AUGMENTED MAXIMUM POWER TRANSFER THEOREM

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ABSTRACT

The classic maximum power transfer theorem is augmented to cope with electrical power transmission requirement in conventional power systems. The proposed technique can be applied for analysis and design of power systems to achieve optimum power transfer and transmission efficiency. The attractive feature of this method is that it does not require engineers' experience to specify the relative importance of the power transfer and the power transmission efficiency to the load.

***Key Words** : power transfer ratio, transmission efficiency*

I. INTRODUCTION

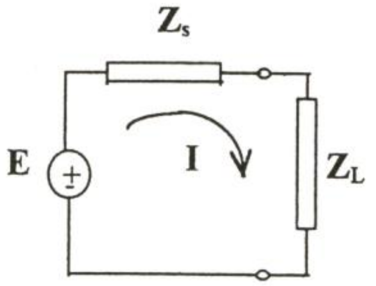


Figure 1 Simplified model of an electric power system

The classic maximum power transfer theorem such as stated in [1-3] considers an electric power system simply modeled as the circuit depicted in figure 1. The impedances $\mathbf{Z}_s$ and $\mathbf{Z}_L$ represent the transmission network and the load, respectively. In case $\mathbf{Z}_s$ and $\mathbf{Z}_L$ are purely resistive, the power transfer is maximum when $R_L = R_s$. Generally $\mathbf{Z}_L = \mathbf{Z}_s^*$ yields the maximum transfer of eletrical power. The concept has been applied to cope with supplying an arbitrary load by a nonlinear source, practically a photovoltaic array [4]. A practical work was conducted [9] to operate a DC separately excited motor optimally. The application of maximum power transfer concept for the design of nonlinear high-frequency mixers and power amplifiers was also reported [5]. Some achievements of maximum power transfer capability for AC Transmission systems under voltage stability constraint are described in [6-8] by different authors. Recently, the study of power transfer capability of DC systems having AC loads has been reported [11]. Mello et.al. also report their approach to evaluate the maximum simultaneous power transfer of large interconnected power systems having more than 1,000 buses [12]. None of these works consider the efficiency of power transmission in the network. Even though the classic theorem does not take account of the importance of power transmission efficiency particularly for electric power systems, the subject was treated by Kong [10] who introduced a parameter α indicating the relative importance between the power transfer and the transmission efficiency. α can be subjectively defined by an engineer to be $0 < \alpha \leq 1$. This parameter appears in Kong's objective function, Q, expressed by

$$Q = \alpha P_L - (1-\alpha)P_{loss} \ ; \ 0 < \alpha \leq 1 \quad (1)$$

where P_L and P_{loss} stand for load power and power losses in the system, respectively. A power system can be operated under maximized Q with appropriately selected α. The selection of α is based upon an engineer's experience, albeit.

This article proposes a different technique to search for an optimum operating condition of a power system. Under this condition, the objective function (J) expressed by

$$J = \mu\eta \quad (2)$$

where η stands for power transmission efficiency, and

μ stands for power transfer ratio $= \dfrac{P_L}{P_{L,\,max}}$ is maximized. This technique does not require a subjective selection of either μ or η.

II. FORMULATION OF POWER TRANSFER RATIO AND TRANSMISSION EFFICIENCY RELATIONSHIP

For a power system represented by the circuit shown in figure 1, it is commonly known that $\eta = P_L/(P_L+P_s)$. On the basis of the maximum power transfer theorem [1-3], $R_L = R_s$ yields the maximum load power that is

$$P_{L,max} = \frac{R_L E^2}{(R_L+R_S)^2} = \frac{E^2}{4R_S}$$

while the transmission efficiency is 50% (η= 0.5). In order to achieve high transmission efficiency, the load resistance must be increased to be much higher than the source resistance ($R_L >> R_S$). This results in lower load power that is $P_L \cong E^2/R_L$. In our consideration, the power transfer ratio (μ) defined by $P_L/P_{L,max}$ is more useful than the load power alone because it reflects the relative achievement of the power transfer to the load. Generally, one can easily derive

$$\mu = \frac{P_L}{P_{L,max}} = \frac{4R_L R_S}{(R_L + R_S)^2 + (X_L + X_S)^2} \quad (3)$$

