Wind-Induced Vibration Control of High-Rise Building: Part I by Tuned Mass and Tuned Liquid Dampers

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Abstract

This study presents a design technique for various passive dampers for vibration control of high-rise buildings under wind loads. The study includes tuned mass damper (TMD), multiple tuned mass damper (MTMD), tuned liquid damper (TLD) and multiple tuned liquid damper (MTLD). Firstly, the design formulas of optimal equivalent damping ratio of TMD, MTMD, TLD, and MTLD are established. Then, the wind-induced displacement and acceleration responses and equivalent static wind loads are computed by using National Building Code of Canada 1995 with the optimal passive damper characteristics obtained in this study. Finally, three examples of high- and very high-rise buildings, namely, 183 m, 317.5 m, and 400 m high, are studied to illustrate the effectiveness of the various passive damper design. The results show that the wind-induced displacement and acceleration responses and equivalent static wind loads can be significantly reduced by applying the passive dampers. The results also indicate that, for the same mass ratio, the effectiveness of dynamic dampers can be ranked as: MTMD > TMD > MTLD > TLD.

1. Introduction

The response of high-rise building to dynamic wind loads in along-and across-wind directions has been an increasingly interesting subject of study for many decades. For modern high-rise buildings, which are very low in damping and light in weight, the natural frequencies of the occurrence of powerful gust and large resonant motions induced by wind must be taken into consideration in design. The resonant amplification of civil-engineering structure response to force induced by atmospheric turbulent wind flow near the ground was proposed by spectrum approach by Davenport [1], among others. Other detailed developments for estimating along-wind response are summarized in [2]. Based on the Davenport approach, the wind loads on building and responses have been further developed by National Building Code of Canada 1995 (NBC) [3]. The NBC considers: (1) the more realistic wind spectrum than the white noise one, (2) the gust effect factor, (3) the exposure factor, (4) the external pressure coefficient, and (5) the first mode response of structures with linear mode shape.

For increasing the levels of structural safety and occupant comfort, it is necessary to reduce the level of wind-induced responses in high-rise building by using aerodynamic and/or structural control means. There are many methods to control the vibration of high-rise building under wind loads, such as the base isolation, the viscous elastic damper, the added damping and stiffness device, the structural bracing and tendon system. All of these techniques have some of their own restrictions and disadvantages; therefore it is somewhat difficult to convince people to accept them for practical application. There exist other ways to control the vibration by using the tuned mass damper (TMD) [4] and the tuned liquid damper (TLD). However, there are some disadvantages using the TMD. The sensitivity of the effectiveness of the TMD to a fluctuation in the natural frequency of the structure and/or that in the damping ratio of the TMD is one of the disadvantages. The effectiveness of a TMD is decreased significantly by the mistuning or the off-optimum damping of the TMD. In order to improve the robustness of TMD, the concept of multiple tuned mass damper (MTMD) together with an optimization procedure was investigated by Yamaguchi and Harnpornchai [5], among others.

Similar in concept to TMD, the tuned liquid damper (TLD) imparts direct damping to the system and thus improves structural performance. A TLD absorbs structural energy by means of viscous actions of fluid and wave breaking. For small amplitude oscillations, the added damping is highly dependent on the ratio of structure to sloshing frequencies, with maximum at the ratio of about one. For larger amplitudes, the additional damping in the system is reduced and is almost constant for any frequency ratio. The performance of TLD was investigated by Fujino et al. [6], among others. These TLDs, have equal water depths. Recently, similar to MTMD, the distribution of natural frequency tanks, namely, multiple tuned liquid dampers (MTLD) was studied by Fujino et al. [7], among others.

This study presents a design technique for various passive dampers for vibration control of high-rise buildings under wind loads. Then, the wind-induced displacement and acceleration responses and the equivalent wind loads in high-rise building are computed by using the National Building Code of Canada 1995 [3] with the optimal passive damper characteristics obtained in this study. The details of wind-induced responses and wind loads are given by Boonyapinyo and Kawakun [8]. Finally, three examples of high- and very high-rise buildings, namely, 183 m, 317.5 m, and 400 m high, are studied to illustrate the various passive damper designs.

