

## A 28-day Compressive Strength Model for Fly Ash Concrete

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### Abstract

This paper presents a model for predicting 28-day compressive strength of concrete incorporating fly ash. The model was formulated based on quantities of concrete constituents, chemical composition and physical characteristics of binder materials. Relative volume of paste to volume of void in compacted aggregate phase was taken into account in the model. In addition, influences of water-reducing admixture and loss on ignition of fly ash were also considered. It was found that the proposed model could be used to predict compressive strength of concretes incorporating fly ash with various chemical compositions and physical properties and various types of water-reducing admixture with satisfactory accuracy.

### 1. Introduction

Compressive strength is considered one of the important properties of structural concrete. This property depends on the quantities and qualities of its ingredients and also external factors like temperature and moisture condition. For concrete with cement only, relationship between concrete constituents and compressive strength has been intensively studied by many researchers. The most practically known relationship was the Abrams's Water to Cement Ratio Law by Duff Abrams [1]. Abrams' Law was formulated based on the investigation that an increase in water-to-cement ratio decreases the concrete strength.

However, cement is nowadays no longer the only cementitious material contained in concrete. Fly ash is one of the widely used supplementary cementitious materials in the

concrete. Many prediction models were developed to determine the compressive strength of fly ash concrete. However, there still remain a number of necessary parameters that were not taken into account according to practical work such as water-reducing admixture, etc. In addition, the effects of particle packing of fly ash and loss on ignition of fly ash on the compressive strength have been rarely included as the parameters in the literatures.

In this study, the chemical compositions of binder materials, water-to-binder ratio, fineness of fly ash, relative volume of paste to volume of void in compacted aggregate phase, loss on ignition of fly ash, and water-reducing admixture are the parameters used for formulating the model to predict compressive strength of concrete at 28 days.

### 2. Model Formulation

The 28-day compressive strength prediction model was developed by reasonably assuming that the 28-day compressive strength varies with effective calcium oxide content in binders, water-to-binder ratio, loss on ignition of fly ash, ratio of paste volume to void volume in compacted aggregate phase, effects of particle filling of fly ash, and water-reducing admixture.

#### 2.1 Effects of Effective Calcium Oxide Content, Fineness of Fly Ash and Water-to-Binder Ratio

It is known that the use of fly ash with high CaO content and high fineness leads to concrete with higher 28-day compressive strength, so the effectiveness of fly ash on 28-day compressive strength was considered in

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this study to be functions of CaO content and fineness of fly ash. Here, the effective calcium oxide content in binders is defined from the fact that calcium oxide in fly ash is in different mineral forms from calcium oxide in cement. Therefore, it is assumed that fly ash gives different effectiveness from cement on the 28-day compressive strength. The effectiveness of fly ash on the 28-day compressive strength is defined as (see Fig.1):

$$\phi = \frac{1 - e^{-\kappa \cdot (CaO_f)}}{1 + e^{-\kappa \cdot (CaO_f)}} \quad (1)$$

whereas

$$\kappa = 0.0048 \left( \frac{S_f}{3000} \right)^{3.07} + 0.0245 \quad (2)$$

where  $\phi$  is the effectiveness of fly ash on 28-day compressive strength and  $S_f$  is the Blaine fineness of fly ash ( $cm^2/g$ ).

The effective calcium oxide content in binders is defined as:

$$CaO_{eff} = \frac{(CaO_c \times W_c) + \phi \cdot (CaO_f \times W_f)}{100} \quad (3)$$

where  $CaO_{eff}$  is effective calcium oxide content in the total binders ( $kg/m^3$  of concrete),  $CaO_c$  and  $CaO_f$  are the calcium oxide contents in cement and fly ash (% by weight), respectively.  $W_c$  and  $W_f$  are cement and fly ash contents in concrete ( $kg/m^3$  of concrete), respectively.

It was found that the 28-day compressive strength increases linearly with logarithm of effective calcium oxide content in the total binders and also increases as water-to-binder ratio decreases. Fig. 2 shows the relationships for concrete incorporating fly ash with fineness of  $2600 \text{ cm}^2/g$ . This relationship was also found to be similar in concrete containing fly ash with other fineness.

