SHEAR CAPACITY OF REINFORCED CONCRETE BEAMS WITH STIRRUPS AS INFLUENCED BY THE POSITION OF INFLEXURE POINTS

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ABSTRACT

Experimental and analytical studies on the shear capacity of reinforced concrete beams with inclined stirrups were carried out to clarify the effect of inflexure point on the shear span. All the results obtained from analytical study closely resembled with experimental results. It was found the presence of inflexure point on the shear span increases the ultimate shear capacity as much as 34% when a/d = 0.5-4.0, where a is the distance of the inflexure point from the nearest support, d is effective depth, A formula is proposed for the shear capacity, and ultimate shear capacity computed from proposed formula is compared with experimental values and various current codes of practices. It was found that the proposed formula gives the best fit to the experimental results in comparison to the selected current codes of practices.

Key Words: *inclined stirrup, inflexure point, nonlinear analysis, ultimate shear capacity*

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INTRODUCTION

Most of the current shear design equations for reinforced concrete beams are based on the empirical results obtained from the two-point loading test on simply supported beams. However, in the real structures, the distributions of shear force and bending moment are not as simple as those of the two-point loading test, and the point of zero moment called as the inflexure point usually exists. The inflexure point in the shear span of the RC beams has a remarkable influence on the ultimate shear capacity of such beams, and in the case of RC beams without shear reinforcements, the presence of inflexure point on the shear span of RC beams increases the ultimate shear capacity as much as 70% in comparison to the beams without inflexure points on their shear span [1]. Among the current design codes viz. ACI, CEB-FIP, JSCE, AS3600, all of them give too conservative prediction of the ultimate shear capacity of RC beams without shear reinforcements. Aoyagi, et al. [1] proposed a new design formula based on the experimental studies on the beams which takes inflexure point into account. In the present paper, the influence of inflexure point on the shear capacity of beams with shear reinforcement is investigated both experimentally and analytically. The experimental procedure consisted of full scale tests on 12 RC beams with inclined stirrups and the analytical study is based on a nonlinear finite element program for reinforced concrete structures. The validity of the analytical procedure was confirmed by the comparison with the test results. Finally, the prediction of the ultimate shear capacity of the beams with shear reinforcement was proposed.

EXPERIMENTAL PROCEDURE

A total of 12 beam specimens were tested in this study. In Fig. 1, the beam configurations were the rectangular cross-section of size 16 cm. (b) x 25 cm.(h) and length of 340 cm. except the two basic beams (without the inflexure points), whose lengths were 240 cm. The beams were so designed that the shear failure preceded the flexural failure in the predicted failure region. As can be seen in Fig. 1, the longitudinal bars of 20 mm. dia. SD40-type with a yield strength about 4560 ksc (kgf/cm^2) were provided with 4-bars at the bottom and 3-bars at the top of the section. The shear reinforcement was provided with the inclined 6 mm. dia. SR24-type bars with a yield strength of about 2900 ksc at an angle of 45° to the beam axis at 10 cm. c/c in the predicted shear failure zone, and with the vertical 9 mm. dia. SR24-type bar with yield strength of about 3000 ksc at 5 cm. c/c in the rest part except in the middle span of 60 cm. where spacing was maintained at 10 cm. c/c as shown in Fig. 1. The compressive strength of concrete at the time of test ranged from 264 to 307 ksc.

The beams were supported on simple supports with one end roller and another end hinged. The middle span in all beams was kept equal to 220 cm. Two-point loading was applied to the basic beams and four-point loading was applied to the other beams. The concentrated loads were applied by using hydraulic jacks, and steel I beams were used to tranfer the loads to the beams as illustrated in Fig. 2. The position of inflexure points were moved by changing the distance between the load with a constant value of 1.6P and the support in the overhang portion of the beams while keeping the locations of the load in the middle span constant. As indicated in Fig. 3, tests on the beams with different a/d ratios i.e. 0, 0.5, 0.75, 1, 2, 3 and 4 were carried out, where "a" is the distance of the inflexure point from the nearest support and "d" is the effective depth of the beam.

