A MODIFIED ONE-PARAMETER TAYLOR METHOD FOR THE DISCRETE ARRAY APPLICATION

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ABSTRACT

This paper presents the modification of the one-parameter Taylor method for the discrete source application instead of the conventional one-parameter Taylor method which can be applied only for the continuous source. The weighting parameter is determined to eliminate the degradation of the nearest minor lobe level. The pattern characteristics of the modified and the conventional one-parameter Taylor methods are compared. The discussion on the use of continuous and discrete sources models are also included.

1. INTRODUCTION

In many applications such as point to point communication, it is desirable for the antenna to be highly directive in a particular direction, narrow beamwidth, maximum gain and low side lobe level to meet the demand of the long distance communication. An array antenna is one typical antenna that can fulfill this requirement. It is formed by assembling antenna elements with appropriate electrical and geometrical configurations. The discrete array [1-10] is a type of the array which each of the elements are placed separately. For the large discrete array and very closed spacing of the elements, the continuous source can be approximated to keep away from the tedious calculation. However, in the case of the large spacing of each elements, that approximation is not accurate.

One-parameter Taylor method is a

method to synthesize the array pattern which was initially introduced by Taylor and is widely investigated by many authors [11-14]. The advantage of this method is the minor lobes decreases monotonically. For some applications such as radar and low noise system, this method is preferable because interfering or spurious signals would be reduced when they enter through the decaying minor lobes. Unfortunately, one-parameter Taylor method can be applied only to the continuous source distribution. However, Balanis [14] described the application of the one-parameter Taylor method to the discrete array by means of the source distribution expression. In that fashion, it is obvious that for a specified side lobe level, the array pattern possesses degradation of that specified side lobe level. In this paper, the authors propose a new expression for the weighting parameter calculation to achieve the array pattern with the specified side lobe level using the least square polynomial regression curve fitting of the third order method. The modified and the conventional one-parameter Taylor methods are compared in this paper. The discussion on the use of continuous and discrete arrays models are also included.

2. ARRAY AND SPACE FACTORS CONSIDERATION

2.1 Array factor of the discrete source

To synthesize the broadside linear array pattern, at first, the array factor will be considered. Let us assume that there is a linear array of isotropic elements. The elements are aligned along the z-axis, symmetry with the center of the array and have equidistance. When the number of the elements is even, an array factor (AF) can be written as [14]

$$AF_{2N}(\theta) = \sum_{n=1}^{N} I_n cos[(2n-1)\frac{\pi d}{\lambda}cos\theta]. \qquad (1a)$$

An array factor of the odd number of the elements can be expressed as

$$AF_{2N+1}(\theta) = \sum_{n=1}^{N+1} I_n cos[2(n-1)\frac{\pi d}{\lambda}cos\theta]. \qquad (1b)$$

where I_n is the amplitude current excitation coefficient, $2N$, $2N+1$ is the number of even and odd elements, respectively, d is the spacing between each elements, λ is the wavelength of the operating frequency and θ is the angle between the field direction of the z-axis.

Then the summation of the cosine term for the case of even and odd elements will be expanded. The order of harmonic cosine term is equal to the total number of the element minus one and the argument of the cosine term is the positive integer times the fundamental frequency. It can be written in the form

$m=k;$

$$cos(mu) = cos^k(u) - (\frac{k}{2})cos^{k-2}(u)sin^2(u)+(\frac{k}{4}) cos^{k-4}(u) \quad sin^4(u)-..-(\frac{k}{k-2})cos^2(u) sin^{k-2}(u) + sin^k(u), \qquad (2)$$

where $(\frac{k}{n}) = \frac{k!}{n!\ (k-n)!}$ and $sin^2(u)=1-cos^2(u)$.

2.2 Space factor of the continuous source

For the large array with closed spacing between each elements, the discrete source can be approximated as the continuous source. In the analysis of the continuous source, the total field is equal to the product of the element factor and the space factor by means of the pattern multiplication. The element factor is the individual pattern of each element and the space factor means the pattern which can be obtained from the integration of the current source distribution times the far field phase kernel throughout the entire elements configuration. To consider the continuous source in the one dimension case or the line source distribution, assume that there is a linear discrete array placed along the z-axis. When the number of the elements are increased whereas the length of the array is fixed, the discrete source can be assumed as the continuous source. The summation in the array factor becomes the integration in the space factor. For a line source distribution of the length l placed symmetry along the z-axis, the space factor (SF) for the uniform phase distribution is given by using a finite Fourier transform as [14]

$$SF\ (\theta)\ = \int_{-l/2}^{l/2} I_n(z')e^{\,j\frac{2\pi}{\lambda}z'\cos\theta}dz'. \tag{3}$$