2. Design of Tuned Mass Damper (TMD)

2.1 Mathematical Model and Responses

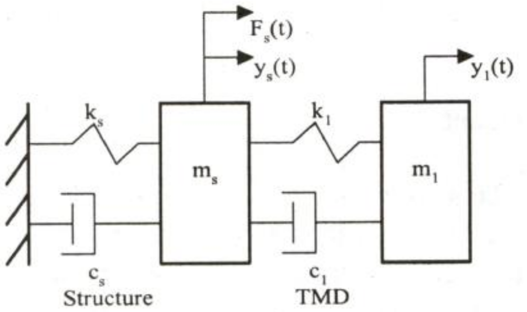

Fig. 1 Mathematical model of a first mode structure attached with TMD.

The TMD consists of a relatively small vibration system (mass, spring, and dashpot) as shown in Fig. 1, attached to a structure whose vibration is designed to mitigate. The following assumptions have been used in this study: 1) the high-rise building is represented adequately by its motion of first mode, 2) the response of structure and TMD is small and linear and 3) a random load with a white noise spectrum density S_o is applied to the main structure (m_s). Then the effect of TMD can be viewed as being equivalent to changing the damping ratio of structure (ξ_s) to equivalent damping ratio (ξ_e). Thus the dynamic response of the first mode of structure can be calculated by using ξ_e, which can be expressed as [9],

$$\xi_e = \frac{\pi \ S_0 \omega_S}{2E\left[H_{ys}^2(\omega)\right]} \tag{1}$$

where

$$E\left[H_{ys}^2(\omega)\right]=S_0\int_{-\infty}^{\infty}\left|H_{ys}(\omega)\right|^2 d\omega \quad (2)$$

$$H_{ys}(\omega)=\frac{\left(-\omega_s^2\omega+2\xi_1\omega_s^2\omega_1 i\omega+\omega_s^2\omega_1^2\right)}{\Delta_\Omega} \quad (3)$$

$$\begin{aligned}\Delta_\Omega=\omega^4-i\omega^3\left(2\xi_1\omega_1+2\xi_s\omega_s+2\mu\xi_1\omega_1\right)\\ -\omega^2\left(\omega_1^2+4\xi_s\xi_1\omega_s\omega_1+\omega_s^2+\mu\omega_1^2\right)\\ +i\omega\left(2\xi_s\omega_s\omega_1^2+2\xi_1\omega_s^2\omega_1\right)+\omega_s^2\omega_1^2\end{aligned} \quad (4)$$

where $H_{ys}(\omega)$ is complex frequency response of first mode structure attached with TMD. $H_{ys}(\omega)$ can be obtained by using Laplace or Fourier transformation since the system is a time-invariant one and can be computed by using MATLAB. $E\ [H_{ys}^2]$ is the mean square of $H_{ys}(\omega)$. m_1 is the mass of TMD and m_s is the modal mass of first mode structure. ω_1 and ω_s are natural frequencies of TMD and the first mode structure in Hz, respectively. ω is the frequency in Hz of applied force. $\mu = m_1 / m_s$ is mass ratio and is usually in the order of 0.5% to 3% of the total mass of structure. ξ_1 and ξ_s are damping ratios of TMD and first mode structure, respectively.

2.2 Optimal Damping Ratio

By neglecting the structural damping ratio to simplify the analysis, optimal damping ratio (ξ_1) and tuned parameter ($f_1=\omega_1/\omega_s$) can be obtained as [10]

$$\xi_{1opt}=\sqrt{\frac{\mu(3\mu+4)}{8(\mu+1)(\mu+2)}} \quad (5)$$

$$f_{1opt}=\sqrt{\frac{\mu+2}{2(\mu+2)^2}} \quad (6)$$

Alternately, Luft [11] shows that, with negligible errors, the following approximate relation can be used.

$$\xi_{1opt}=\frac{\sqrt{\mu}}{2} \quad (7)$$

$$f_{1opt}=\frac{1}{1+\mu} \quad (8)$$

$$\xi_{eopt}\approx\frac{\sqrt{\mu}}{4}+0.8\xi_s>\xi_s \quad (9)$$

2.3 Displacement of TMD Mass

In designing a TMD system, allowance must be made for the peak displacement (stroke) of TMD mass. Let the displacement of TMD with respect to mass m_s be denoted by $x_1=y_1-y_s$. The complex frequency response function of x_1 can be written as [12]

