An inexpensive solution to limit state problems with applications in the Pipeline industry

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Abstract

This paper outlines a general approach to calculating approximate failure probabilities in systems using limit state analysis. The approach described here relies on a relatively inexpensive standard mathematical analysis software package, namely Mathcad, and does not require the use of specialized proprietary software. The margin between "safe" and "unsafe" or "failed" states is first defined by a limit state function. Then, solving what is essentially a constrained minimization problem, an estimate of failure probability is obtained. A practical example, which examines the effect on the failure probabilities of a gas pipeline system when up-rating to higher operating pressures, is presented in order to illustrate the scope of the method.

1. Introduction

The engineering codes and standards, which allow for the safe design and operation of pipelines, have evolved over many years. The safety factors incorporated into such regulations are based on judgement and experience and may result in a conservative design, which can reduce potential revenue from a pipeline. Recently, the benefits of a risk-based approach to pipeline design and operation have been recognised by a number of worldwide regulatory authorities [1,2].

Risk is usually expressed as a function of failure probability, or failure frequency, together with the consequences of failure. It should also be noted that "failure" does not necessarily imply a leak or rapture in a pipeline. Failure can also be interpreted as an undesirable event, such as metal loss reaching 80% wall thickness. By defining "failure" in this way, it is possible for pipeline operators to target expenditure on maintenance and rehabilitation in a proactive manner, thus avoiding the greater costs associated with remedial work following a leak or rupture.

2. Limit State Approach

Our approach to estimating failure probability is based on a limit state analysis. Using a relatively inexpensive mathematical/algebraic software package: Mathcad [3], a PC based limit state analysis can be readily formulated. This avoids the need for specialised and expensive proprietary software analysis tools.

The limit state approach is based on the definition of an appropriate state function, which represents the margin between the "safe" and the "failed" states of the system. The limit state, or "failure surface", represents the transition between the "safe" and "failed" states of the system. A well-established measure of system safety is given by the safety-index β [4], which represents the minimum distance from the origin to the "failure surface" in state space. The closest point on the "failure surface" to the origin represents the most probable failure point of the system. For the implementation of the limit state approach used here, the problem is formulated in a Mathcad worksheet as a constrained optimization problem.

In the context of the pipeline industry application, described in Section 3, the design pressure (P_D) is a function of design factor (f), nominal wall thickness (t), specified minimum yield strength (σ) and outside diameter of pipe (D); expressed by the deterministic equation:

$$P_D = 20.\ t.\ \sigma .f / D \qquad (1)$$

A probabilistic representation of the same equation takes the form:

$$P_D = 20.\ (F_t(x1).\ F_\sigma(x2).\ f)/ F_D(x3) \qquad (2)$$

where $F_t(xl)$, $F_\sigma(x2)$ and $F_D(x3)$ represent the statistically distributed variables associated with t, σ, and D respectively. xl, x2 and x3 represent equivalent standard Normal values for each of these distributed variables.

A state function representing the safety margin is defined as:

$$g(xl,x2,x3) = P_D(F_t(x1),\ F_\sigma(x2), F_D(x3), f) - MPOP \qquad (3)$$

where MPOP represents the maximum permissible operating pressure.

The limit state, or "failure surface", which represents the transition between the "safe" and "failed" states in "x1, x2, x3" space is given by:

$$g(xl,x2,x3) = 0. \qquad (4)$$

By minmizing $x1^2+x2^2+x3^2$ subject to the constraint that the solution lies on the failure surface represented by equation (3), an estimate of the safety-index β is obtained, where $\beta^2= x1^2+x2^2+x3^2$. An estimate of the approximate failure probability is then calculated from tabulated values of the standard Normal variable. Such a constrained optimization problem is easily formulated and solved in Mathcad. An overview of the approach is shown in Figure 1.

3. Up-rating Pipeline Pressures

As an example of how limit state analysis can be used in practice, the following example compares failure probabilities in a gas pipeline network comprising 5 high-pressure lines. Estimates of the likely failure probabilities if an operator wishes to operate his pipelines at higher pressures are included in this example. Operating at higher pressures, a process called up-rating, is a means of providing additional transmission capacity in a network with a consequent increase in revenue. Details of the 5 pipelines are given in Table 1. All are assumed to operate with a design factor f=0.72. Values of design pressure (P_D) have been calculated using equation (1).

For the purpose of this example, all pipelines are currently assumed to operate at MPOP = 75 bar with the operator wishing to up-rate the MPOP to a higher pressure-value.