3. ONE-PARAMETER TAYLOR METHOD

3.1 Conventional one-parameter Taylor method for the continuous source

For the continuous source distribution, to improve the ideal space factor which is derived from Dolph-Tschebyscheff discrete source distribution, Taylor suggested the one-parameter Taylor method. The advantage of this method is that it provides tapered minor lobe distribution whereas the Dolph-Tschebyscheff ideal space factor for the continuous source yields a pattern with equal ripple minor lobes.

After some mathematical manipulations by using Gegenbauer's finite integral and Gegenbauer polynomial, the space factor of the one-parameter Taylor line source can be written as [11-14]

$$SF\ (\theta)\ = \begin{cases} \dfrac{\sinh[\sqrt{(\pi B)^2-(\frac{\pi l}{\lambda}\cos\theta)^2}]}{\sqrt{(\pi B)^2-(\frac{\pi l}{\lambda}\cos\theta)^2}} \\[2ex] \dfrac{\sin\ [\sqrt{(\frac{\pi l}{\lambda}\cos\theta)^2\ -(\pi B)^2}]}{\sqrt{(\frac{\pi l}{\lambda}\cos\theta)^2\ -(\pi B)^2}} \end{cases} \tag{4}$$

The upper and the lower equations are used, respectively, when $(\frac{\pi l}{\lambda}\cos\theta)^2$ less and greater than $(\pi B)^2$. B is the constant which can be determined from the side lobe level. The B parameter is also called weighting parameter or one-parameter. For the specified side lobe level, the formulation for the relation between that side lobe level, $R_n(dB)$ and the weighting parameter is given as [11-14]

$$R_n(dB)\ =\ 13.26\ +\ 20\log_{10}(\frac{\sinh\pi B}{\pi B}), \tag{5}$$

where the cardinal number 13.26 means the side lobe level of the uniform distribution which occurs when the value of the weighting parameter vanishes.

Generally, in the design of the radiation pattern, we would start with the required side lobe level, then determine the weighting parameter and the space factor, subsequently. From this reason, (5) is not appropriate for computing the weighting parameter because it is an inverse problem. Blanton [13] presented the alternative expression for solving the weighting parameter as the straightforward problem from the hyperbola equation as

$$B = b\sqrt{\frac{(R_n - c)^2}{a^2} - 1}, \tag{6}$$

where a and b represent the hyperbola's semi-transverse and semi-conjugate axes, respectively. c is the displacement (in dB) of the hyperbola's center from the origin. In the case of the line source distribution the values of a, b and c are, respectively, *22.96*, *0.9067* and *9.7*.

3.2 Modified one-parameter Taylor method for the discrete source

The normalized amplitude current excitation coefficient must be known to modify the one-parameter Taylor method for the discrete array application. By substituting (4) in (3) and using the inverse transform pairs of (4), the amplitude current excitation can be written after some intricate mathematical manipulations in a relatively simple form as [11-14]

$$I_n = I_0\left(\pi B\sqrt{1-\xi^2}\right), \tag{7}$$

where I_n denotes the amplitude current excitation coefficient, $I_0(x)$ is the modified Bessel function of the first kind of order zero which is related to the ordinary Bessel function of the first kind of order zero $(J_0(x))$ as

$$I_0(x) = J_0(jx), \tag{8}$$

and ξ is the normalized distance along the overall source which is defined as

$$\xi = \frac{z'}{l/2}, \tag{9}$$

where z' is the dimension along the array, with the origin at the array's midpoint, and l is the array's total length. The constrained value of ξ is between -1 and 1.

The complete expression of the array factor can be determined by substituting (7) into (1a) or (1b) according to the number of the elements. Unfortunately, from the array pattern, it is found that for a certain specified side lobe level, the error of that side lobe level occurs. The array pattern always keep the side lobe level less than the specified value by *2.00* dB By this way, the authors propose a new expression for the weighting parameter term calculation to achieve the array pattern with the specified side lobe level. This parameter is determined by using the least square polynomial regression of the third order curve fitting from the data after the array pattern was estimated for that specified side lobe level. The relation between the weighting parameter and the side lobe level after calculation as discribed can be written as

$$B = 0.0000769287R_n^3 - 0.00575009R_n^2 + 0.1834R_n - 1.33104 \tag{10}$$

where B is the weighting parameter to be determined, R_n is the specified side lobe level (dB) in the design and the accuracy of the calculation is quite sensitive to the value of the constant coefficient of the polynomial, so it should neither be rounded nor truncated. The amplitude current excitation coefficients can be calculated from this new expression.