$$H_{x1}(\omega)=\frac{\omega_s^2\omega^2}{\Delta_\Omega} \quad (10)$$

Since the maximum roof displacement of the building due to dynamic effect only can be determined by $\Delta_g = C_g - 1$, the peak stroke of TMD can be obtained as

$$\Delta_{TMD}=\Delta_g\left(E\left[H_{x1}^2(\omega)\right]/E\left[H_{ys}^2(\omega)\right]\right)^{1/2} \quad (11)$$

where $\left(E\left[H_{x1}^2(\omega)\right]/E\left[H_{ys}^2(\omega)\right]\right)^{1/2}$ is the ratio of the standard deviation of the stroke of TMD and the roof displacement of the building.

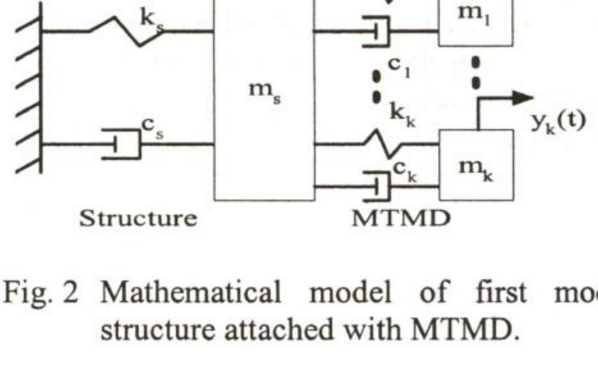

Fig. 2 Mathematical model of first mode structure attached with MTMD.

3. Design of Multiple Tuned Mass Damper (MTMD)

3.1 Mathematical Model and Responses

The MTMD consists of a number of TMDs attached to a structure whose vibration is designed to mitigate, as shown in Fig. 2. The following additional assumptions from TMD have been used in this study: 1) natural frequencies of structure are not closely space, 2) mass and damping ratio of each TMD are the same, 3) frequencies of MTMD are equally distributed as shown in Fig 3; and 4) the frequency of central TMD is equal to the structure frequency. ξ_e can be obtained in Eq. (1) similar to the case of TMD. $|H_{ys}(\omega)|$ for the case of the structure attached with MTMD was given by Yamaguchi and Harnpornchai [5].

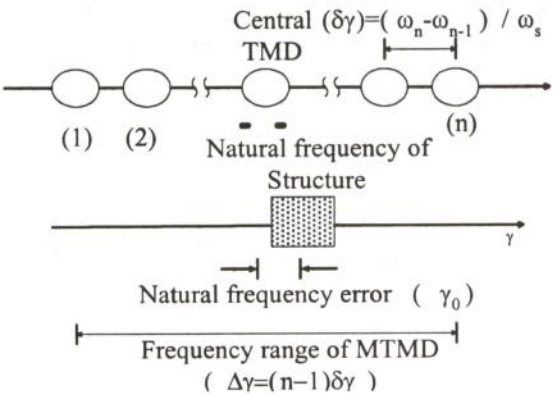

Fig.3 Frequency distribution of MTMD around structural natural frequency.

3.2 Optimal Damping Ratio

The procedure to determine optimal MTMD can be summarized as follows: (1) The appropriate frequency range ($\Delta\gamma$) is selected to reduce the flatness of the response in the wide range of resonant region. $\Delta\gamma$ is usually in the range of about 0.1-0.2 for 1 % total mass [5]. (2) The appropriate number of TMDs are selected by the following two combinations: (a) a small number of TMDs (which is identical with larger spacing of frequency) with a large damping ratio; and (b) larger numbers of TMDs (which identical with smaller spacing) with a smaller damping ratio. The damping ratio exists in the MTMD in the same manner as TMD. The damping plays a role of reducing the secondary peak response if the damping ratio is smaller than the optimal value. However, the excessive damping increases the primary peak response in the frequency response curve [5].

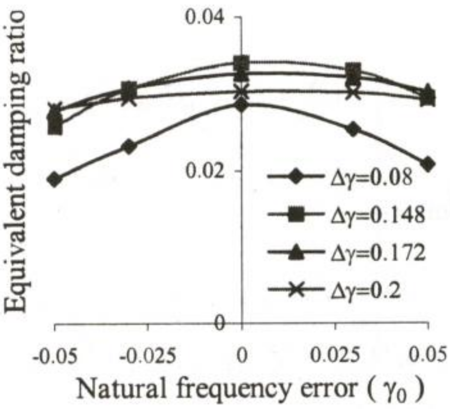

Fig.4 Effectiveness and robustness of MTMD for 400 m. high-rise Building.