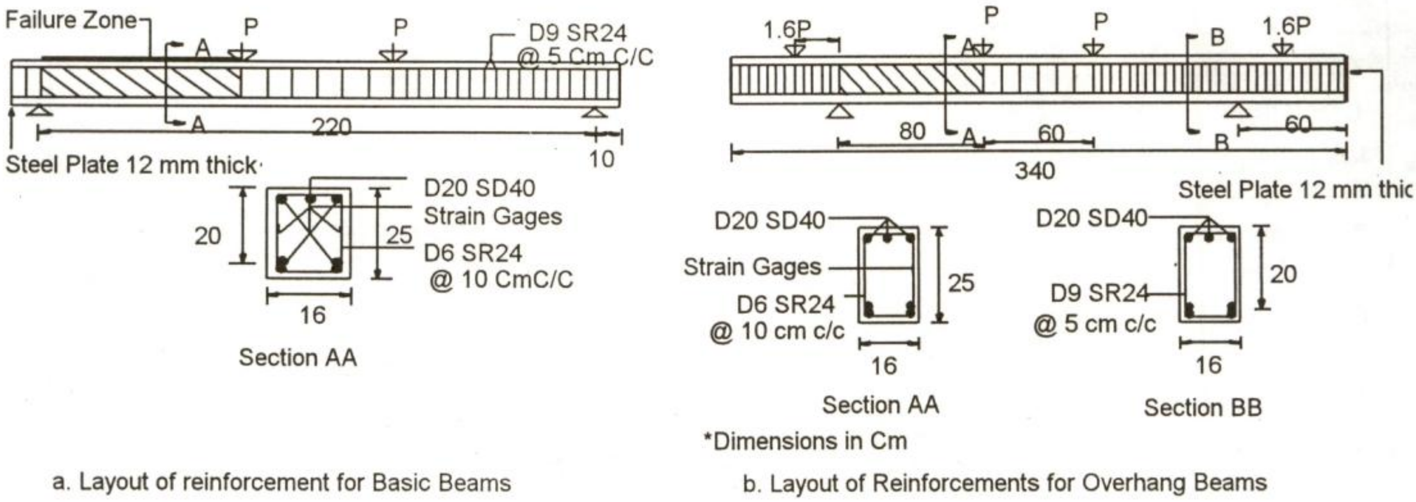


Figure 1 Beam details

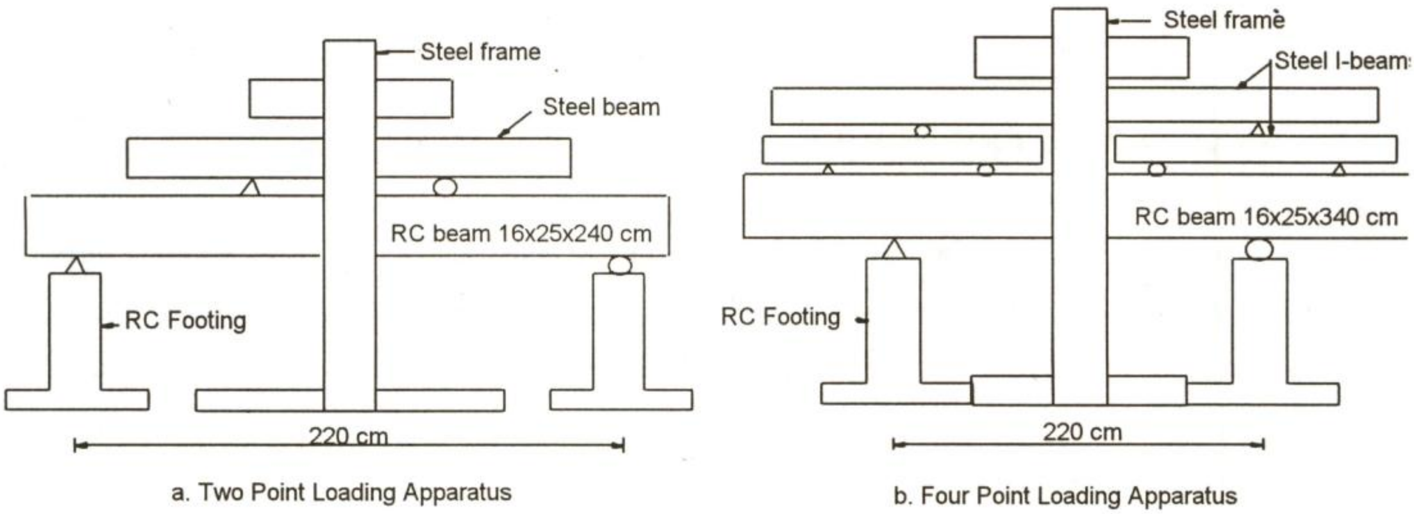


Figure 2 Loading apparatus

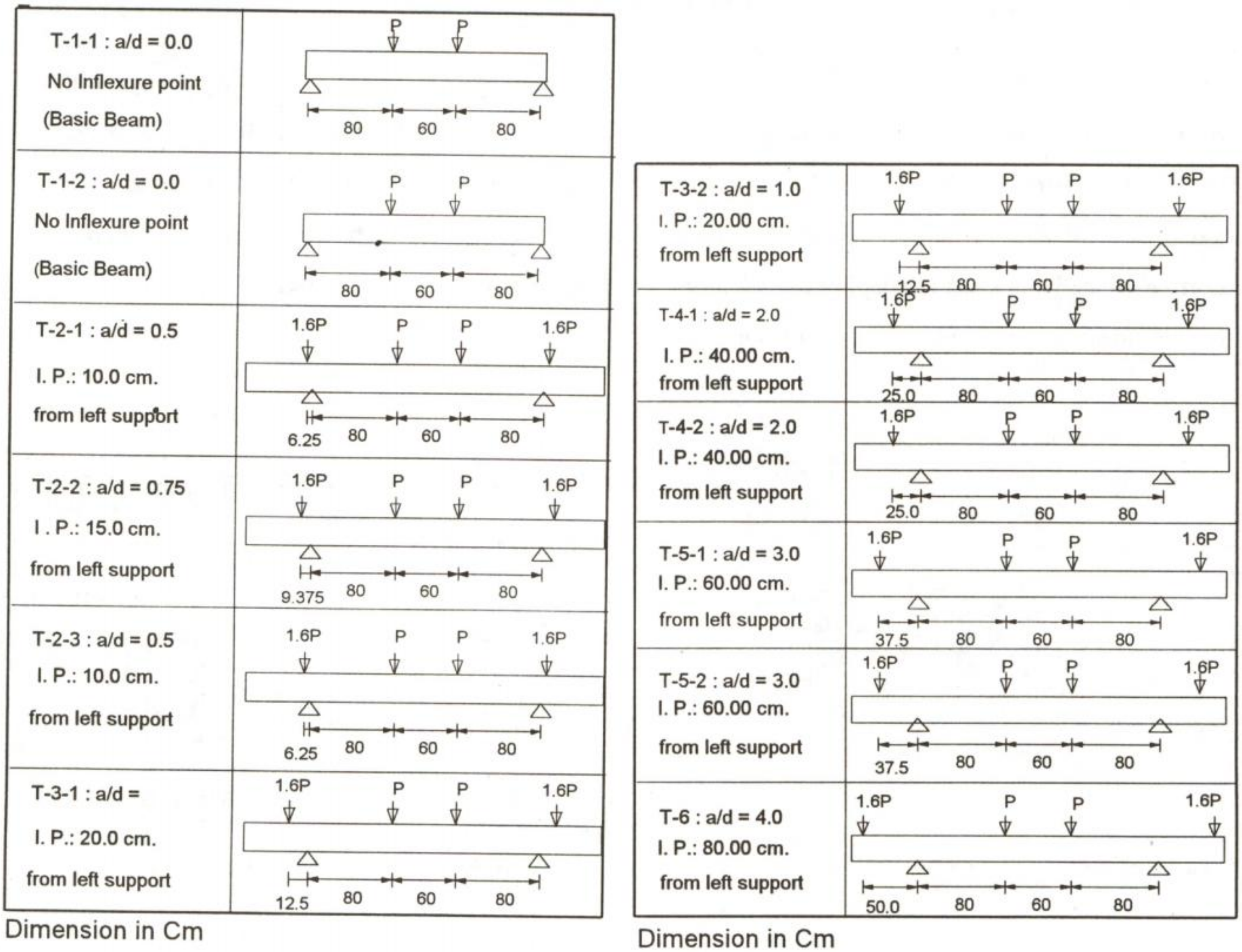


Figure 3 Test program

ANALYTICAL PROCEDURE

The present study adopts a nonlinear finite element program for two dimensional reinforced concrete structures (WCOMR) which was originally developed by Okamura and Maekawa [2], and further modified by Chaisomphob, et al. [3] in order to deal with the problem of multi-directional reinforcements. The program is based on smeared crack modeling which deals a reinforced concrete element macroscopically by expressing the average stress and stain relationshp in the element.

All the beams were discretized into a number of 4-node rectangular elements. Since the geometry of the beams and loading condition were symmetrical, only half portion was analyzed. As shown