If the condition $X_L = -X_S$ holds, the equation (3) is reduced to

$$\mu = \frac{4R_L R_S}{(R_L + R_S)^2} \quad (4)$$

Let $a = R_L/R_S$, the equation (4) can be rewritten as

$$\mu = \frac{4a}{(a+1)^2} \quad (5)$$

One can easily obtain

$$\eta = \frac{a}{a+1} \quad (6)$$

$$\mu = 4\eta\ (1-\eta) \quad (7)$$

and $$a = \frac{\eta}{1-\eta} \quad (8)$$

The curve shown in figure 2 illustrates the parabolic relationship between the power transfer and the transmission efficiency. It is clearly seen that the maximum power transfer ($\mu = 1$) can be achieved when the transmission efficiency (η) is only 50%. This curve can be used to determine the tradeoff between μ and η or the equation (7) may be used instead.

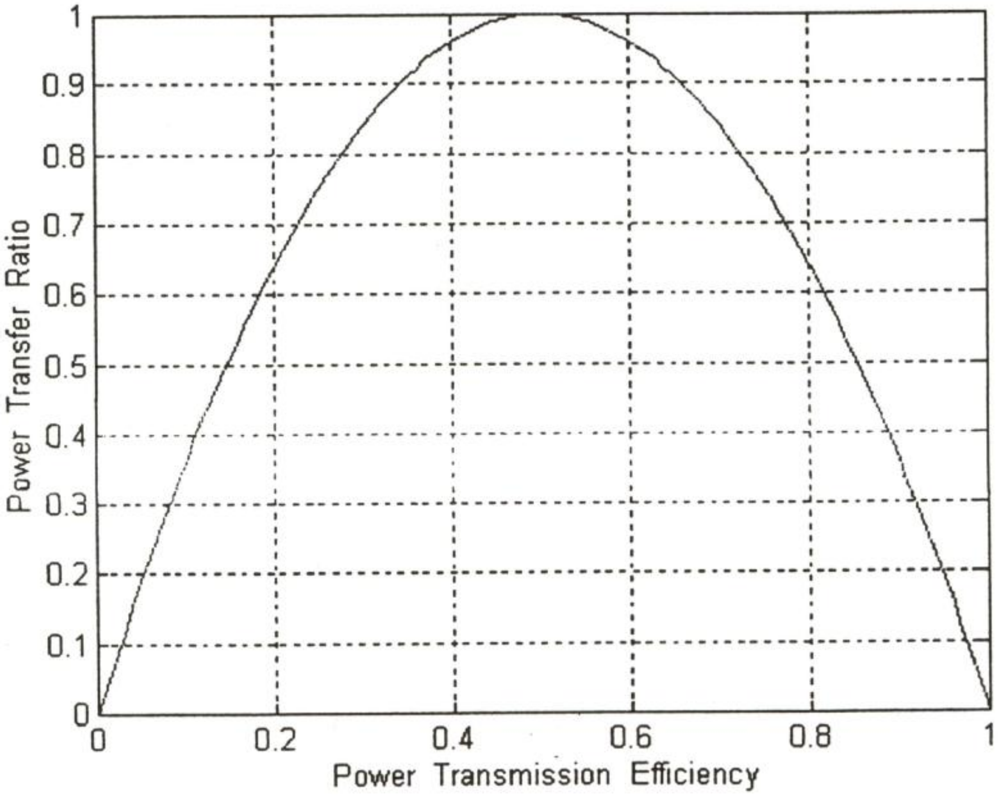


Figure 2 Parabolic relationship between μ and η

μ and η can be plotted against the resistance ratio (a) by applying the equations (5) and (6), respectively. The curves are shown in figure 3. It can be observed from figure 3 that 100% μ can be achieved at 50% η with a = 1 or $R_L=R_s$, and that with a = 3 or $R_L= 3R_s$ one can achieve 75% μ and 75% η.

