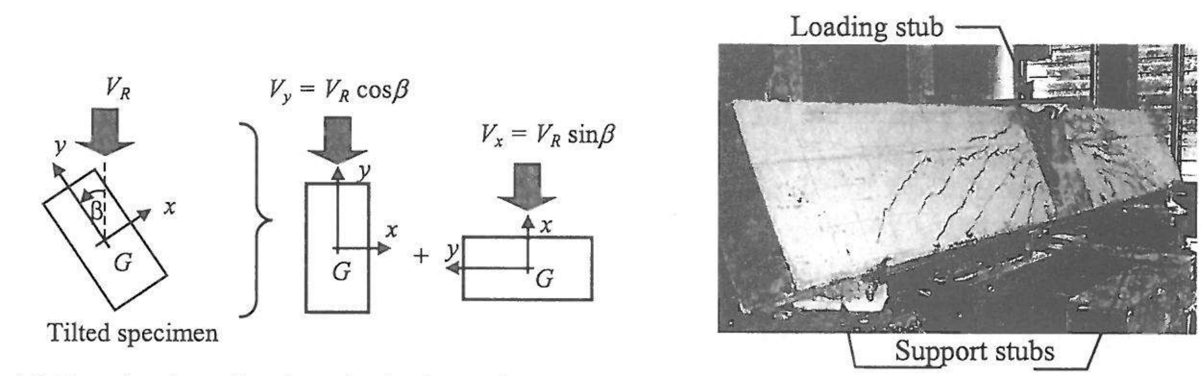


Fig.5 Anisotropy of tension softening [2]

3. Test of Bi-Axial Loading

Fig.6 shows the test of reinforced concrete beam subjected to bi-axial loading [4]. Rather than using the multi-hydraulic jacks, two lateral loads are applied to the beam specimen by tilting the beam at a certain angle (β). Since the load from one hydraulic jack applied in vertical direction does not coincide with the principal axes (x,y) of the beam cross-section, two lateral loads can be simultaneously applied to the beam. As shown in Fig. 6(a), the applied vertical load V_R which passes through shear center G is decomposed into two lateral loads acting in x and y directions, i.e., V_x and V_y respectively. Hence, ratio of the loads in x and y directions can be changed in accordance with the tilt angle of the beam (β) between the principal axes and line of the apply load, V_R. The beam specimen is simply supported and subjected to a concentrated load at midspan. In order to achieve such loading scheme, one loading stub at midspan and two support stubs at ends are provided as can be seen in Fig. 6 (b).

There are four reinforced concrete beam specimens in bi-axial loading test performed by the authors [5]. In Fig.7 and Table 1, all specimens have the same dimension, i.e.

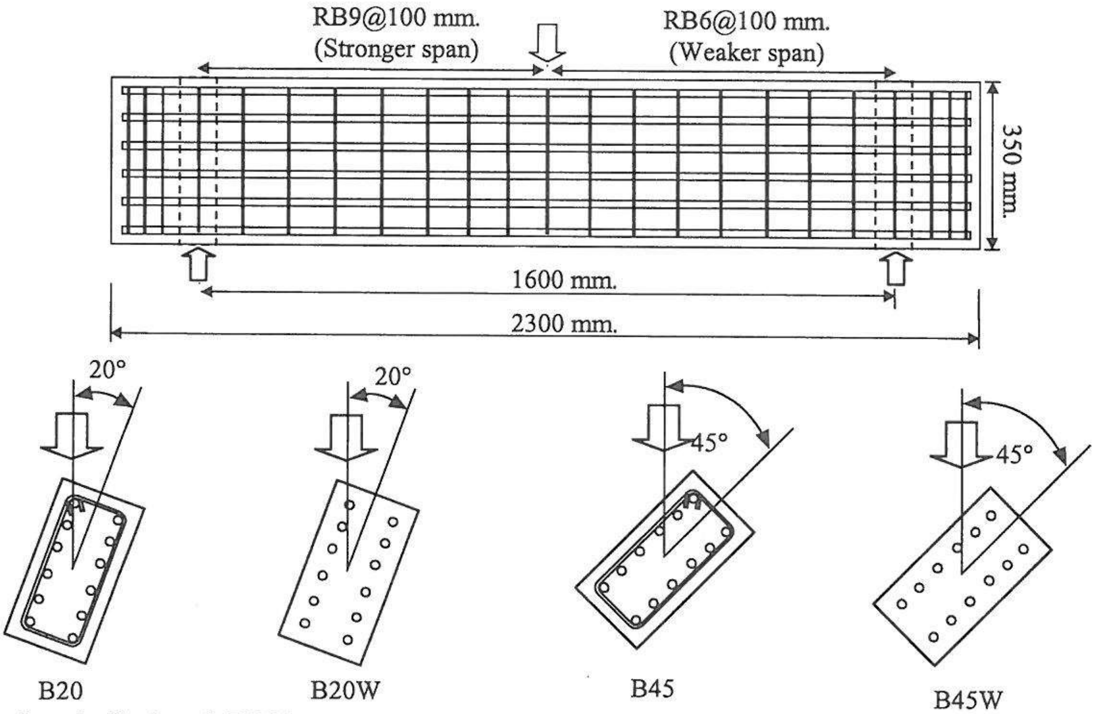


(a) Two loads acting in principal x and y axes (b) Test specimen tilted at 20 degree