in Fig. 4, the basic beams were discretized into 48 elements and all other beams into 68 elements. A nonlinear analysis was carried out by using the load control scheme with a convergence tolerance set to be 10^{-2} for an unbalanced force. Loads were applied at nodal point in the steps of 500 kgf for the basic beams and 1 tonf for the rest of the beams up to failure.

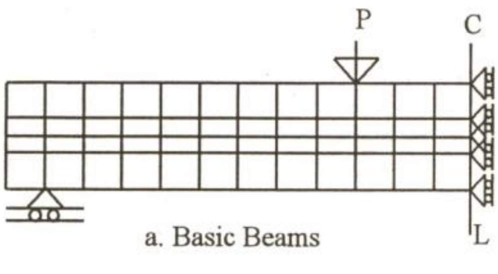
RESULTS AND DISCUSSIONS

(1) Load-deflection responses

The deflections at the mid-point of the middle span of all the beams were recorded by dial gages in the experimental program and those from analytical results were also obtained. The relations of load vs. deflection for two typical cases are shown in Fig. 5. In most of the cases analytical results closely resembled with experimental results, but in some cases analytical results were found to be stiffer than the experimental ones. This might be due to an overestimation of the initial stiffness of the beams by the present analytical procedure.

(2) Cracking patterns

All the beams failed in shear in the predicted failure region. The failure of the beam is preceded by yielding of one or more stirrups in all the cases. First a shear crack appeared in the beam at an angle varying from 40-45 degree. Then a number of cracks were generated with further increase of the load. The width of one or two major cracks gradually increased with sudden increase in the strains on the strrups crossing these cracks. In most cases either one or two major cracks led to the failure. The cracking patterns of two typical beams obtained from experimental and analytical analyses are shown in Fig. 6. It can be seen that the analytical cracking patterns were very much similar to the experimental cracking patterns.