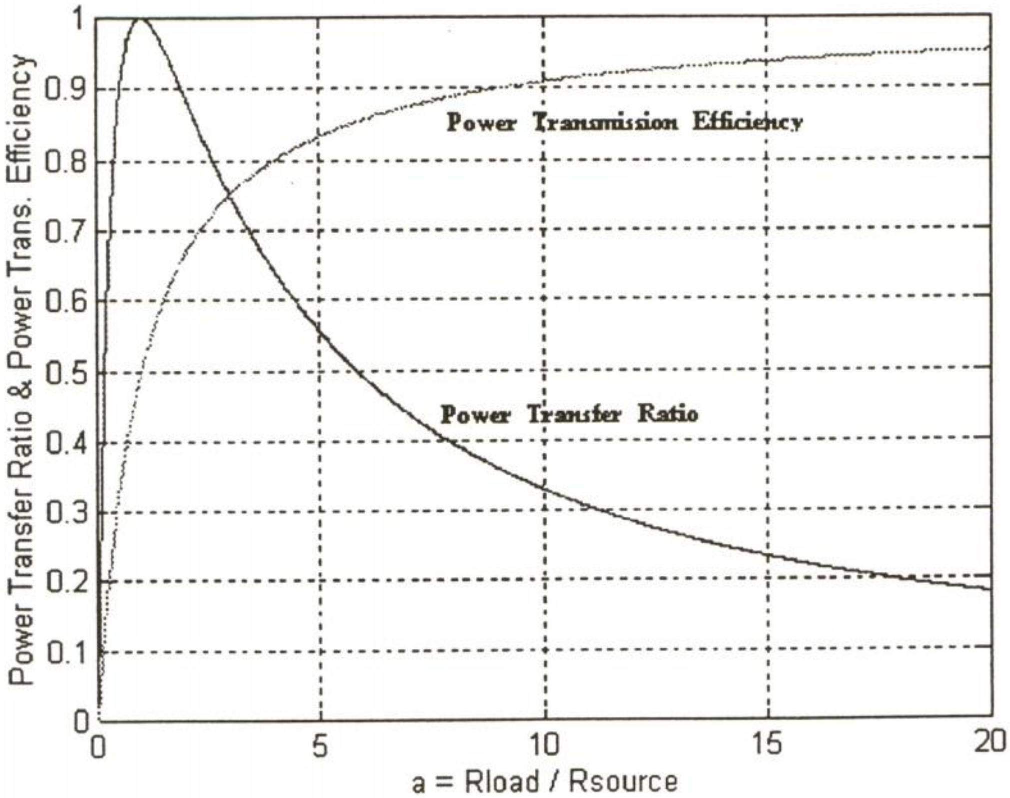


Figure 3 Plots of μ and η against resistance ratio (a)

In order to ensure an optimum operating condition, an objective fuction in the form of multiplied parameters is used. The objective function is expressed as :

$$J = \mu\eta = \frac{4a^2}{(a+1)^3} \tag{9}$$

Maximize J : $\frac{dJ}{da} = 0 = \frac{4a(2-a)}{(a+1)^4}$

yields a = 2. If we assume that $X_L = -X_s$ holds, this gives $R_L = 2R_s$ and $\mathbf{Z}_L = 2R_s - jX_s$. This means that the power system operates under the optimum condition of maximized J with μ = 88.89%, and η = 66.67%. If the condition $X_L = -X_s$ does not hold, we have

$$\eta = \frac{R_L}{(R_L + R_s)} \tag{10}$$

and μ in the equation (3) must be reformulated as

$$\mu = \frac{4R_S^2\eta(1-\eta)}{R_S^2 + \{(1-\eta)(X_L + X_S)\}^2} \tag{11}$$

Substitute η in (10) into (11), we obtain

$$\mu = \frac{4R_S R_L}{(R_L + R_S)^2 + (X_L + X_S)^2} \tag{12}$$

Equation (12) can be applied to find information about source and load impedances. For example, at the optimum μ with - respect - to R_L we have

$$\frac{\partial\mu}{\partial R_L} = \frac{\partial}{\partial R_L}\left[\frac{4R_S R_L}{(R_L + R_S)^2 + (X_L + X_S)^2}\right]$$