Fig. 6 Bi-axial loading test with one hydraulic jack providing load in vertical direction

Table 1 General details of specimens

Specimen	Section $h_x \times h_y$ (mm×mm)	Tilt angle β (Degree)	Shear span a (mm)	Beam span L (mm.)	Concrete strength f_c' (MPa)	Longitudinal reinforcement: yield strength (MPa)	Shear reinforcement: yield strength (MPa)
B20	150×350	20	800	1600	28	DB25: 440	RB6: 370
B20W							
B45		45					
B45W							



Longitudinal steel: DB25
Stirrup steel: RB6

Fig. 7 Dimension of test specimens

rectangular cross-section of 150 by 350 mm. and span length of 1600 mm. In the test, the ratio of the two lateral loads or the tilt angle β is considered as one parameter. There are two cases: 1) specimens B20 and B20W for $\beta = 20$ degrees, and 2) specimens B45 and B45W for $\beta = 45$ degrees. Another parameter considered in the test is the stirrup steel. Similarly, there are two cases: 1) specimens B20 and B45 with stirrup steel, and 2) specimens B20W and B45W without stirrup steel.

4. Numerical Results

In order to predict the nonlinear behavior of the tested beams, three-dimensional nonlinear finite element analysis by using isotropic 20-node brick element and Hauke and Maekawa's constitutive model is performed. Due to the symmetrical condition at the midspan of the beam, only a half span of the beam is analyzed. In modeling, the beam is discretized into 254 brick elements and 1,513 nodes. Fig.8 shows the finite element mesh of the beam. In the nonlinear analysis, the displacement at midspan is incrementally controlled.

From the results of nonlinear finite element analysis, Figs. 9 (a), (b), (c), (d) show the relationship between vertical shear force and displacements, i.e. vertical and lateral displacements, at midspan for the specimens B20, B20W, B45 and B45W, respectively. In the range of ascending branch of the curve, fair agreement between analytical results and experimental results can be seen for all results of Fig. 9. However, in the descending branch after the peak point of the curve, the analytical results deviate from the experimental ones. The reason is that since the large amount of longitudinal reinforcement is provided for preventing flexural failure of the tested beams, the experimental results exhibits ductile behavior, i.e. large deformation in this descending branch. On the other hand, in the analysis, due to the lost of numerical stability after the peak point, the load suddenly drops, and hence the ductile behavior cannot be well predicted.

Regarding the ultimate shear capacity or the peak load of the curves in Fig. 9, comparisons between those obtained from the tests and the analytical results are made in Table 2. It can be seen that the present finite element analysis gives the calculated ultimate shear capacity quite close to the tested results, i.e. the difference is in the range of (–1%)-(6%).

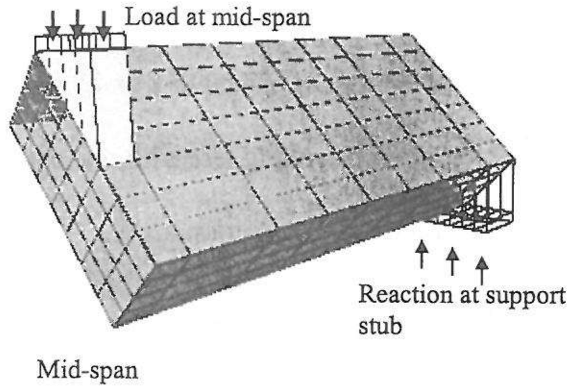


Fig.8 Finite element model of reinforced concrete beam specimens subjected to bi-axial shear (Half specimen)

5. Conclusions

In this study, the nonlinear behavior of four reinforced concrete beams subjected to bi-axial loading is simulated by performing three-dimensional finite element analysis. A half of the beam is modeled by 254 three-dimensional brick elements with 1,513 nodes. The present analysis adopts the constitutive models of concrete and reinforcement in the three-dimensional cracking state proposed by Hauke and Maekawa [2]. It can be seen that the numerical results obtained especially in terms of ultimate bi-axial shear capacity agree well with the test results of all four beams. Therefore, the three-dimensional finite element analysis with the proposed constitutive model by Hauke and Maekawa [2] could be used to predict the ultimate capacity of reinforced concrete structures subjected to complicated bi-axial loading. As a result, in the prediction of ultimate capacity, this type of numerical approach should be encouraged to be used associated with the experimental approach. Hence, the number of test can be significantly reduced leading to the reduction of time and cost consumed.

