Cracking Localization Analysis Using a Specially Treated Smeared Crack Finite Element Model with Energy Consideration

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Abstract

In this paper, an analysis method to analyze problems involving cracking localization is proposed. The proposed analysis method employed the well-known smeared crack model. Nevertheless, in the finite element formulation, a mixed formulation that discretizes not only displacements but also crack strains is used. This is to allow stability consideration of crack patterns to be done efficiently. Stability analysis of crack patterns is done by performing eigenvalue analysis of Hessian matrices obtained from the mixed finite element formulation. At each bifurcation point identified by the stability analysis, the actual equilibrium path is incrementally traced by searching for a crack pattern with the minimum total potential energy increment. Search algorithms employed include an exhaustive search algorithm and a genetic algorithm. Finally, the proposed analysis method is used to analyze the four-point bending problem of plain concrete and the results are discussed.

1. Introduction

Tensile failure of quasi-brittle materials such as concrete is commonly known to start from formation of cracks, and propagation of the newly formed cracks or existing defects. After that, these cracks will localize into one or a few cracks. This will subsequently lead to the final failure. In order to capture the ultimate capacity of such materials in a structure, consideration of cracking localization cannot generally be neglected. However, the consideration of cracking localization needs a very expensive computation because solution methods for solving localization problems involve checking stability and bifurcation of many different equilibrium paths. Consequently, many researchers avoid the consideration of cracking localization by either allowing many cracks to grow without the consideration of localization [1-3] or by assuming positions for localized cracks [4]. The first approach is not realistic and can generally lead to inaccurate results. Having many cracks without localization allows an incorrect amount of energy to dissipate from the domain. Thus, obtained results will also be inaccurate. However, in some cases where the gradient of stress is very high, it is possible that major cracks will finally prevail and other cracks will undergo elastic unloading even when cracking localization is not considered. The second approach, which assumes positions of localized cracks prior to analysis, may yield reasonable results in some cases. These include cases where assumed positions of localized cracks are reasonably or undoubtedly correct and cases where solutions are not sensitive to locations of localized cracks. Nevertheless, the approach is not appropriate for general cases since locations of localized cracks may not be easily predicted or solutions may be sensitive to locations of cracks.

Consideration of stability and bifurcation of equilibrium states is one of the major tasks to be done in the analysis of cracking localization. Many researchers have considered stability and bifurcation of equilibrium states by investigating definiteness of stiffness matrices [5-6]. When a stiffness matrix is positive-definite, an equilibrium state is considered stable. The same theory can be applied to the

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analysis of cracking localization. However, cracking is an irreversible process. In this case, stability and bifurcation of equilibrium states can be determined by investigating definiteness of stiffness matrices (Hessian matrices) constructed with respect to irreversible parameters [7]. These irreversible parameters can be crack opening displacements in the discrete crack approach or crack strains in the smeared crack approach. Investigating definiteness of Hessian matrices will provide information on stability of equilibrium paths. Consequently, bifurcation points can be located. Nevertheless, tracing the actual equilibrium path needs some more effort.

Employing Gibbs' statement of the second law of thermodynamics, Nemat-Nasser [8] pointed out that the equilibrium path that makes the total potential energy an absolute minimum would also render the elastic energy an absolute minimum. In addition, this path will also be the actual equilibrium path [9]. Employing the same concept, Brocca [10] used crack opening displacements in the discrete crack finite element analysis as irreversible parameters in the analysis of cracking localization. In his work, Hessian matrices constructed with respect to irreversible crack opening displacements are used to investigate stability and bifurcation of crack patterns. In addition, the equilibrium path is also traced by using the Simplex method to find the path with the minimum total potential energy. From his work, it is clear that Hessian matrices constructed with respect to irreversible parameters can easily be obtained when the discrete crack approach is employed because irreversible parameters are discrete. Nevertheless, the discrete crack approach is not suitable for problems with many cracks in the domain. Usually, in the cracking localization analysis, there will be many cracks occurring in the domain. As the number of cracks increases, the mesh topology may have to be changed to cope with the new crack patterns and this leads to more degrees of freedom. On the other hand, the smeared crack approach, which is more suitable for problems with many cracks, does not provide any discrete irreversible parameters for construction of Hessian matrices. Another disadvantage of the smeared crack approach is that, with this approach, it is necessary to define the crack-band width or the crack characteristic length. For fairly regular meshes, the characteristic length is frequently determined in an intuitive way which is difficult to generalize in a formal manner for irregular meshes and arbitrary crack directions. However, for two-dimensional domains, this problem can be overcome. Oliver [11] proposed a general approach for calculation of the characteristic length. In his study, a crack is modeled as a limiting case of two singular lines that coincide with the boundary of elements covering the crack path. The expression for the characteristic length is obtained by analyzing the energy dissipated from the band bounded by these two singular lines.