$= 0$, and hence $R_L = \sqrt{R_S^2 + (X_L + X_S)^2}$ (13)

In general, $\mathbf{Z}_S$ is known and fixed. From the equation (13), we can conclude that the load reactance could be adjusted, e.g. by normal practice of LC combination, etc., to achieve R_L that results in optimum operating condition of the power system. In some situations, trades must be made between $\mathbf{Z}_L$ and $\mathbf{Z}_s$ to achieve a satisfactory operating condition. The equation (13) could be one among others to be used for surface generation. Eventually, an optimization technique will be applied to search for an optimum or merely a satisfactory condition over the surface.

III. SOME APPLICATIONS

Four problems are exemplified herein to demonstrate the usefulness of the proposed method. Firstly, we consider the same problem as Kong did [10] in order to show that the same conclusion is reached.

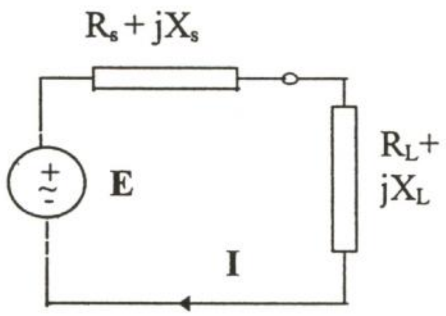


Figure 4 Simple circuit represents a power system

A 20 kV power system is assumed to be modeled as shown in figure 4. Load power needs to be determined assuming that the system is operated under minimum $\eta = 90\%$, $R_S = 0.4\Omega$, and $X_L = -X_S$.

By applying the equation (7) with $\eta = 90\%$, we obtain $\mu = 4\eta\ (1-\eta) = 0.36$ Hence, $P_{L,max} = E^2/(4R_S) = 250$ MW, and load power $P_L = \mu\ P_{L,max} = 90$ MW. The same figure for load power (P_L) is obtained as in [10] for $\alpha = 0.2$.

In [10], α is assumed to be 0.1 in order to increase η to 95% for a required load of 120 MW. It is interested to know E. Kong solved for E and obtained 32 kV. Using $\eta = 1\text{-}0.5\alpha$ [10] in conjunction with the proposed method, we firstly obtain $\mu = 2\alpha\ (1\text{-}0.5\alpha) = 0.19$. Then, $P_{L,max} = P_L/\mu = 632$ MW, and $E = \sqrt{4R_S P_{L,max}} = 32$ kV.

The second example demonstrates that our method can be helpful to the selection of transmission line. Given two transmission lines : (a) bare aluminum 25 mm², $\mathbf{Z} = 1.18 + j0.4\ \Omega/km$; (b) bare copper 25 mm², $\mathbf{Z} = 0.74 + j\ 0.4\ \Omega/km$. We need to determine which transmission line is preferable for a power system having load of 300 kVA, 0.8 pf lagging. This load is 10 km away from the source and required to have 3.5 kV across it.

From the data available, we obtain for the aluminum transmission line $\mathbf{Z}_a = 11.8 + j\ 4.0\ \Omega$, and for the copper one $\mathbf{Z}_b = 7.4 + j\ 4.0\ \Omega$. The load impedance is

$$\mathbf{Z}_L = \frac{(3.5kV)^2(0.8)+j(3.5kV)^2(0.6)}{300kVA} = 32.7 + j\,24.5\,\Omega.$$

By applying the equations (3) and (10), respectively, we could obtain :

$\mu_a = 0.5527$, $\eta_a = 0.7348$ and $\mu_b = 0.40$, $\eta_b = 0.8155$. Hence, $J_a = \mu_a\eta_a = 0.4061$, and $J_b = \mu_b\eta_b = 0.3262$. Since the transmission line "a" possesses a greater value of J, it is preferable.