The distributions described in Table 2 were then used as inputs to the state function for this pipeline. The particular distributions and ranges used in Table 2 were selected as typical values based on the experience of the current authors. The mean value of a parameter refers to the value shown in Table 1.

The effects on the safety index β, and on the estimated approximate failure probability for up-rating to higher-pressure loadings, are shown in Table 3 for the five pipelines. Network failure probability is the probability that one or more of the 5 pipelines will fail at its operating pressure. Network failure probabilities are also shown in Table 3.

The results demonstrate that should the operator wish to up-rate his pipeline to 85 bar, there would be no significant increase in risk. For the individual pipelines, the safety index β ranges from 5.96 up to 25.68, At 75 bar the network failure probability is calculated to be negligible ($<10^{-15}$). Up-rating the network to 85 bar will increase the calculated approximate failure probability to only 1.3×10^{-7} %. Whilst up-rating the network to 90 bar, increases the calculated failure probability to 0.32 %. For 100 bar, the network failure probability rises to 46%.

The type of analysis performed here would assist pipeline operators in identifying those sections of the network where additional expenditure may be needed in order to reduce the risk associated with the network to an acceptable level.

A more detailed comparison of the pipelines *A* to *E* is shown in Figure 2. The "Critical" line shown in Figure 3 represents a failure probability of 10^{-4} (β=2.7)

For up-rating to 85 bar, there is a negligible probability of failure in all the pipelines. However, up-rating the pipelines even further to 90 bar, increases the probability of failure in pipeline *B* to what might be considered an unacceptable level.

4. Further Applications in the Pipeline Industry

A more comprehensive analysis of pipeline risk analysis than that which is shown above, would require consideration of additional modes of failure, such as corrosion, fatigue, denting, gouging, buckling, loss of ground support, etc. For the limit state functions corresponding to these different failure modes, additional input parameters, some of them time dependent, would also be required for the analysis. For example, we have incorporated this limit state approach into a methodology for estimating current and future pipeline failure probabilities from corrosion defects reported by on-line inspection pigs, including a strategy for handling large numbers of defects. The results will be reported in a future publication.

5. Conclusion

The limit state approach to estimating failure probabilities presented in this paper has a number of attractive features. In particular, it can be implemented with ease in a relatively inexpensive PC software package such as Mathcad. The analyst also has at his disposal the full flexibility of such a general software package for the post-processing and graphing of results.

Estimating failure probability is an essential component of any quantified risk analysis. The outcome of such an analysis can impact not only on the safety of a system, but also on its economic operation, including maintenance expenditure. These features are particularly important in the operation of pipeline systems, as demonstrated in the example of up-rating pressures shown here.

6. References

[11 Anon. (1994) Draft Appendix C, "Oil and Gas Pipeline Systems", Canadian Standards Association, CSA Z662-94, 1994.

[2] Anon. (1996) "The Pipeline Safety Regulations 1996", SI1996 No.825, HMSO, UK, 1996.

[3]Mathcad.Website:http:Hwww.mathsoft.com/mathcad/.

[4) Ang, Alfredo H.S, Tang, Wilson H. (1984) "Probability Concepts in Engineering Planning and Design -Volume II Decision, Risk and . Reliability". John Wiley and Sons, Inc., 1984.

Table 1: Details of Pipe lines in Network

Pipe ID	Thickness (t mm)	Pipe Grade	Yield stress (σ N/mm^2)	Outside Diameter. (D mm)	Design Pressure (P_D bar)
A	8.7	X52	358	406.4	110.4
B	7.1	X46	317	323.9	100.1
C	17.5	X65	448	1067.0	105.8
D	15.9	X60	414	762.0	124.4
E	9.5	X52	358	457.0	107.2

Table 2: Statistical Distributions of Variables

Parameter	Distribution	Range
D	Uniform	(Mean -5%) to (Mean+ 10%)
t	Uniform	(Mean -5%) to (Mean+ 5%)
σ	Normal	Mean, Standard deviation = 4 N/mm^2

Table 3: Safety Indices and Approximate Failure probabilities for Up-rating

Pipe ID	Safety Index (β)					Pipeline Failure Proability (P_f)				
	MPOP =75	MPOP =85	MPOP =90	MPOP =100	MPOP =105	MPOP =75	MPOP =85	MPOP =90	MPOP =100	MPOP =105
A	22.65	13.88	9.61	2.49	1.14	≈ 0	≈ 0	≈ 0	0.0063	0.13
B	14.14	5.96	2.73	0.36	-0.31	≈ 0	1.3E-9	0.0032	0.36	0.62
C	24.62	13.19	7.70	1.29	0.46	≈ 0	≈ 0	≈ 0	0.099	0.32
D	34.78	25.68	21.15	12.20	7.90	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0
E	20.67	11.70	7.39	1.56	0.65	≈ 0	≈ 0	≈ 0	0.060	0.26
Network Failure Probability =						≈ 0	1.3E-9	0.0032	0.46	0.83

Figure 1: Overview of Limit State Approach.