Fig. 4 shows the effectiveness and robustness of MTMD for 400-m tall building (ξ_s=0.01, ω_s=0.09) attached with 5 TMDs for four cases. Each MTMD consists of $\delta\mu$=0.002, $\delta\xi_\kappa$=0.01. The frequency ranges of the four cases of MTMD are $\Delta\gamma$=0.08, 0.148, 0.172 and 0.2. It is clearly seen that too high or too small frequency range results in small value of ξ_e. In addition, when there is too small frequency range, ξ_e is sensitive to the error of natural frequency of structure. The result shows that $\Delta\gamma$=0.148 for 5 TMDs is optimal. The result also shows that the total optimal damping ratio in MTMD with 1% total mass is about twice of that in TMD.

4. Design of Tuned Liquid Damper (TLD)

4.1 Mathematical Model and Response

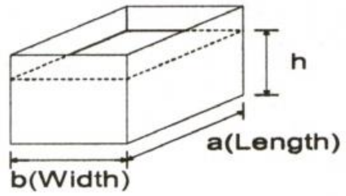

Fig. 5 Geometric of tuned liquid dampers.

The advantages of TLD over TMD can be summarized as follows: (1) Maintenance cost is minimized. This is because the simple physical concepts, on which the restoring force is provided in TLD requires, no activation mechanism. (2) TLD systems are at all times active, avoiding problems due to an inadequate activation system, while the mechanism activating a TMD must reach a certain threshold level of excitation. (3) For large amplitude of oscillation, the system is not very sensitive to the actual frequency ratio between primary and secondary system. (4) For structures with different fundamental frequencies in the two major directions, tuning may be accomplished by using rectangular tanks as shown in Fig. 5.

For the rectangular tank, the sloshing natural frequency in rad/s can be obtained from the application of linear wave theory [6] to be

$$\omega_R = \left(\frac{g}{\mathrm{a}}\pi\tanh\left(\pi\frac{h}{\mathrm{a}}\right)\right)^{0.5} \qquad (12)$$

where h is the height of water and a is the length of water tank.

4.2 Optimal Damping Ratio

For the rectangular tank, damping ratio of liquid sloshing can be approximately calculated from linear boundary layer theory as [6]

$$\xi_R = \frac{1}{2h}\sqrt{\frac{v}{\pi f_R}}\left(1+\frac{h}{b}\right) \qquad (13)$$

where f_R is the natural frequency of TLD in Hz and v is the kinematic viscosity = 0.01 cm^2/sec for water at room temperature and b is the width of water tank.

In case of optimally tuning TLD, the same principal as TMD can be applied but uses the effective TLD mass [13]. The damping ratio of TLD in Eq. (13) is usually lower than its optimal value and can be increased by using the liquid with the higher viscosity and/or adding the slice into the tank.

The size and number of TLD tank are calculated from frequency and mass ratio requirements. The large tank is preferred so that the total number of tanks becomes small (low cost) to attain the required mass ratio. However, the water depth should be small, say $h/a \approx 0.1 - 0.3$ to avoid the dead-mass effects in liquid motion.

5. Design of Multiple Tuned Liquid Damper (MTLD)

5.1 Mathematical Model and Response

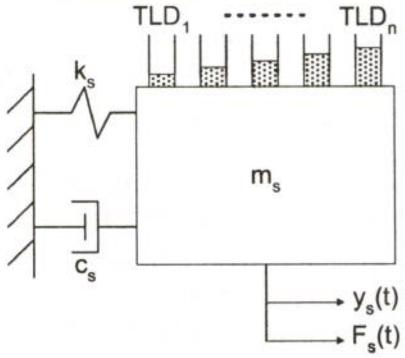

Fig. 6 Mathematical model of first mode structure with MTLD.

The MTLD consists of a number of TLDs as shown in Fig. 6. Each natural frequency of each TLD is calculated by changing the water depth. The equation of motion of MTLD is similar to MTMD except that the damping ratio of each TLD can be changed consequently from the water depth and

Table 1 Structural properties of three high-rise buildings.