## 2.2 Filling Effect of Fly Ash

When incorporating fly ash in concrete, some very fine fly ash particles can fill in the voids among cement particles. This leads to dense packing of particles of binder materials

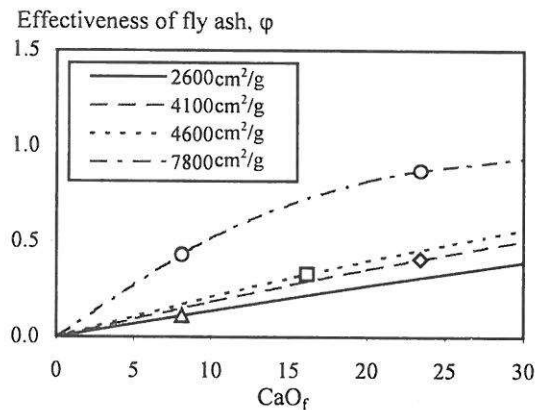


Fig. 1 Relationship between effectiveness of fly ash on 28-day compressive strength and percentage of calcium oxide in fly ash as well as fineness of fly ash

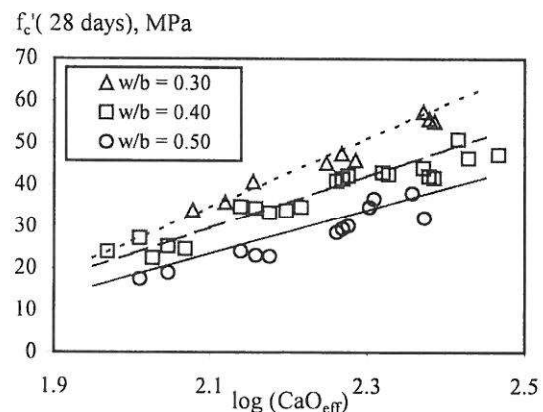


Fig. 2 Relationship between 28-day compressive strength and  $\log(CaO_{eff})$  of mixtures with various w/b

and affects concrete strength positively. The ability to fill voids among cement particles increases as the particle size of fly ash reduces. Moreover, it was found that particle shape of fly ash also influences the ability to fill voids [2]. Spherical particles are likely to have a better ability to fill voids among cement particles than non-spherical particles. Tangtermsirikul et al. (2001) presented the filling coefficient  $F$  as functions of specific surface area ratio, shape factor of fly ash, and replacement ratio to investigate its effect on workability of fresh concrete [2]. The filling coefficient  $F$  was defined as the ratio of the volume of fine fly ash in the voids among

cement particles per volume of cement. For example, a filling coefficient  $F$  of 0.2 means that fly ash with a volume that is 20 percent of the volume of cement is fillable in the voids among cement particles. Fig. 3 shows the relationships between the specific surface area ratio of fly ash to cement and filling coefficient  $F$  [2]. In this study, the filling coefficient was adopted to explain the role of particle filling of fly ash on 28-day compressive strength. The filling coefficient is determined from Eq. (4) as

$$F = 0.25 - \frac{0.69}{\exp \left[ 1 + 3 \left( \frac{R-1}{\Psi^{3.3}} \right) \right]^{0.6r^{0.25}}} \quad (4)$$

in which

$$R = \frac{S_f}{S_c} \quad (5)$$

where  $R$  is the specific surface area ratio of fly ash to cement, respectively.  $S_f$  and  $S_c$  are the specific surface areas of fly ash and cement, respectively,  $r$  is the replacement ratio of fly ash.  $\Psi$  is the shape factor of fly ash, recommended to be 1.05 for original or air-classified spherical-shape fly ash and 1.20 for ground fly ash ( $\Psi = 1.0$  for a perfectly spherical particle, and  $\Psi > 1.0$  when particles become more irregular).