To allow the consideration of cracking localization in the smeared crack model, Nanakorn and Soparat [12] proposed an analysis method that uses the smeared crack finite element model with a mixed formulation. In their work, the discretization is performed not only on the displacement field but also on the crack strain field. The newly introduced discrete nodal crack strain variables serve as the discrete irreversible variables needed for the localization analysis. However, their work is limited to stability analysis of crack patterns, and there is no attempt to trace the complete equilibrium path.

In this study, an analysis method for cracking localization is proposed. In the proposed method, stability of crack patterns is investigated by employing the analysis method proposed by Nanakorn and Soparat [12]. When the current crack pattern becomes unstable, the stable crack pattern with the minimum total potential energy is searched for and selected as the solution path [8]. In the search for the stable crack pattern with the minimum total potential energy, an exhaustive search algorithm and a genetic algorithm are used. The proposed analysis method is used to solve the cracking localization problem of a four-point bending beam of plain concrete. Finally, the obtained results are discussed.

2. Smeared Crack Model with a Mixed Finite Element Formulation

In the smeared crack model, the total strain increment $\Delta\varepsilon$ is decomposed into the strain increment of the intact elastic solid $\Delta\varepsilon^{o}$ and the strain increment of the cracked solid $\Delta\varepsilon^{cr}$, i.e.,

$$\Delta\varepsilon = \Delta\varepsilon^{o} + \Delta\varepsilon^{cr}. \tag{1}$$

The relationship between the global crack strain increment $\Delta\varepsilon^{cr}$ and the local crack strain increment $\Delta\hat{\varepsilon}^{cr}$ is expressed as

$$\Delta\varepsilon^{cr} = \mathbf{T}\Delta\hat{\varepsilon}^{cr} \tag{2}$$

where $\mathbf{T}$ is the transformation matrix, which can be written as a function of the angle between the vector normal to the crack surfaces and the global x-axis.

By following Nanakorn and Soparat [12], the total potential energy increment of a cracked domain V is expressed as

$$\begin{aligned}\Delta\Pi &= \Delta\Pi^{M} + \Delta\Pi^{D} \\ &= \left[\begin{array}{l}\dfrac{1}{2}\displaystyle\int_V \Delta\varepsilon^{oT}\mathbf{D}^{o}\Delta\varepsilon^{o}\,dV \\ -\displaystyle\int_V \Delta\mathbf{u}^{T}\Delta\mathbf{f}dV - \int_S \Delta\mathbf{u}^{T}\Delta\mathbf{t}dS\end{array}\right] \\ &\quad + \left[\dfrac{1}{2}\displaystyle\int_V \Delta\hat{\varepsilon}^{crT}\hat{\mathbf{D}}^{cr}\Delta\hat{\varepsilon}^{cr}dV\right].\end{aligned} \tag{3}$$

The total potential energy increment $\Delta\Pi$ shown above consists of two parts that are the mechanical potential energy increment $\Delta\Pi^{M}$ and the dissipated energy increment $\Delta\Pi^{D}$. Here, $\Delta\mathbf{u}$ denotes the displacement increment vector. In addition, $\mathbf{D}^{o}$ and $\hat{\mathbf{D}}^{cr}$ denote the constitutive matrices for the intact elastic solid and the cracked solid, respectively. Finally, $\Delta\mathbf{f}$ and $\Delta\mathbf{t}$ represent the body force increment vector and the surface traction increment

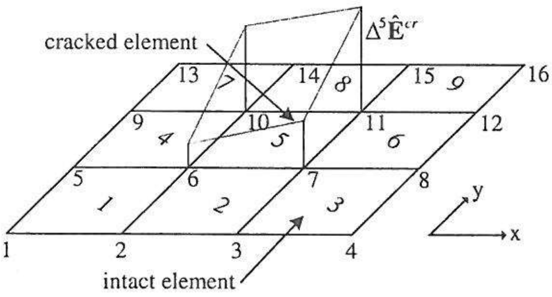


Fig. 1 A problem with one cracked element surrounded by intact elastic elements [12]

vector, respectively.

Discretizing both displacement and local crack strain increments, we have for the i^{th} element in the finite element analysis [12]

$$\Delta\mathbf{u} = \mathbf{N}\Delta^{i}\mathbf{U}, \tag{4a}$$

$$\Delta\hat{\varepsilon}^{cr} = \mathbf{N}^{cr}\Delta^{i}\hat{\mathbf{E}}^{cr} \tag{4b}$$

where $\mathbf{N}$ and $\mathbf{N}^{cr}$ represent the shape function matrices for the displacement increment and the local crack strain increment, respectively. In addition, $\Delta^{i}\mathbf{U}$ and $\Delta^{i}\hat{\mathbf{E}}^{cr}$ represent the nodal displacement increment and the nodal local crack strain increment, respectively. Note that the local crack strain increments are not continuous across elements and the nodal local crack strain increments of the same node for different elements can be different. One example is a problem with one cracked element surrounded by uncracked elements (see Fig. 1). In the cracked element including its boundary, non-zero crack strain increments can be expected. However, in the surrounding uncracked elements, the crack strain increments are expected to be zero because there is no crack in those elements. On the contrary, the total displacement increments must be continuous across all the elements.