Height (m)	Structural properties							
	Depth (m)	Width (m)	along-wind ω (Hz)	across-wind ω (Hz)	ρ_b (kg/m^3)	m_s (ton)	ξ_s	Wind exposure
183.0	30.5	30.5	0.263	0.263	200	11349	0.015	B
317.5	38.1	38.1	0.1	0.1	192	29497	0.010	B
400.0	45.0	66.0	0.09	0.1	153	60588	0.010	A,B,C

Table 2 Dynamic damper design.

Design Parameter	Dynamic Dampers			
	TLD	MTLD	TMD	MTMD
Building: 183 m high				
μ	0.01	0.01	0.01	0.01
γ	0.993	-	0.990	-
Δγ, δγ	-	0.14 , 0.035	-	0.14 , 0.035
total ξ	0.041	0.08	0.050	0.100
h/a (central ω tank)	0.257	0.160	-	-
h (m)	1.955	0.833	-	-
a , b (m)	7.62 , 7.62	5.22 , 5.22	-	-
Building: 317.5 m high				
μ	-	-	0.010	0.010
γ	-	-	0.990	-
Δγ, δγ	-	-	-	0.14 , 0.035
total ξ	-	-	0.050	0.100
Building: 400 m high				
μ	-	-	0.010	0.010
γ	-	-	0.990	-
Δγ, δγ	-	-	-	0.14 , 0.035
total ξ	-	-	0.050	0.100

the mass in each TLD is different. The advantage of MTLD over TLD is that the equivalent damping ratio of structure with MTLD is higher than that with TLD, even though the mass ratio and the liquid damping are the same [7], because of small phase differences among the liquid motion in each TLD of the MTLD. Similar to TLD case, the damping ratio of liquid sloshing in rectangular tank can be calculated from the linear boundary layer principle as previously shown.

5.2 Optimization Damping Ratio

The procedure to determine optimal MTLD is similar to the case of MTMD. In this study, the damping ratio in each TLD of the MTLD is assumed to be equal.

6. Numerical Examples

To illustrate the usefulness of the various dynamic dampers and assess their

Table 3 Comparison of vibration responses for three high-rise buildings with various dynamic dampers.

Dynamic damper	Parameters / Responses					
	Optimal ξe	Cg	Roof pressure (kg/m^2)	Δ (m)	Peak roof acceleration/g	
					Along wind	Across wind
Building: 183 m high in exposure B						
Uncontrol	0.015	2.147	218.767	0.174	0.021	0.040
TLD	0.032	1.917	195.286	0.155	0.014	0.027
MTLD	0.035	1.900	193.552	0.154	0.013	0.026
TMD	0.037	1.888	192.362	0.153	0.013	0.025
MTMD	0.039	1.877	191.226	0.152	0.013	0.024
Building: 317.5 m high in exposure B						
Uncontrol	0.010	3.047	408.826	1.876	0.049	0.159
TMD	0.033	2.195	294.492	1.351	0.027	0.087
MTMD	0.035	2.158	289.559	1.329	0.026	0.084
Building: 400 m high in exposure A						
Uncontrol	0.010	2.567	386.69	2.364	0.046	0.093
TMD	0.033	1.913	288.15	1.762	0.025	0.051
MTMD	0.035	1.885	283.89	1.736	0.024	0.049
Building: 400 m high in exposure B						
Uncontrol	0.010	2.753	393.16	2.311	0.047	0.093
TMD	0.033	2.021	288.68	1.697	0.026	0.051
MTMD	0.035	1.989	284.16	1.671	0.025	0.049
Building: 400 m high in exposure C						
Uncontrol	0.010	3.075	407.784	2.357	0.050	0.092
TMD	0.033	2.209	292.962	1.694	0.028	0.051
MTMD	0.035	2.172	288.003	1.665	0.027	0.049

effectiveness, three examples of high- and very high-rise buildings namely 183-m, 317.5-m, 400-m are studied. The structural properties and wind exposures are shown in Table 1. The results include equivalent static pressure, maximum along-wind roof displacement, peak along- and across-wind accelerations at the top of buildings.