It was found from the relationship between 28-day compressive strength and  $\log(\text{CaO}_{\text{eff}})$  that for concrete with the same water-to-binder ratio, concrete containing fly ash with higher value of filling coefficient  $F$  results in higher value of y-intercept in the curve representing the relationship between 28-day compressive strength and  $\log(\text{CaO}_{\text{eff}})$  as shown in Fig. 4. The influence of filling coefficient on y-intercept of the curve representing the relationship between 28-day compressive strength and  $\log(\text{CaO}_{\text{eff}})$  is presented in Fig. 5 and the filling effect of fly ash on 28-day compressive strength was formulated as

$$\lambda_F = \frac{1}{1 + (0.25\Psi^{-4.91}) \tan^{-1}(357F^{3.24})} \quad (6)$$

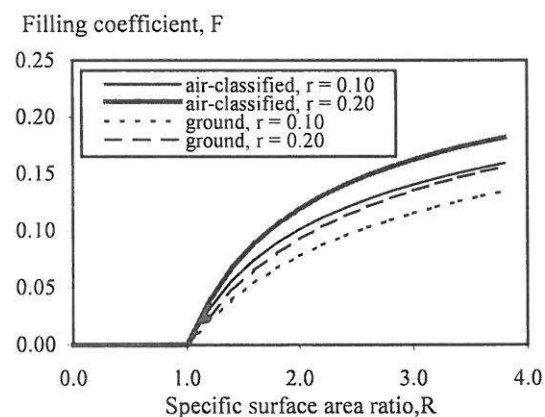


Fig. 3 Relationship between specific surface area ratio of fly ash to cement and filling coefficients of ground and classified fly ashes

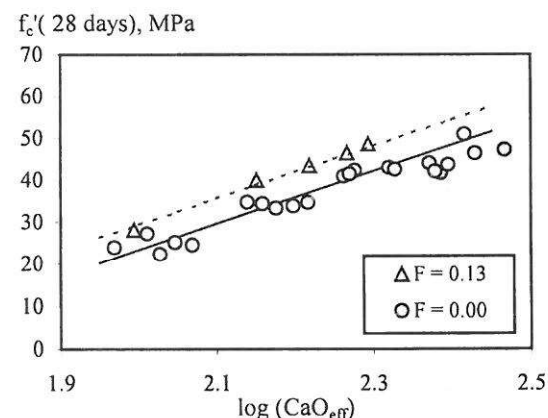


Fig. 4 Relationship between  $f'_c(28 \text{ days})$  of concrete with  $w/b$  of 0.40 and  $\log(\text{CaO}_{\text{eff}})$  with fly ashes of different filling coefficients

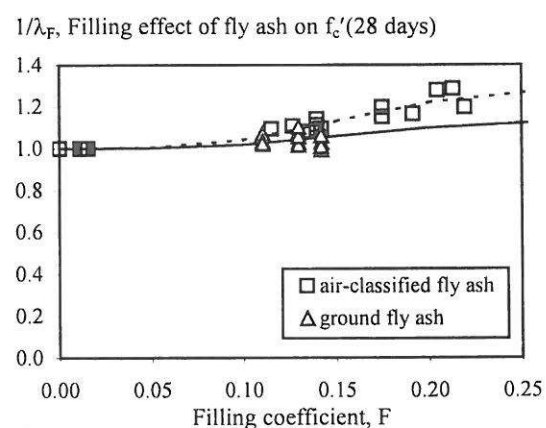


Fig. 5 Relationship between filling coefficient of processed fly ashes and their effect on y-intercept of the  $f'_c(28 \text{ days})$  and  $\log(\text{CaO}_{\text{eff}})$  relationship ( $1/\lambda_F$ )

## 2.3 Effect of Ratio of Paste Volume to Void Volume in Compacted Aggregate Phase

The relative volumes of paste and voids in compacted aggregate phase was considered to influence 28-day compressive strength of normal concrete. Normal concrete here means concrete with conventional aggregate and is not high strength concrete (28-day compressive strength up to 60 MPa is considered in this model). It was obvious that normal concrete with much less paste, in comparison with voids among aggregates, leads to inadequate paste to fill voids and results in a poor aggregate-paste interface bond. Consequently, poor compressive strength concrete is obtained. On the contrary, too much paste content, in comparison with voids content, also results in lower compressive strength since generally paste is more porous than aggregate. Replacing the volume of aggregates by paste leads to more porous portions in concrete. Nevertheless, the reduction of the strength while increasing  $\gamma$  beyond its optimum value becomes less significant in concrete with lower w/b as the result of its higher strength of paste (Fig. 6).