From (2) and (4), the total strain increment and the global crack strain increment are expressed as

$$\Delta\varepsilon = \mathbf{B}\Delta^{i}\mathbf{U}, \tag{5a}$$

$$\Delta\varepsilon^{cr} = \mathbf{T}\mathbf{N}^{cr}\Delta^{i}\hat{\mathbf{E}}^{cr}. \tag{5b}$$

From (1), (3) and (5), the total potential energy increment can be expressed as

$$\begin{aligned}\Delta\Pi = &\frac{1}{2}\int_V \Delta^i\mathbf{U}^T\mathbf{B}^T\mathbf{D}^0\mathbf{B}\Delta^i\mathbf{U}dV \\ &-\frac{1}{2}\int_V \Delta^i\mathbf{U}^T\mathbf{B}^T\mathbf{D}^0\mathbf{T}\mathbf{N}^{cr}\Delta^i\hat{\mathbf{E}}^{cr}dV \\ &-\frac{1}{2}\int_V \Delta^i\hat{\mathbf{E}}^{cr^T}\mathbf{N}^{cr^T}\mathbf{T}^T\mathbf{D}^0\mathbf{B}\Delta^i\mathbf{U}dV \\ &+\frac{1}{2}\int_V \Delta^i\hat{\mathbf{E}}^{cr^T}\mathbf{N}^{cr^T}\mathbf{T}^T\mathbf{D}^0\mathbf{T}\mathbf{N}^{cr}\Delta^i\hat{\mathbf{E}}^{cr}dV \\ &+\frac{1}{2}\int_V \Delta^i\hat{\mathbf{E}}^{cr^T}\mathbf{N}^{cr^T}\mathbf{D}^{cr}\mathbf{N}^{cr}\Delta^i\hat{\mathbf{E}}^{cr}dV \\ &-\int_V \Delta^i\mathbf{U}^T\mathbf{N}^T\Delta\mathbf{f}dV - \int_S \Delta^i\mathbf{U}^T\mathbf{N}^T\Delta\mathbf{t}dS.\end{aligned} \quad (6)$$

Applying the stationary condition $\delta(\Delta\Pi)=0$, we obtain the element stiffness equation for the i^{th} element as

$$\begin{bmatrix}\mathbf{k}_{11} & \mathbf{k}_{12}\\ \mathbf{k}_{21} & \mathbf{k}_{22}\end{bmatrix}\begin{Bmatrix}\Delta^i\mathbf{U}\\ \Delta^i\hat{\mathbf{E}}^{cr}\end{Bmatrix}=\begin{Bmatrix}\Delta\mathbf{r}\\ 0\end{Bmatrix} \quad (7)$$

where

$$\begin{aligned}\mathbf{k}_{11} &= \int_V \mathbf{B}^T\mathbf{D}^o\mathbf{B}dV, \\ \mathbf{k}_{12} &= -\int_V \mathbf{B}^T\mathbf{D}^o\mathbf{T}\mathbf{N}^{cr}dV, \\ \mathbf{k}_{21} &= -\int_V \mathbf{N}^{cr^T}\mathbf{T}^T\mathbf{D}^o\mathbf{B}dV, \\ \mathbf{k}_{22} &= \int_V \mathbf{N}^{cr^T}\left(\hat{\mathbf{D}}^{cr}+\mathbf{T}^T\mathbf{D}^o\mathbf{T}\right)\mathbf{N}^{cr}dV, \\ \Delta\mathbf{r} &= \int_V \mathbf{N}^T\Delta\mathbf{f}dV + \int_S \mathbf{N}^T\Delta\mathbf{t}dS.\end{aligned}$$

After assembling all element stiffness equations and applying prescribed displacements and forces, the system stiffness equation is arranged as

$$\begin{bmatrix}\mathbf{K}_{11} & \mathbf{K}_{12}\\ \mathbf{K}_{21} & \mathbf{K}_{22}\end{bmatrix}\begin{Bmatrix}\Delta\mathbf{U}\\ \Delta\hat{\mathbf{E}}^{cr}\end{Bmatrix}=\begin{Bmatrix}\Delta\mathbf{R}_1\\ \Delta\mathbf{R}_2\end{Bmatrix} \quad (8)$$

where $\Delta\mathbf{U}$ and $\Delta\hat{\mathbf{E}}^{cr}$ are the nodal displacement increment and the nodal local crack strain increment of the system, respectively.