6.1 Design of Dynamic Dampers

The mass ratio of each kind of dynamic damper is selected to be equal where natural frequencies and damping ratios are different to tune each kind of dynamic damper to the optimum values. Design parameters of TMD, MTMD, TLD, and MTLD, for 183-m. high-rise building are summarized in Table 2. For 317.5 m. and 400 m. high building, TLD and MTLD are not applied because the low natural frequency and the high mass of building cause h/a less than 0.1. Design parameters of TMD, MTMD for 317.5-m. and 400-m. high-rise buildings are also summarized in Table 2.

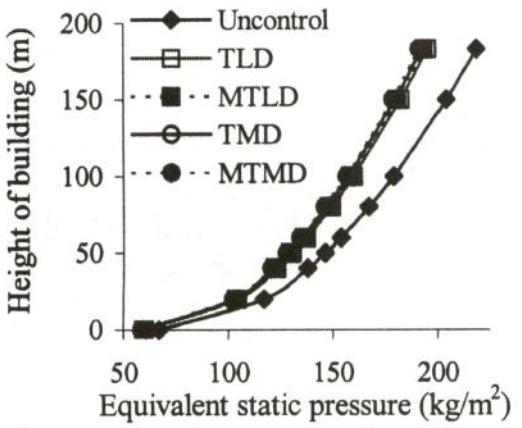

Fig. 7 Equivalent static pressure for 183-m building (Exposure B).

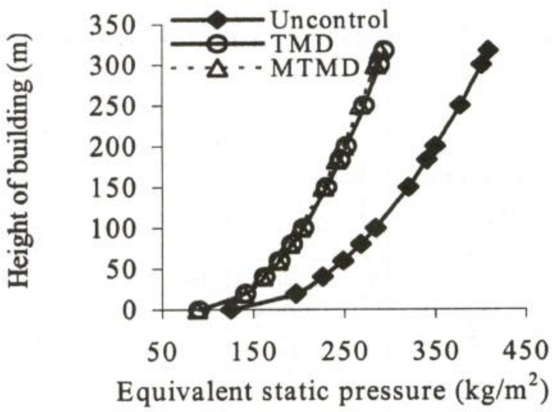

Fig. 8 Equivalent static pressure for 317.5-m building (Exposure B).

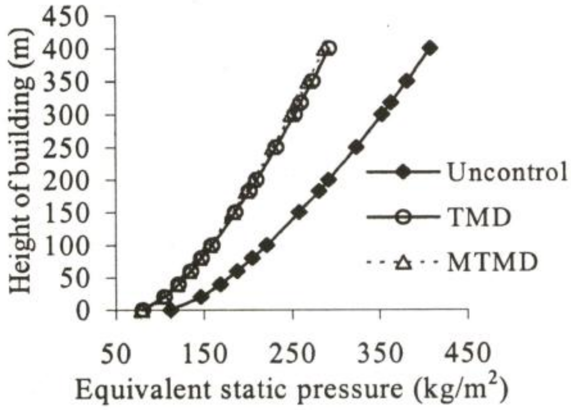

Fig. 9 Equivalent static pressure for 400-m building (Exposure C).

6.2 Effects of Dynamic Dampers on Reduction in Wind Loads and Responses

In the study, the equivalent damping ratio method is used to investigate the responses of the high-rise buildings with the dynamic dampers. Table 3 shows the comparison of vibration responses for three buildings with various dynamic dampers and wind exposures. In Table 3, Δ is the maximum displacement at the top of building in along-wind direction and C_g is gust effect factor [3,8]. Figs. 7-9 show comparisons of equivalent wind loads for three buildings with and without dynamic dampers and various wind exposures. From these results, the followings can be noted. (1) Building with dynamic dampers leads to increase in equivalent damping ratio and then results in significant reduction in gust effect factors, equivalent wind loads, deflection, and acceleration of the building. (2) The peak along- and across-wind acceleration can be reduced by about 45% for 317.5-m. high building attached with 1% mass ratio of TMD. However, the maximum lateral deflection at the top building can be reduced by only 28% for the same building and TMD, because the maximum lateral deflection consists of mean (static) and dynamic parts, and the dynamic damper can only effect the dynamic part. (3) For these high-rise buildings, the peak accelerations in across-wind direction are greater than those in along-wind direction. (4) For the same mass ratio, the effectiveness of dynamic dampers can be ranked as MTMD>TMD>MTLD>TLD. (5) The equivalent wind load and maximum lateral deflection for wind exposure A are the largest among three wind exposures. However, no significant difference in along- and across-wind accelerations for three wind exposures is found.