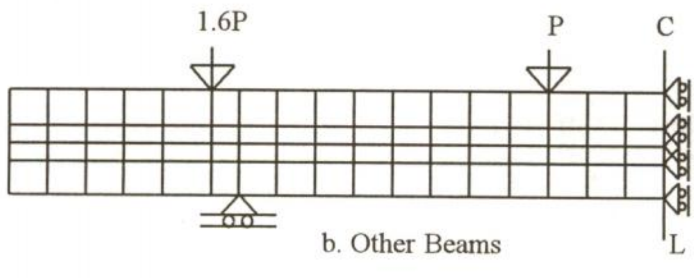
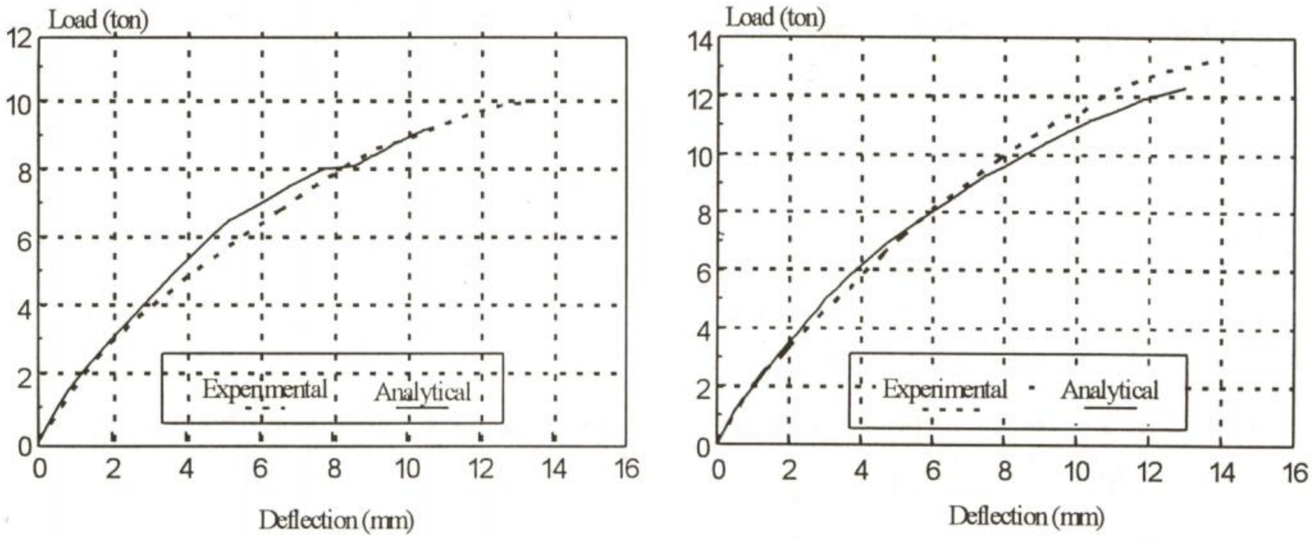