The static condensation is then used to remove the nodal displacement increment from the obtained system matrix equation. Consequently, the equation can be written in the following form, i.e.,

$$\mathbf{K}^{cr}\Delta\hat{\mathbf{E}}^{cr} = \Delta\mathbf{R}^{cr} \quad (9)$$

where $\mathbf{K}^{cr}$ and $\Delta\mathbf{R}^{cr}$ are defined as

$$\mathbf{K}^{cr} = \mathbf{K}_{22} - \mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12}, \quad (10a)$$

$$\Delta\mathbf{R}^{cr} = \Delta\mathbf{R}_2 - \mathbf{K}_{21}\mathbf{K}_{11}^{-1}\Delta\mathbf{R}_1. \quad (10b)$$

In the consideration of stability of crack patterns, the eigenvalue analysis of $\mathbf{K}^{cr}$ is performed. If all the eigenvalues are positive, then it means that the stationary solution in (9) is stable with respect to the current crack pattern. Otherwise, the stationary solution is unstable and bifurcation occurs. Note that this scheme is only used for stability analysis of crack patterns, not for obtaining the displacement solution. The displacement solution will be obtained from the original smeared crack model where the basic unknowns are the nodal displacement increments.

3. Equilibrium Path with The Minimum Total Potential Energy

When the equilibrium path reaches a bifurcation point, a fan of many possible equilibrium paths emanates from the bifurcation point. In fact, if instability occurs in the real system, the actual equilibrium path is the path that contains the minimum total potential energy [9] or the minimum elastic strain energy [8]. These two conditions are actually the same [8], if one defines the total potential energy in the usual way. In this study, the minimum total potential energy criterion is employed. However, since the analysis is performed incrementally, and the total potential energy is written in the incremental form [see (3)], the stable path with the minimum total potential energy increment is

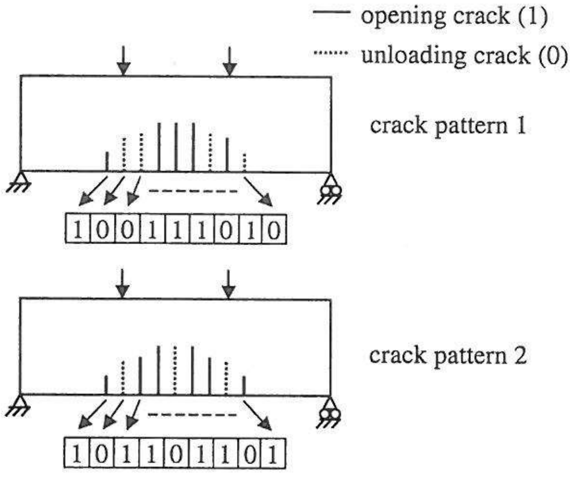


Fig. 2 Examples of coding of crack patterns

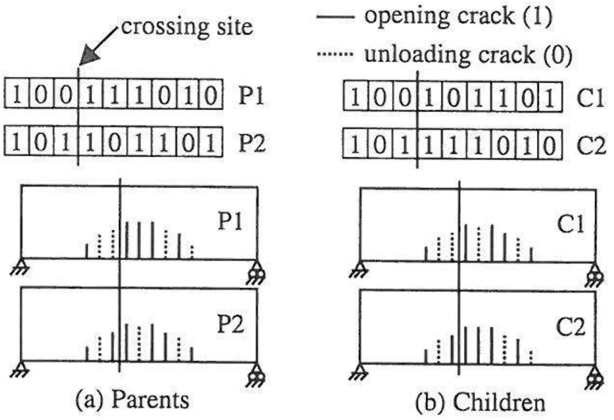


Fig. 3 One-point crossover

the desired solution path.

In order to obtain the solution path with the minimum total potential energy increment, energy increments of all possible equilibrium paths, which depend on their crack patterns, can be compared. This approach of comparing all possible solutions is essentially an exhaustive search. The algorithm for this search approach is simple and straightforward. Nevertheless, it is obvious that the technique is expensive and suitable only for small problems where the complete search is still possible. In the case of larger problems where many cracks occur in the domain and, as a result, many crack patterns are possible, the exhaustive search may not be practical and it is advisable to employ an appropriate optimization technique to find the minimum energy path. In this study, a genetic algorithm (GA) [13] is used for this purpose because this optimization technique is suitable for problems with discrete variables. Variables in the minimization problem of the total potential energy increment are discrete statuses of cracks that can be either opening or unloading. Since GAs do not require the evaluation of the gradient of the function being minimized or maximized, the evaluation of the total potential energy increment is enough for the minimization process.

In this study, the simple GA is employed. It is composed of three different operators, i.e., reproduction, crossover and mutation operators. These three operators are based on the same basic elements in the real natural genetics. The details of this technique can be found in the literature [13-14].