4. DEMONSTRATION OF THE CONVENTIONAL AND THE MODIFIED ONE-PARAMETER TAYLOR METHODS

The pattern characteristics of the conventional and the modified one-parameter Taylor methods are demonstrated to compare the side lobe level. The side lobe level of the conventional one-parameter Taylor method is obtained by substituting R_n in (5) or (6) and then solved for the B parameter. When the current distribution is substituted in (1a) or (1b), the array factor can be plotted in fig.1. Alternatively, for the case of the modified one-parameter Taylor method, (10) is used instead of (5) or (6) and the array factor is also illustrated in fig. 1. From this graph, it is obvious that the modified one-parameter Taylor method is realized to achieve the side lobe level at *20.00* dB whereas the conventional one-parameter Taylor method possesses *22.00* dB side lobe level. Fig. 2 shows the radiation pattern of the fixed length discrete array at the spacing 0.5λ, 0.4λ, 0.3λ and continuous array at *20.00* dB side lobe level. Fig. 3 shows the normalized amplitude current excitation coefficient of the one-parameter Taylor method.

5. DISCUSSION ON THE DISCRETE AND CONTINUOUS SOURCES

In this section, the guidelines to apply to the continuous and discrete sources models will be discussed. Normally, the study of the source distribution would be started with the discrete source. The isotropic radiator is used as the array elements to simplify the calculation. For the non-isotropic antenna, that individual pattern would be multiplied by the array factor by means of the multiplication of the radiation pattern. When the number of the element increases as the large array, the tedious calculation which is a significant problem would occur. To avoid this problem, when the spacing is very close compared with the array dimension, the discrete source will be approximated as the continuous source. The summation becomes the integration which implies that the array factor becomes the space factor. The directivity of the continuous source and various discrete sources at different spacings for the fixed length (4.5λ) are illustrated in fig.4. The plot of the discrete source was started with the 0.5λ spacing and followed by 0.45λ, $0.4,\lambda$ 0.35λ, 0.3λ spacings and the continuous source, respectively. From this graph, it is found that the characteristics such as directivity and beamwidth of the discrete source, with close spacing approaches the continuous source. It is noticed that for the spacing less than *1/15* of the dimension length, the discrete source can be approximated as the continuous source accurately. Directivity in the case of large spacing discrete array is higher than the close spacing discrete and continuous sources because of their narrower beamwidth property.

6. CONCLUSION

One-parameter Taylor method is modified for applying to the discrete source application with the desired side lobe level. From the characteristics comparison between the conventional one-parameter Taylor method for the continuous source and the modified one-parameter Taylor method for the discrete source applications, it is obvious that the discrete source could be approximated as the continuous source when the spacing of each elements less than *1/15* of the dimension length. The approximation should be used when the array dimension is large with close spacing between each elements to keep away from tedious calculation problem.

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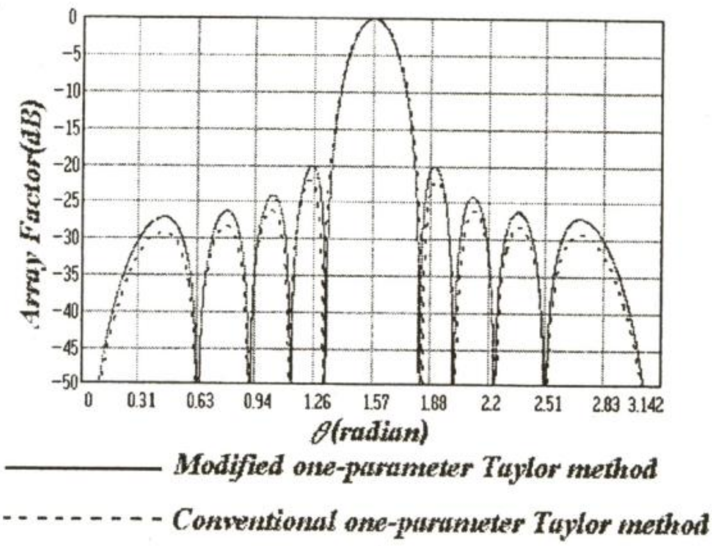