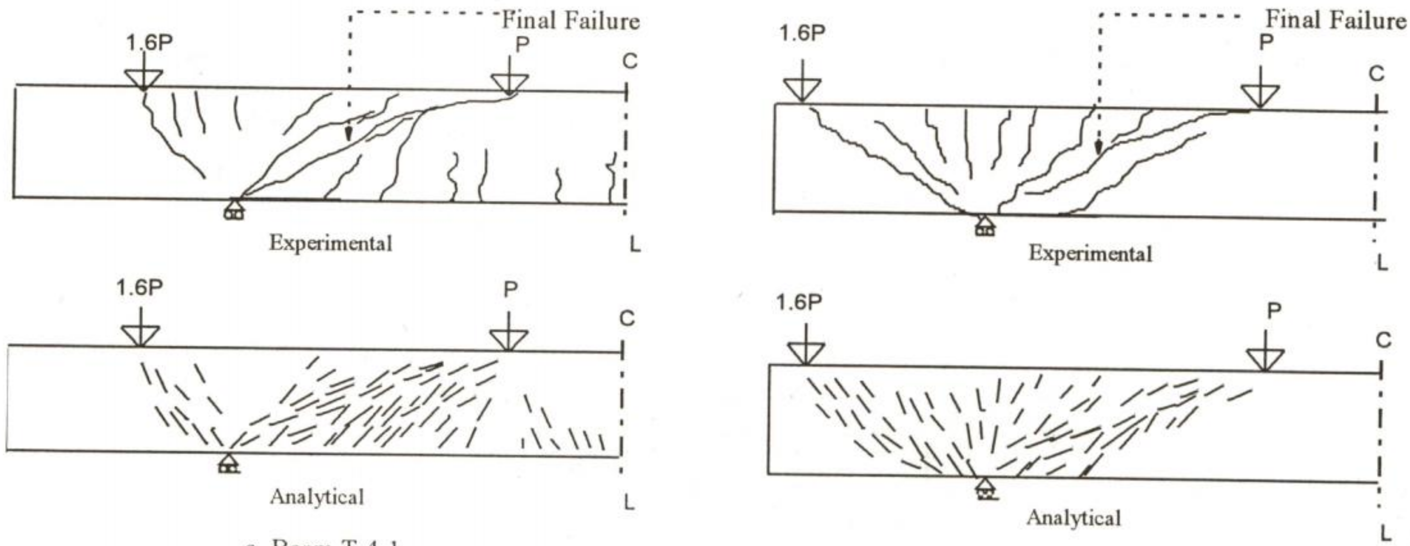
Figure 4 FEM mesh



a. Load vs. Middle deflection for Beam T-1-2

b. Load vs. Middle deflection for Beam T-2-2

Figure 5 Load-deflection relations



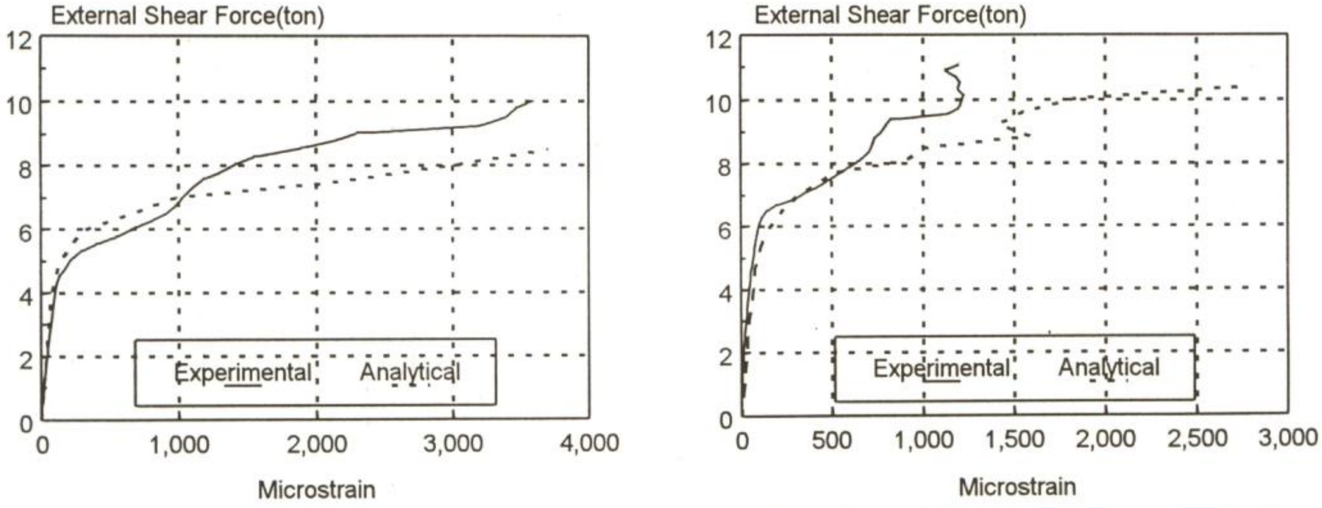
a. Beam T-4-1

b. Beam T-6

Figure 6 Cracking patterns

(3) Load-strain behaviors of stirrups

Strains were measured during the test program from each of the stirrups. Strains in the stirrups at the measured locations were also obtained from the FEM analysis. The values of strains are plotted against external shear force on the span for two typical cases in Fig. 7. It can be seen that strain remained negligible before the diagonal cracking of concrete, then the strain started to increase in the post diagonal crack region. It can be attributed to the fact that once concrete cracks, the stress in the stirrups crossing the crack starts to increase due to the formation of truss mechanism. It was also recognized that strains obtained from FEM analysis were higher than the experimental ones. The main reason seems to be the bilinear constitutive model of reinforcing bars adopted in the WCOMR program.



a. Shear Force vs. Strain in Beam T-1-1, Stirrup -3

b. Shear Force vs. Strain in Beam T-4-1, Stirrup -2

Figure 7 Load vs. strain relations

(4) Shear capacity of beams

From the experimental program and the FEM analysis, it can be observed that the beams carried additional loads after the first yielding of stirrups, and when more stirrups yielded the beams failed at the ultimate load defined by the maximum point of the load-deflection curves. Fig. 8 shows different shear capacities from the experimental program. It is noted that the average values of the loads of the beams with the same a/d are plotted in Fig. 8. It can be seen that shear capacities increase with the presence of inflexure points on the shear span of the beams.