In general, GAs do not directly work with the parameters themselves. The algorithms start with coding of the parameter set. For coding, binary strings are most popular and convenient. Each point in a search space, often called "individual" in the GA terminology, is represented by a single string of number 0's and 1's. The optimization problem of this study is to minimize the total potential energy increment. The total potential energy increment to be minimized is a function of crack patterns. Therefore, each crack pattern will be coded as a binary string. The idea of the coding is to have each bit in a binary string represent the status of one particular crack. If the value of the bit is one (1), it indicates that its corresponding crack is opening. If the value of the bit is zero (0), the corresponding crack is unloading. Fig. 2 shows examples of the coding of two different crack patterns. The number of bits used in the string is equal to the number of the existing crack paths.

In GAs, the reproduction operator defines a process in which individuals are selected for mating based on their fitness values relative to that of the population. Fitness is defined as a figure of merit. Individuals with higher fitness values have higher probabilities of being selected for mating and subsequent genetic

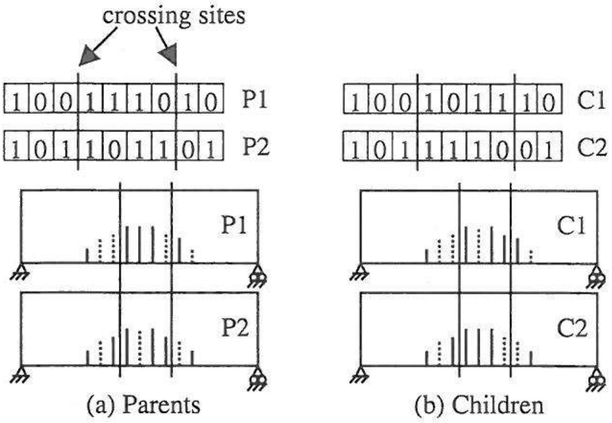


Fig. 4 Two-point crossover

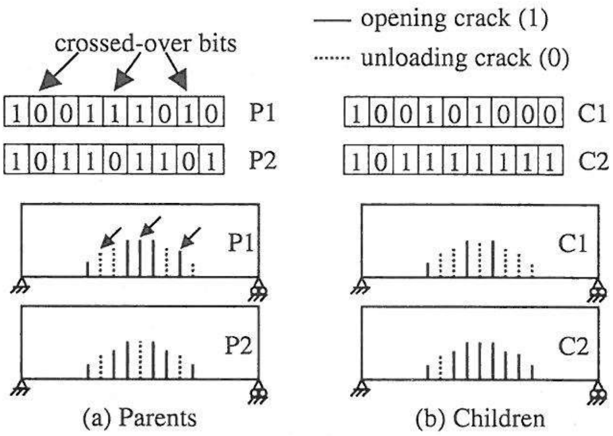


Fig. 5 Uniform crossover

actions. Consequently, highly fit individuals live and reproduce, and less fit individuals die. In this study, a crack pattern that results in a smaller total potential energy increment will be given a higher fitness value.

In the crossover operator, new strings are created by exchanging information among strings. Many crossover operators exist in the literature [13]. Generally, two strings are selected at random as a crossover pair and some portions of the two strings are exchanged. The two strings participating in the crossover are known as parent strings and the resulting strings are known as children strings. In this study, three types of crossover operator are employed, i.e., one-point, two-point and uniform crossover operators. Fig. 3 shows an example of the one-point crossover. In this study, the one-point crossover is performed by randomly selecting a crossing site along the parent strings and by exchanging all bits on the right hand side of the selected crossing site. In the case of the two-point crossover, two crossing sites are randomly selected and all the bits between the two crossing sites of the two parent strings are exchanged as shown in Fig. 4. For the uniform crossover, the number of bits to be crossed over and their positions are randomly determined. Fig. 5 shows an example of this type of crossover in this study.

It is clear that the crossover operator may yield better or worse children strings. To be able to adjust the degree of the uncertainty of the crossover phase, it is not necessary to use all individuals in the mating pool in the operator. This is done by adjusting the probability that a crossover is performed (crossover probability).

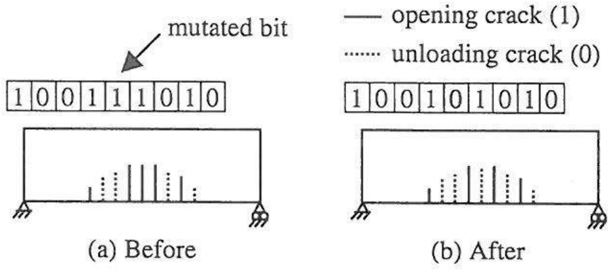
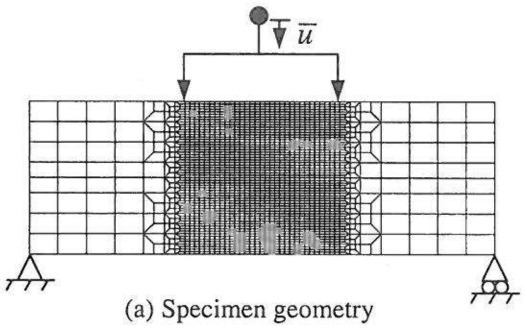