The ratio of paste volume to void volume in compacted aggregate phase ( $\gamma$ ) is defined as follows:

$$\gamma = \frac{V_{\text{paste}}}{V_{\text{void}}} \quad (7)$$

in which

$$V_{\text{paste}} = V_c + V_f + V_w + V_{\text{air}} \quad (8)$$

where  $\gamma$  denotes the ratio of paste volume to void volume in compacted aggregate phase.  $V_{\text{paste}}$ ,  $V_c$ ,  $V_f$ ,  $V_w$ ,  $V_{\text{air}}$ , and  $V_{\text{void}}$  are the volumes of paste, cement, fly ash, water, air, and voids in compacted aggregate phase in  $\text{m}^3/\text{m}^3$  of concrete, respectively.

In addition, it can be seen from Fig. 6 that there must be some optimum values of  $\gamma$  for each w/b that gives rise to the highest 28-day compressive strength. Concrete with higher w/b is expected to have a lower optimum value

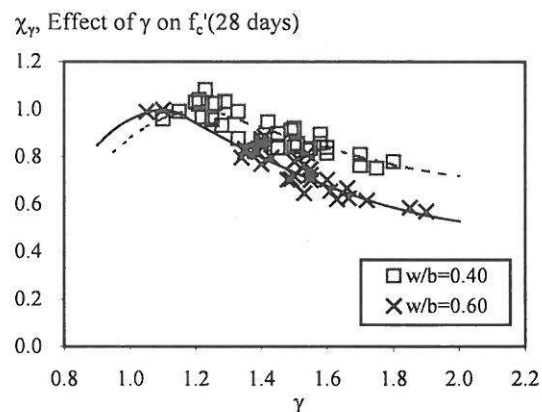


Fig. 6 Relationship between ratio of paste volume to void volume in compacted aggregate phase ( $\gamma$ ) and its effect on 28-day compressive strength

of  $\gamma$  than that with lower w/b. This can be explained that concrete with appropriately higher w/b requires lower paste content to achieve fully compacted concrete because of its high fluidity and vice versa.

The optimum  $\gamma$  for each w/b was derived as follows:

$$\gamma_{\text{opt}} = 0.59^{1.74(w/b)} + 0.55 \quad (9)$$

where  $\gamma_{\text{opt}}$  denotes the optimum ratio of paste volume to void volume in compacted aggregate phase. w/b is the water-to-binder ratio.

The relationship between  $\gamma$  and its effect on 28-day compressive strength is shown in Eq. (10).

For  $\gamma < \gamma_{\text{opt}}$

$$\chi_\gamma = 1 - [10.64(w/b)^{3.68} + 2.38] \cdot (\gamma_{\text{opt}} - \gamma)^{2.21} \quad (10a)$$

For  $\gamma \geq \gamma_{\text{opt}}$

$$\chi_\gamma = 1 - \frac{[11.97(w/b)^{1.12}] \cdot (\gamma - \gamma_{\text{opt}})}{7.57 + \exp[1.83(\gamma - \gamma_{\text{opt}})]} \quad (10b)$$



where  $\chi_\gamma$  represents the effect of  $\gamma$  on 28-day compressive strength and  $w/b$  is the water-to-binder ratio.

However, the proposed relationship is for conventional concrete with conventional aggregate, which means strength of aggregate is much higher than strength of paste. Therefore, if the relative strength between paste and aggregate changes, the effect of  $\gamma$  on 28-day compressive strength is expected to be different. Concrete with high strength paste and/or low strength aggregate is expected to have less reduction of strength with increase of  $\gamma$  or may even have improved strength with increase of  $\gamma$ .

## 2.4 Effect of Loss on Ignition of Fly Ash

In this model, LOI of fly ash is regarded as a nonreactive part and a low strength portion in concrete. Majority of LOI in fly ash is unburned carbon.