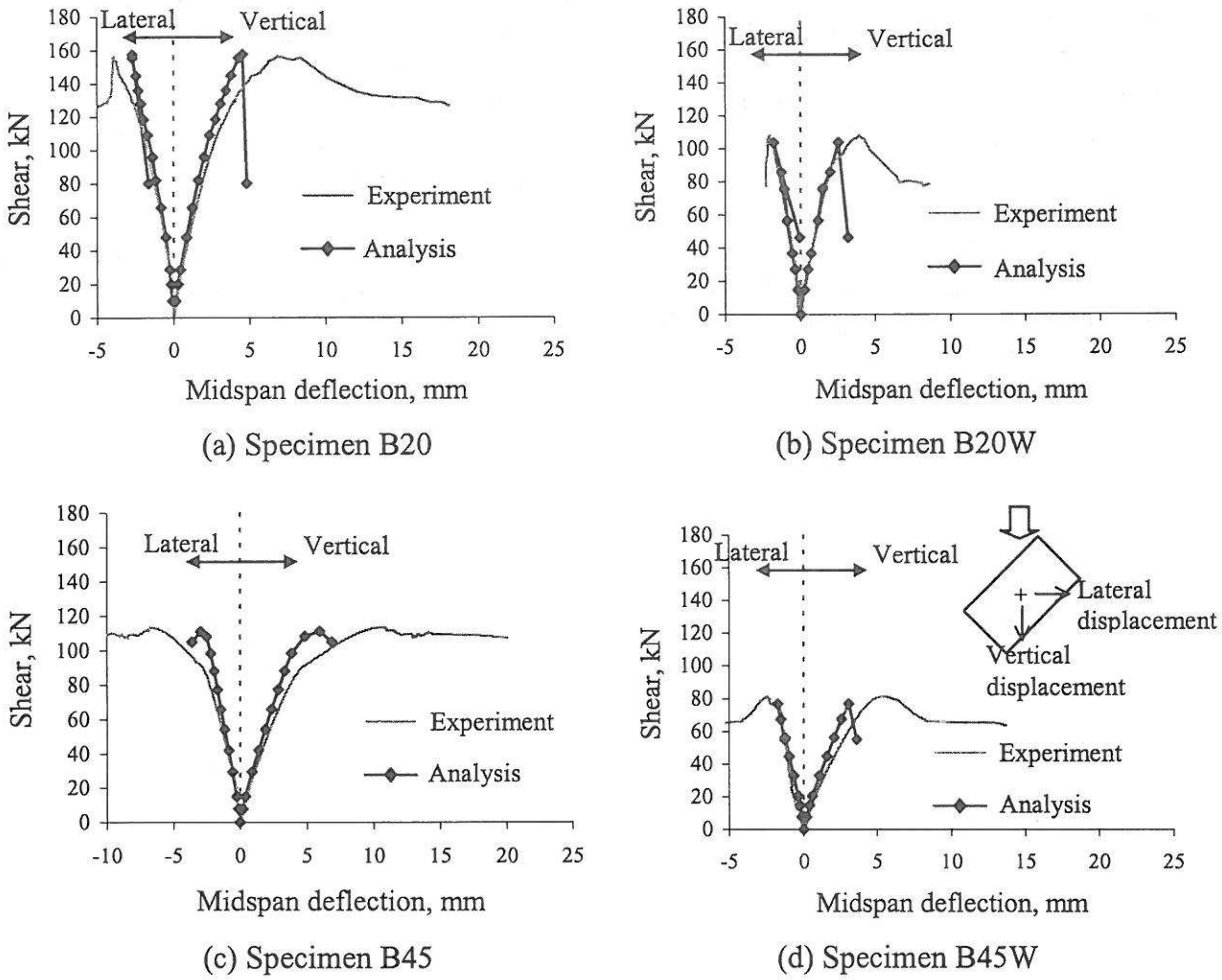


Fig.9 Relationships between Vertical shear force-displacements at midspan

Table 2 Comparisons of ultimate shear capacity obtained from tests and analyses

Specimen	Experiment (kN) V_u	Analysis (kN) V_u	Exp./Ana. V_u
B20	156.5	157.5	0.99
B20W	108.0	103.7	1.04
B45	113.4	111.0	1.02
B45W	81.3	76.8	1.06

6. Acknowledgments

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