Fig. 6 Mutation

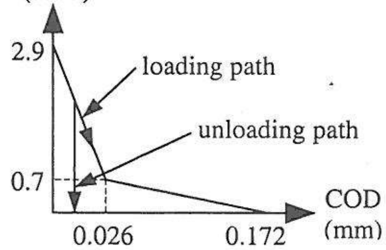
The last genetic algorithm operator is the mutation operator. Fig. 6 shows an example of the mutation operator employed in this study. The mutation operator changes 1 to 0 and vice versa at a randomly chosen bit. The operator is used sparingly with a small probability (mutation probability).

4. Analysis Procedure

In the analysis, the specimen under consideration is analyzed by using the conventional smeared crack model. Cracks are initiated when the maximum tensile stress reaches the tensile strength of the material. After that, the cracks follow the tension-softening curve, which is treated as one of the material properties. The tension-softening curve is the relationship between the tensile



(a) Specimen geometry



(b) Tension-softening curve

Fig. 7 Four-point bending problem

stress transferred across the crack surfaces and the crack opening displacement. Note that, in this study, shear retention of cracks is assumed negligible. As mentioned earlier, the analysis is done incrementally. In each step, the stability of the obtained crack pattern will be investigated by performing eigenvalue analysis of the matrix $\mathbf{K}^{cr}$ obtained from the mixed smeared crack finite element formulation [12]. If the crack pattern is found to be stable, the analysis is continued to the next step. However, if the crack pattern is unstable, the search for the crack pattern with the minimum total potential energy increment must be performed. Here, if the number of possible crack patterns is not very large, an exhaustive search can be employed; otherwise, a GA will be used, instead. It must be noted that, if a GA or another optimization technique is used, the obtained crack pattern may have a near-minimum total potential energy increment, not the true minimum one for the finite element discretization being currently used. In order to compare total potential energy increments of different crack patterns, the energy for cases with different crack patterns must be evaluated under the same controlled parameter. In this study, the controlled displacement is used. After the crack pattern with the minimum or near-minimum total potential energy increment is obtained, the analysis is carried on to the next step. The same process is then repeated and the actual equilibrium path can be traced.

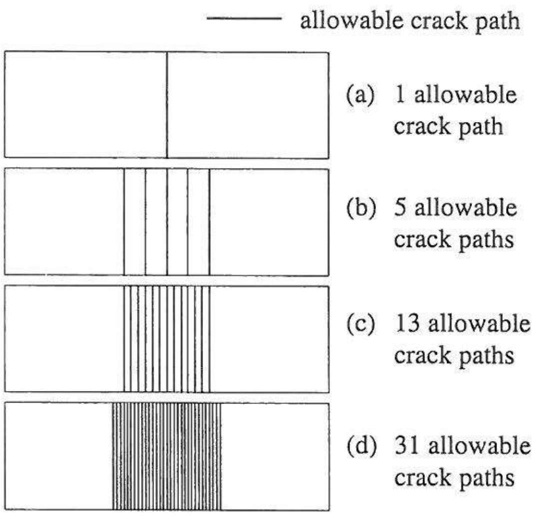
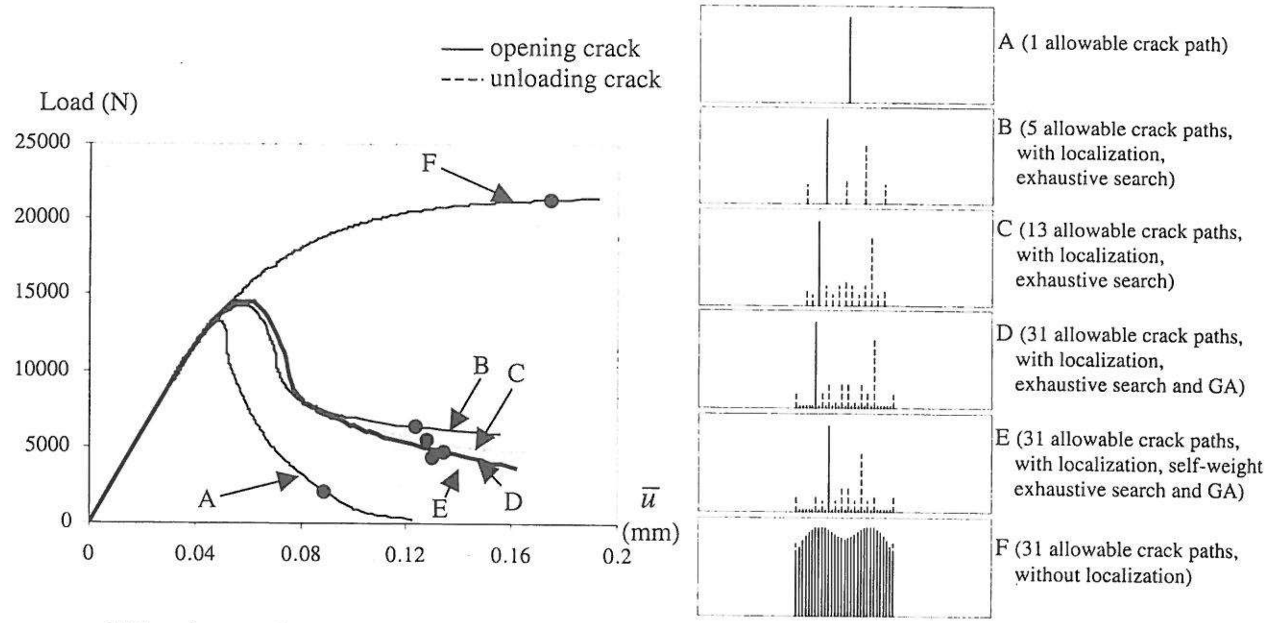