To consider the effect of LOI on 28-day compressive strength, the term  $\eta$  was introduced, as the ratio of equivalent volume of ignited materials (mainly unburned carbon) in fly ash to the volume of paste as shown in Eq. (11). Higher value of  $\eta$  resulted in lower compressive strength as shown in Fig. 7. In other words, fly ash with higher LOI gives lower 28-day compressive strength than that with lower LOI when the replacement contents are the same. However, this effect is smaller if fly ash has a higher calcium oxide content.

$$\eta = \frac{V_{LOI}}{V_{paste}} \quad (11)$$

in which

$$V_{LOI} = \frac{(\%LOI \times W_f / \rho_{uc})}{100} \approx \frac{(\%LOI \times W_f / \rho_f)}{100} \quad (12)$$

where  $\eta$  denotes the ratio of equivalent volume of ignited materials (mainly unburned carbon) in fly ash to volume of paste,  $V_{LOI}$  is the equivalent volume of ignited materials in fly ash ( $m^3/m^3$  of concrete),  $V_{paste}$  is the volume of

$\chi_{LOI}$ : Effect of LOI of fly ash on  $f'_c$  (28 days)

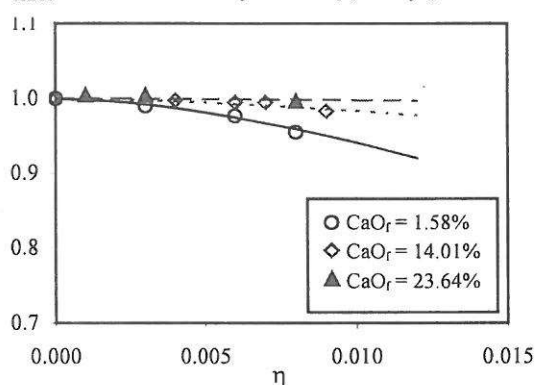


Fig. 7 Relationship between ratio of equivalent volume of LOI in fly ash to volume of paste ( $\eta$ ) and its effect on 28-day compressive strength

paste ( $m^3/m^3$  of concrete), %LOI is the percentage of LOI in fly ash,  $W_f$  is the weight of fly ash ( $kg/m^3$  of concrete),  $\rho_{uc}$ , and  $\rho_f$  are the specific gravities of unburned carbon, and fly ash, respectively.

The specific gravity of unburned carbon was assumed to be equal to the specific gravity of fly ash for simplicity. The effect of LOI on 28-day compressive strength is then derived from Fig. 7 as:

$$\chi_{LOI} = 1 - 155.75 [\exp(-0.15 \cdot CaO_f)] \eta^{1.66} \quad (13)$$

where  $\chi_{LOI}$  represents the effect of LOI of fly ash on 28-day compressive strength,  $CaO_f$  is the percentage by weight of calcium oxide in fly ash.

## 2.5 Effect of Water-Reducing Admixture

Water-Reducing Admixtures, also known as Plasticizers or Superplasticizers, are normally added to a concrete to improve its workability. However, water-reducing admixture was also found to affect the strength positively due to its ability to disperse cementitious materials. In this study, only water-reducing admixtures with no retarder were studied whereas the effect of retarder is still left for future study. From the study, the incorporation of water-reducing admixture was

found to affect the compressive strength positively especially at early age since water-reducing admixture contributes to dispersion of the particles of binders in the fresh state, which improves the reactivity within the paste. Nevertheless, the water-reducing admixture influences the improvement of the strength differently when considering concretes with the same binder contents but different fly ash replacement; the effect of water-reducing admixture on compressive strength is smaller in concrete with higher fly ash replacement. Hence,  $\nu$  was introduced as the percentage of admixture to the effective binder by weight and is derived from Eq.(14) and Eq.(15).

$$\nu = \frac{W_{wr}}{W_{b,eff}} \times 100\% \quad (14)$$

in which

$$W_{b,eff} = W_c + \frac{W_f}{\phi} \quad (15)$$

where  $\nu$  is the percentage of admixture to the effective binder,  $W_{wr}$ ,  $W_{b,eff}$ ,  $W_c$ , and  $W_f$  are the weights of water-reducing admixture, effective binder, cement and fly ash in  $\text{kg/m}^3$  of concrete, respectively.  $\phi$  is the effectiveness of fly ash from Eq. (1).