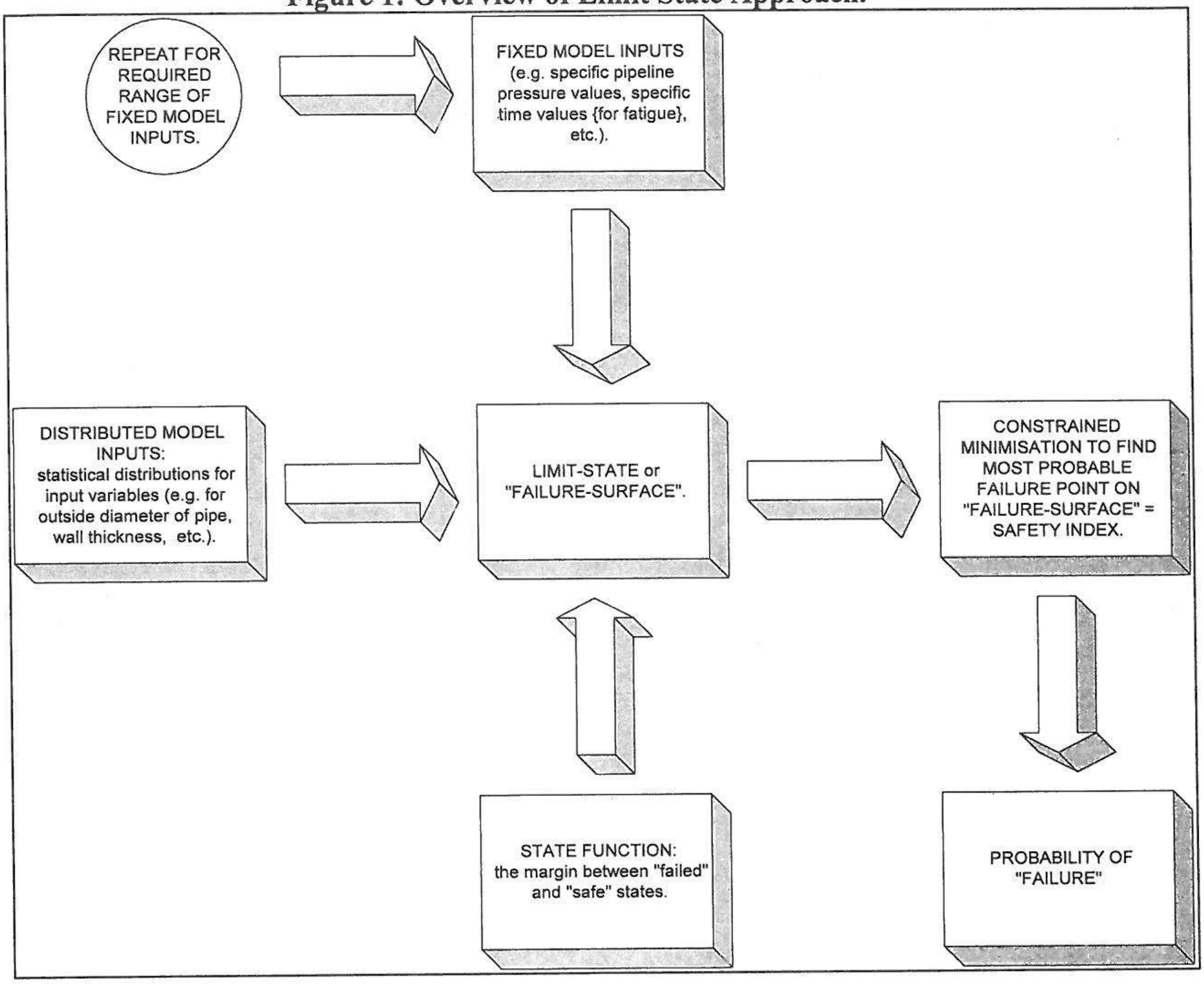

Figure 2: Detailed Comparison of Pipelines.