Fig. 8 Specimen with different numbers of allowable crack paths

5. Results

Here, the classical four-point bending beam test of plain concrete shown in Fig. 7a is investigated. Specimen's dimension is 300×100×100 mm. Controlled displacements are applied at the top of the beam, 100 mm from both ends. Young's modulus and Poison's ratio used are 27.5 GPa and 0.2, respectively. Unit weight of the material is 2,300 kg/m^3. The tension-softening curve used is shown in Fig. 7b. In the analysis, four-noded quadrilateral elements are employed. The finite element mesh consists of 2,232 elements and 2,288 nodes (see Fig. 7a).

For this problem, it can be reasonably assumed that all crack paths are straight. To simplify the problem, cracks will be allowed to occur only on the pre-specified paths. The problem is solved both with and without the specimen's self-weight. When the self-weight is neglected, the problem is solved with various numbers of allowable crack paths as shown in Fig. 8, and, in all of these cases with different allowable crack paths, the equilibrium path



(a) Load-controlled displacement responses (b) Crack patterns

Fig. 9 Load-controlled displacement responses and crack patterns

with the minimum total potential energy increment is traced by employing an exhaustive search. In addition, only for the case with 31 allowable crack paths, a GA is also employed for the search. When the self-weight is considered, the analysis is done only for the case with 31 allowable crack paths, and the equilibrium path with the minimum total potential energy increment is traced by employing both exhaustive search and GA. GA parameters used in the analysis are shown in Table 1.

Fig. 9a shows load-controlled displacement responses for all of the calculations mentioned above. Moreover, it also includes the case with 31 allowable crack paths when the cracking localization is not considered. This additional case is performed without the self-weight and it will allow the importance of the localization analysis to be observed. Fig. 9b shows crack patterns obtained from these different cases at the loading points indicated by black circular

Table 1 GA parameters

Population size	40
Number of generations	40
Crossover probability	0.80
Mutation probability	0.05

markers on every response curve. At these loading points, the main cracks in all cases reach the length of 90 percent of the beam depth. For the case with 31 allowable crack paths with the localization consideration (the cases D and E), it can be seen that the results obtained from the exhaustive search and the GA are exactly the same. Therefore, it is shown that GAs can be used instead of the exhaustive search. It must be noted that the time used by the exhaustive search is very much longer than that used by the GA. For the cases B, C, and D where no self-weight is assumed, it can be seen that the obtained results, both crack patterns and response curves, are not much different. Therefore, for this problem, having only five allowable crack paths that are distributed properly is sufficient for obtaining the converged solution. Since it can be observed from the crack patterns of the cases B, C, and D that there are actually two long cracks in the beam, it may be understood that the response is actually governed by two main localized cracks which are not localized into one crack until at a much later loading stage. Also from the response curves, it is seen that the results of the case A, which assumes one localized crack at the center of the span, and the case F, which does not consider the