It was found that increasing  $\nu$  enhances 28-day compressive strength to a certain level for the reasons given above. However, this effect varies with w/b as shown in Fig. 8. Concrete with higher w/b has less effect of water-reducing admixture on compressive strength than that with lower w/b because binders are relatively well dispersed in concrete with higher w/b. Moreover, the type of water-reducing admixture also affects its contribution to the compressive strength. As mentioned earlier, it is obvious that the compressive strength is enhanced by the degree of dispersion of binder particles which is different from type to type of admixtures. Thus, the model considers the water-reducing efficiency of admixtures, which is defined as the efficiency to reduce water content for obtaining the same workability as those without water

reducer, so that it is applicable for many types of water-reducing admixtures.

In this study, the method for evaluating the water-reducing efficiency of admixtures was introduced by finding the ratio of the difference between the water content in cement paste containing 0.5%-dosage water-reducing admixture and the water content in cement paste without water-reducing admixture for obtaining the same flow diameters of 180 mm to the water content of cement paste without water-reducing admixture. First, for cement paste without water-reducing admixture, the water is added so as to obtain the flow diameter of 180 mm. Then, cement paste with 0.5%-dosage admixture and with the reduced water content used as in that without water-reducing admixture is proportioned until the same flow is obtained. The portion of a mixture is placed in the metal mold (a metal mold in the form of a frustum of a cone with dimensions as follow:  $40 \pm 3$  mm inside diameter at the top,  $90 \pm 3$  mm inside diameter at the bottom, and  $75 \pm 3$  mm in height) until the mold is full. Immediately after filling, remove the mold from the mixture by raising it carefully in a vertical direction and measure the flow diameter of the mixture. Then the water reducing efficiency of admixture is defined as:

$$\Omega = \frac{W_0 - W_{ad}}{W_0} \quad (16)$$

where  $\Omega$  is the water-reducing efficiency of a water reducer,  $W_{ad}$ , and  $W_0$  are the water content in cement paste with 0.5%-dosage water-reducing admixture and water content in the paste without water-reducing admixture for obtaining the same flow diameter of 180 mm as mentioned above, respectively.

Water-reducing efficiency ( $\Omega$ ) implies the ability of water reducer to disperse binder particles when considering concrete with the same w/b. However, it should be noted here that the model is not applicable for predicting concrete incorporating too much water-reducing admixture as segregation in concrete will occur. Fig. 9 represents the effect of a

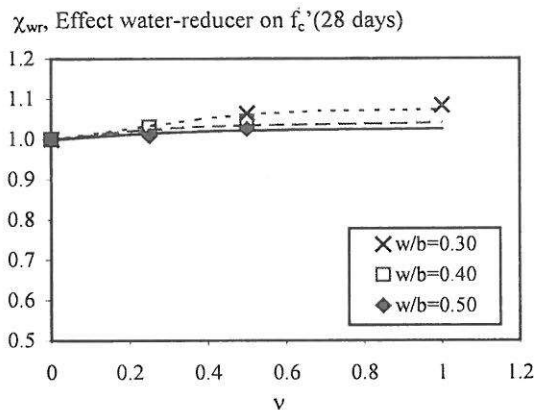


Fig. 8 Relationship between effective dosage of water reducer ( $v$ ) and its effect on 28-day compressive strength of concretes with various  $w/b$  and incorporating a naphthalene-based water reducer with water-reducing efficiency of 0.34