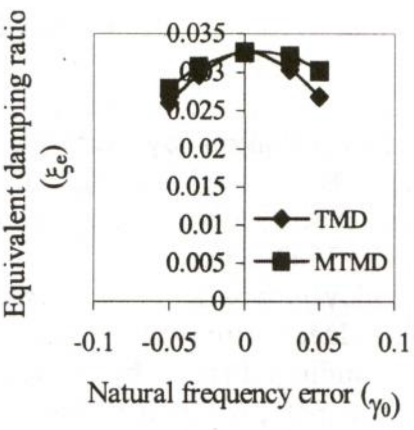

Fig. 10 Robustness of TMD and MTMD on natural frequency error (γo) for 400-m. high-rise building.

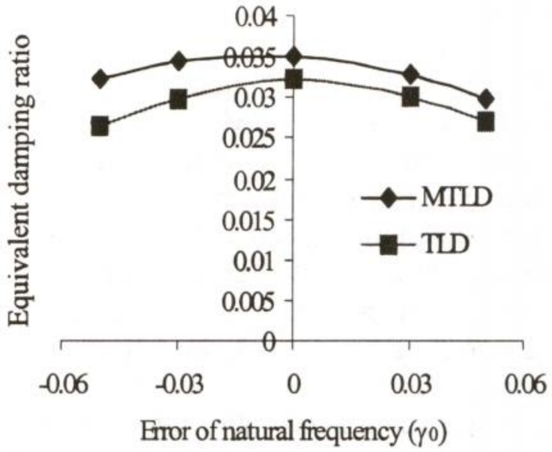

Fig.11 Robustness of TLD and MTLD on natural frequency for 183-m high-rise building.

6.3 Robustness on Natural Frequency Error of MTMD

Usually, the TMD is tuned to the optimal natural frequency which is very close to the natural frequency of the structure. However, off-tuning may occur owing to various reasons, such as the nonlinearity of structure, the error in identifying the natural frequency due to the change of life load. The efficiency of the MTMD is herein studied when the off-tuning exists. The robustness on natural frequency error for the 400-m. high-rise building attached with TMD in Table 1, is compared with that attached with 5 TMDs. TMD has μ=0.01 and ξ_{1opt}=0.05. For 5 TMDs, each TMD consists of $\delta\mu$=0.002, $\delta\xi_\kappa$=0.01 and $\Delta\gamma$=0.172.

Fig. 10 shows that the robustness on natural frequency error in case of MTMD is much better than TMD due to the close interaction of adjacent TMDs through the structural response.

6.4 Robustness on Natural Frequency Error of MTLD

The robustness on natural frequency error of TLD and MTLD for 183-m high building (Table 1), with almost the same total mass ratio, is investigated and compared in Fig. 11. The design parameters of TLD and MTLD is shown in Table 2. The result from Fig. 11 shows that the robustness on natural frequency error of MTLD is much better than optimal TLD. The reasons are: 1) small phase different among the liquid motion in each TLD of the MTLD, 2) lower depth in MTLD results in higher damping ratio for MTLD and 3) the effect of unequal masses for each TLD on the robustness is small. In case of uncertain natural frequency, the MTLD is more effective than TLD.

7. Conclusions

A design technique for various passive dampers for vibration control in high-rise buildings under wind loads is presented by the equivalent damping ratio method. From the numerical examples of three high- and very high-rise buildings with various passive dampers, the conclusions are summarized as follows:

(1) Buildings with dynamic dampers lead to increase in equivalent damping ratio and then result in significant reduction in gust effect factors, equivalent wind loads, deflection and acceleration of the building.

(2) The peak along- and across-wind acceleration can be reduced by about 45% for 317.5-m. high building attached with 1% mass ratio of TMD. However, the maximum lateral deflection at the top building can be reduced by only 28% for the same building and TMD.
(3) For high- and very high-rise buildings, the peak acceleration in across-wind direction is greater than that in along-wind direction.
(4) For the same mass ratio, the effectiveness of dynamic dampers can be ranked as MTMD>TMD>MTLD>TLD.
(5) The equivalent wind load and maximum lateral deflection for wind exposure A are the largest among three wind exposures. However, no significant difference in along- and across-wind accelerations for three wind exposures is found.

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