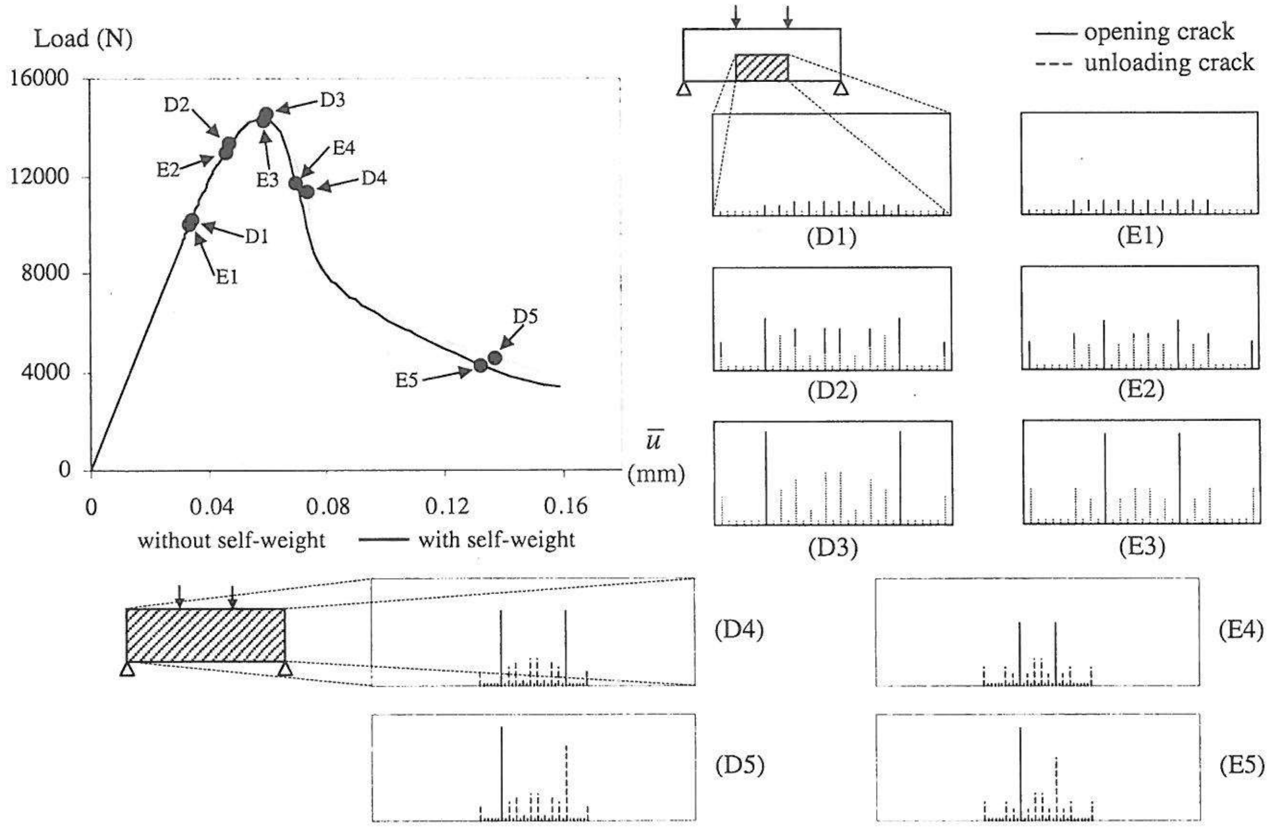


Fig. 10 Crack patterns of specimen with 31 allowable crack paths

localization, are very much different from those of the cases B, C, and D which properly consider the localization. Finally, from a comparison of the results of the cases D (without self-weight) and E (with self-weight), it can be seen that the load-displacement responses of both cases are very similar. Therefore, for this particular problem, neglecting the self-weight does not have a significant effect. Nevertheless, it can also be observed from the obtained crack patterns that the two main cracks are closer to each other when the self-weight is considered. This is expected since the self-weight makes the stress higher at locations closer to the center of the span. Fig. 10 shows the crack patterns of the cases D and E at different loading stages.

6. Conclusions

In this study, an analysis method for cracking localization in quasi-brittle materials is presented. The analysis method employs the smeared crack finite elements with a mixed formulation for the stability investigation of crack patterns. In the mixed formulation, the discretization is performed on not only the displacement field but also the crack strain field. The discretized crack strains will allow the stability analysis of crack patterns to be done more easily. At bifurcation points, the actual equilibrium path is traced incrementally by finding the path with the minimum total potential energy increment. The search for the minimum total potential energy increment is done by employing both the exhaustive and GA search algorithms, depending on the size of the problem being solved. It is found in this study that GAs can be efficiently used for this search. The results obtained from the four-point bending problem of plain concrete clearly show that the true localized solutions are very much different from the solution obtained by assuming one localized crack at the center of the span. Furthermore, the true localized solutions are also very much different from the solution obtained without the localization consideration. It is also found that there are two major localized cracks that are not localized into one crack until at a much later loading stage. The behavior of the beam is therefore governed by these two cracks. This clearly illustrates that assuming only one localized crack from the beginning may lead to erroneous results. Finally, it is found that, for the four-point bending test of plain concrete, neglecting the self-weight does not have significant effect on the obtained results. With self-weight or without self-weight, there are two main localized cracks. Although these two cracks are slightly closer when the self-weight is considered, the difference between the obtained responses from both cases are negligible.

7. Acknowledgements

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