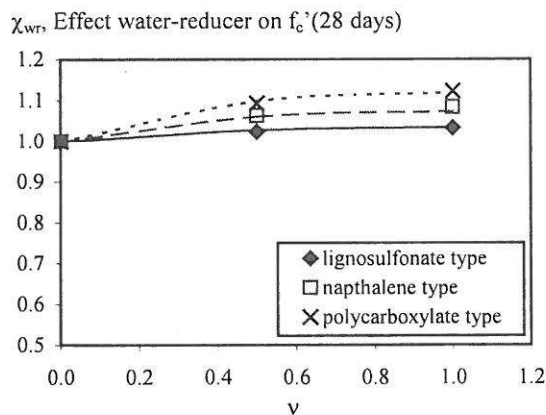


Fig. 9 Relationship between effective dosage of water reducer ( $v$ ) and its effect on 28-day compressive strength of concretes with  $w/b$  of 0.30, 0.40, and 0.50 and incorporating a naphthalene-based water reducer having water-reducing efficiency of 0.34

lignosulfonate-based, a naphthalene-based, and a polycarboxylate-based water-reducing admixture having the water-reducing efficiencies of 0.20, 0.34, and 0.50, respectively, on 28-day compressive strength of concretes with  $w/b$  of 0.30, 0.40, and 0.50. Eq. (17) shows the effect of water-reducing admixture on 28-day compressive strength.

$$\chi_{wr} = 1 + (3.52\Omega - 0.27) \cdot (0.005(w/b)^{-2.07}) \cdot \tan^{-1}(3.90v) \quad (17)$$

Finally, the equation for predicting 28-day compressive strength of concrete with fly ash was formulated as:

$$f'_c(28\text{days}) = [\alpha_1 \cdot \log(\text{CaO}_{\text{eff}}) + \lambda_F \cdot \alpha_2] \cdot \chi_\gamma \cdot \chi_{LOI} \cdot \chi_{wr} \quad (18)$$

in which

$$\alpha_1 = 8.50(w/b)^{-1.54} + 27.73 \quad (19)$$

and

$$\alpha_2 = -0.45(w/b)^{-3.80} + 67.97(w/b) - 114.3 \quad (20)$$

where  $f'_c(28\text{days})$  denotes the 28-day compressive strength in MPa,  $w/b$  is the water-to-binder ratio,  $\text{CaO}_{\text{eff}}$  is the effective calcium oxide content in concrete ( $\text{kg/m}^3$  of concrete) computed from Eq. (3),  $\lambda_F$  represents the effect of particle filling fly ash which can be determined by Eq. (6),  $\chi_\gamma$  is the effect of  $\gamma$  which can be determined by Eq. (10),  $\chi_{LOI}$  is the effect of LOI in fly ash which can be determined by Eq. (13), and  $\chi_{wr}$  is the effect of water-reducing admixture which can be determined by Eq. (17).

### 3. Verifications of the Model

The verification of the 28-day compressive strength model is presented for concretes with various fly ashes having different fineness and using various types of water-reducing admixture. The verification is expressed in the form of comparisons between experimental data and predicted results from the model as shown in Fig. 10. The data for verification were obtained from both the authors' and other researchers' studies [3, 4, 5, 6, 7]

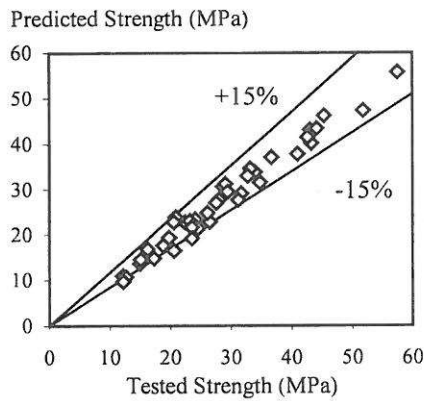


Fig.10 (a) Concretes containing original fly ash with fineness of  $2600 \text{ cm}^2/\text{g}$

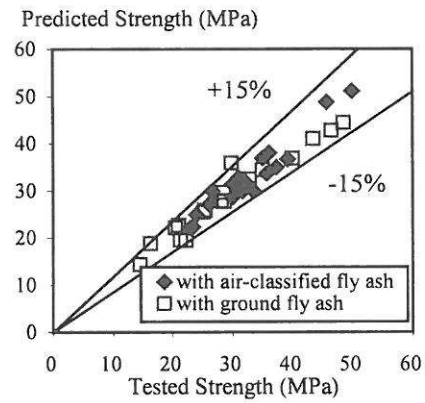


Fig.10 (d) Concretes containing air-classified and ground fly ash with various finenesses