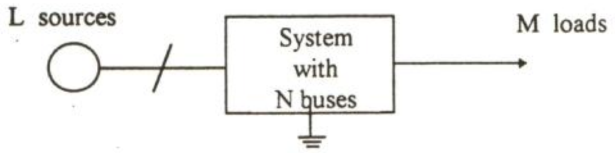


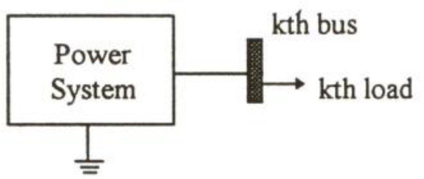
Figure 5 Simplified block diagram represents a complicated power system

The proposed method can be applied to a more complicated power system having L sources, M loads and N buses as shown in figure 5. The calculation of μ, η and power generated can be accomplished by using the following formulae

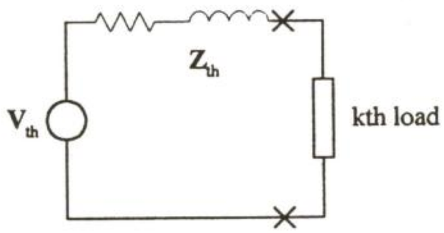
$$\mu = \frac{\sum_{m=1}^{M} P_{Load,m}}{\sum_{m=1}^{M} P_{Load\ (max),\ m}} \quad (14)$$

$$\mu = \frac{\sum_{m=1}^{M} P_{Load,m}}{\sum_{l=1}^{L} P_{Generated,\ l}} \quad (15)$$

and $$\sum_{l=1}^{L} P_{Generated,\ l} = \sum_{m=1}^{M} P_{Load,m} + \sum_{q=1}^{\frac{N(N-1)}{2}} P_{Transmission\text{-}line\ losses,\ q} \quad (16)$$



(a)



(b)

Figure 6 Power system with a load of interest (a) kth load connected to kth bus, (b) Thevenin's equivalent of (a)

Thevenin's equivalent is useful for the analysis. Figure 6 (a) shows the components of interest that are the kth load connected to the kth bus. Its Thevenin's equivalent is shown in figure 6 (b) where $\mathbf{V}_{th} = \mathbf{V}_k$ (voltage across the kth bus) and $\mathbf{Z}_{th} = \mathbf{Z}_{kk}$ (Thevenin impedance, usually called driving point impedance, across the terminals of the kth load when looking into the kth bus) [13]. If an impedance matrix is formed for a power system, these $\mathbf{Z}_{kk}$'s appear as its diagonal elements. Hence, the maximum power of the kth load can be calculated by using equation (17).

$$P_{Load\ (max),\ k} = |\mathbf{V}_k|^2/[4Re(\mathbf{Z}_{kk})] \quad (17)$$

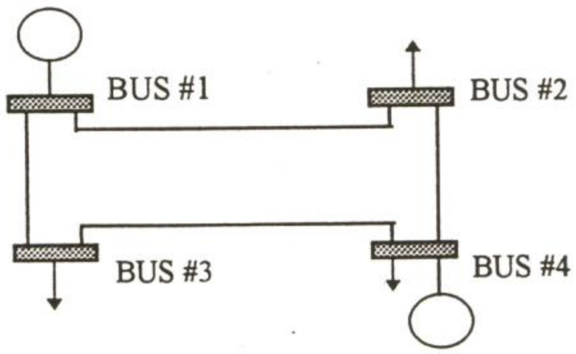


Figure 7 Diagram represents a 100MVA, 230 kV power system [13]

For a complicated system represented by the diagram in figure 7, the relations (14) - (17) can be applied for the load-flow calculation. This is to find the generated power of the generator at BUS #4 (P_{G4}) that maximizes J. The calculation uses the data for transmission lines [13] and buses [13, 14] as shown in tables 1 and 2, respectively.