Fig. 1 Array pattern of *10* element broadside linear array with SLL *20.00* dB and *0.5λ* spacing, of the conventional and the modified one-parameter Taylor methods

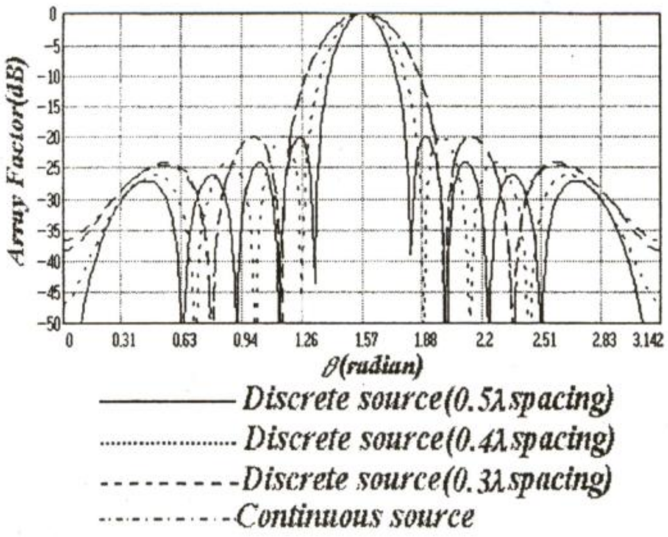

Fig. 2 Array pattern of the *0.5λ, 0.4λ, 0.3λ* spacing discrete and continuous sources with SLL *20.00* dB

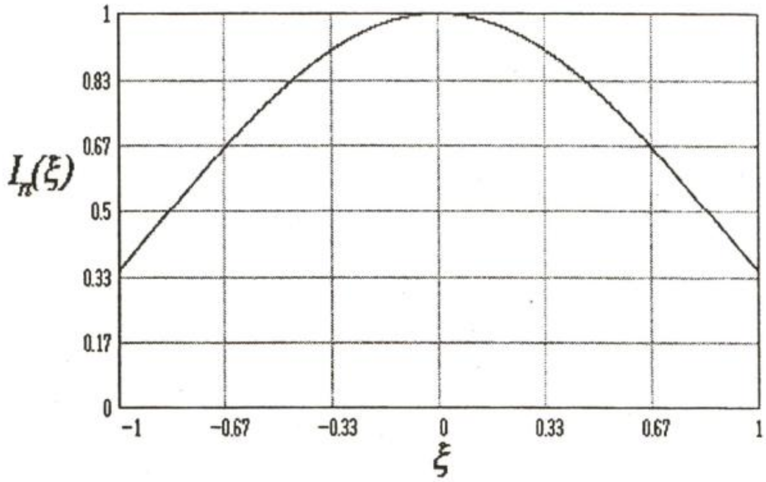

Fig. 3 Normalized amplitude current excitation coefficient of the one-parameter Taylor method

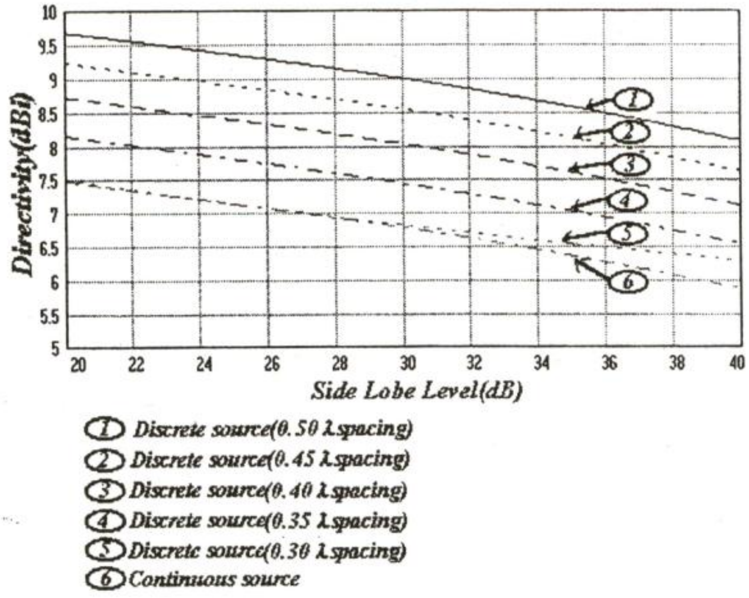

Fig. 4 Directivity of the continuous source and various discrete sources at different spacings for a fixed length *(4.5λ)* array