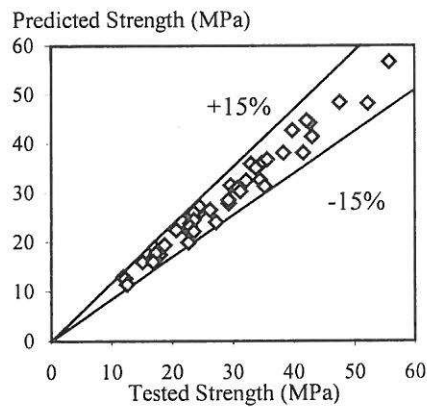


Fig.10 (b) Concretes containing original fly ash with fineness of  $4100 \text{ cm}^2/\text{g}$

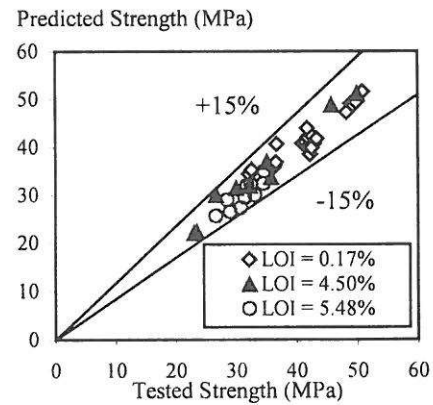


Fig.10 (e) Concretes containing fly ash with various LOI contents

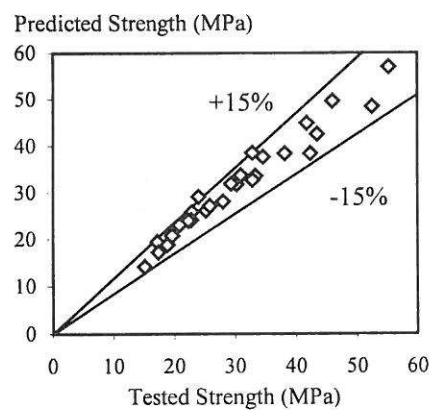


Fig.10 (c) Concretes containing original fly ash with fineness of  $4600 \text{ cm}^2/\text{g}$

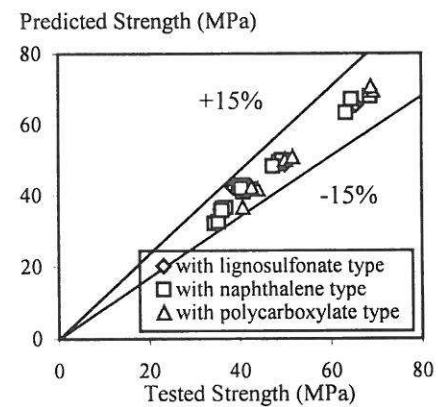


Fig.10 (f) Concretes containing various types of water-reducing admixture



#### 4. Conclusions

Based on the proposed compressive strength models and the verification, the following conclusions can be drawn.

1. The 28-day compressive strength of fly ash concrete has been verified to have a relationship with the logarithm of the effective calcium oxide content in concrete. This relationship varies with different water-to-binder ratio.
2. In this study, it is assumed that calcium oxide in fly ash has different effectiveness from cement on the 28-day compressive strength. The effectiveness of fly ash depends on its fineness and the percentage of calcium oxide in the fly ash.
3. The influences of relative volume of paste to volume of void in compacted aggregate phase, filling effect of fly ash, loss on ignition of fly ash, and water-reducing admixture on 28-day compressive strength were considered in the model.
4. The verification showed that the proposed compressive strength models could be used for predicting 28-day compressive strength of conventional fly ash concrete with a satisfactory accuracy.
5. It is remarked here that the model is applicable for concrete with conventional aggregate, compressive strength at 28 days smaller than 60 MPa, fly ash replacement ratio smaller than 0.70, and fly ash with LOI up to 10%.

#### 5 Acknowledgments

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