Table 1 Transmission line data

Line	Series Z		Series Y		Shunt Y
Bus to Bus	R (per-unit)	X (per-unit)	G (per-unit)	B (per-unit)	Y/2 (per-unit)
(1)-(2)	0.0108	0.0649	2.5	-15	0.0524
(1)-(3)	0.0235	0.0941	2.5	-10	0.0412
(2)-(4)	0.0118	0.0471	5.0	-20	0.03875
(3)-(4)	0.0147	0.0588	4.0	-16	0.06281

Table 2 Bus data

BUS	Generation		Load		$V_{(pu)}$	remarks
	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)		
(1)	-	-	-	-	$1.02\angle 0^{o}$	slack bus
(2)	-	-	70	42	$1.00\angle 0^{o}$	load bus
(3)	-	-	80	50	$1.00\angle 0^{o}$	load bus
(4)	30 - 200	-	65	36	$0.96\angle 0^{o}$	PV bus

A portion of calculated results is depicted in figure 8. The curve clearly illustrates that P_{G4} of 135 MW yields maximum J.

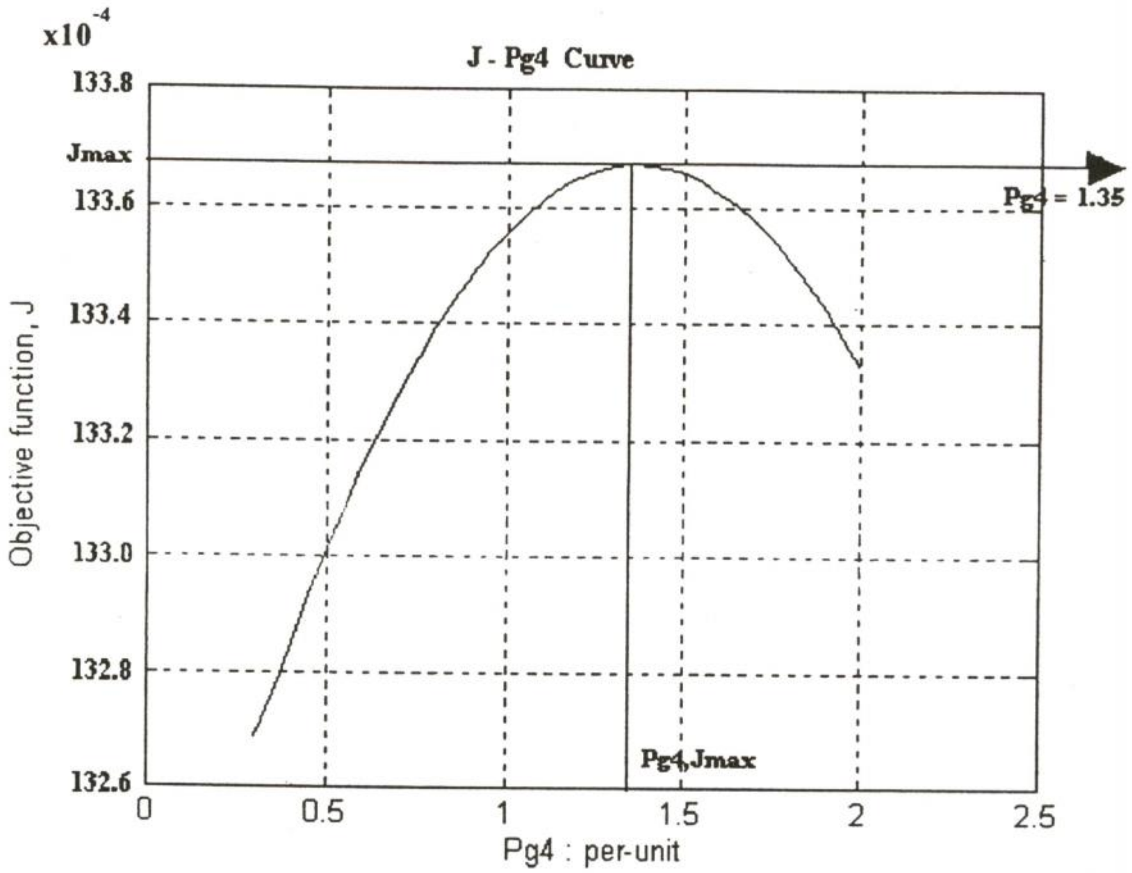


Figure 8 Plot of J against P_{G4}

Another system exemplified herein is represented by the diagram shown in figure 9. The problem is to find transmission lines that optimize J. Table 3 and 4 show the data for transmission lines and buses, respectively.

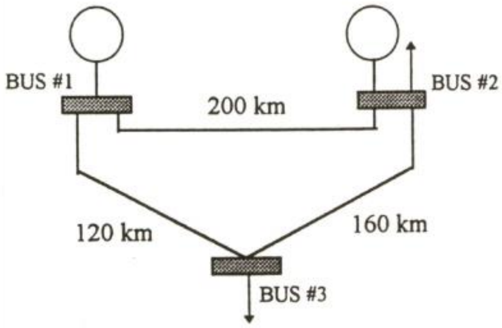


Figure 9 Diagram represents a 100 MVA, 230 kV power system [15]

Table 3 Transmission line data [13]

T_x line	Series Z	Shunt Y (Y/2)
A	0.02666 + j 0.1333 Ω/km	0.4844 μS/km
B	0.01968 + j 0.09839 Ω/km	0.3663 μS/km

Table 4 Bus data [15]

BUS	Generation		Load		$V_{(pu)}$	remarks
	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)		
(1)	-	-	20	-	$1.00\angle 0^o$	slack bus
(2)	80	-	50	-10	$1.05\angle 0^o$	PV bus
(3)	-	-	100	40	$1.00\angle 0^o$	load bus

Table 5 Possible transmission line combinations

Bus to Bus	Combination of transmission lines							
(1) - (2)	A	A	A	A	B	B	B	B
(2) - (3)	A	A	B	B	A	A	B	B
(1) - (3)	A	B	A	B	A	B	A	B

Our calculation using the equations (14) - (17) results in the numerical data tabulated in table 6. We select the transmission line combination "BBA" that maximizes J.

Table 6 Values of μ, η and J corresponding to various combinations of transmission lines

Tx Line comb.	μ	η	J
AAA	0.0039	0.6729	0.0026
ABA	0.0036	0.4516	0.0016
AAB	0.0043	0.0900	0.0004
ABB	0.0032	0.4861	0.0015
BAA	0.0035	0.5150	0.0018
BBA	0.0032	0.8643	0.0027
BAB	0.0034	0.2002	0.0007
BBB	0.0029	0.3405	0.0010

IV. CONCLUSIONS

The work presented in this article augments the maximum power transfer theorem such that power transmission efficiency can be considered. The proposed method uses the power transfer ratio (μ) defined by $\frac{P_L}{P_{L,max}}$ and the objective function $J = \mu\eta$. It is demonstrated that the method is useful for analysis and design of power systems without the requirement of an engineer's experience to specify the relative importance of μ and η. In general condition of $\mathbf{Z}_L \neq \mathbf{Z}_S^*$, the method yields an optimum operating point of locally maximum μ and η respectively. Some applications of transmission line selection are exemplified in section III. The method is also applicable to loads with leading as well as adjustable power factors.

ACKNOWLEDGMENT

The authors wish to thank Manop Rujipakorn and reviewers for their helpful comments, and Pattarawan Sea-Tai for her excellent typing of the manuscript.

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