As can be seen in Fig. 8, the increment in the shear capacity of the beams due to the presence of inflexure points on the shear span is as much as 34% (when a/d = 0.75) in comparison to the beams without inflexure points. The increment in the shear capacity can be explained by the formation of a fictitious support at the inflexure point which divides the total shear span into two, and hence the shorter shear span results in the higher ultimate load. In Fig. 8, the maximum value of the ultimate shear capacity seems to occur when a/d = 2.0 and also a/d ratio = 0.5 and 0.75. In the case of a/d = 2.0, the total shear span is divided into two equal parts by forming artificial hinge at the inflexure point and hence the beam carried the maximum load among other cases [1]. In the case of a/d = 0.5 and 0.75, the large ultimate shear capacities are induced by the local stresses around the support and the arch action since the loads were nearest to the support in those beams.

For the difference of ultimate shear capacities obtained from the FEM analysis and experimental results from Table 1, it was found that the difference was less than 12-14% (in cases of T-2-1 and T-3-2), and the average value was about 5%. It is noted that the experimental result in case of T-3-2 is less than the FEM result, and this might be because the tested beam failed not only by the shear but also by the others such as flexure, bearing at the support of the beam. From the above comparisons between experimental and analytical results, the applicability of WCOMR to simulate nonlinear behavior of the beams with stirrups is verified.

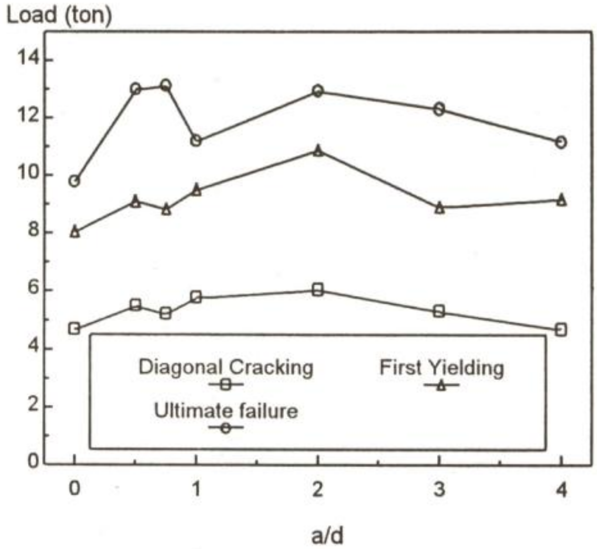


Figure 8 Various shear capacities from experiments

Table 1 Comparison of ultimate shear capacities obtained from experiments and FEM analysis

Beam No.	a/d	Experiment (ton)	FEM (ton)
T-1-1	0.00	9.80	9.00
T-1-2	0.00	9.80	9.75
T-2-1	0.50	13.40	11.74
T-2-2	0.75	13.13	11.90
T-2-3	0.50	12.64	11.70
T-3-1	1.00	12.05	11.55
T-3-2	1.00	10.37	11.84
T-4-1	2.00	13.03	11.94
T-4-2	2.00	12.86	12.85
T-5-1	3.00	12.38	12.32
T-5-2	3.00	12.29	12.18
T-6	4.00	11.20	11.50

PREDICTION OF SHEAR CAPACITY

By extending the proposed formula by Aoyagi, et al. [1] to compute the shear carried by concrete portion and using the concept of truss analogy to compute the shear taken by the shear reinforcement, the proposed formula of the ultimate shear capacity can be expressed as follows:

$$V_u = V_{uc} + V_{us} \quad (1)$$

where, V_{uc} = the ultimate shear force carried by concrete which is obtained by considering the effect of the position of inflexure point, and equal to the ultimate shear capacity of the beam without shear reinforcement obtained by the proposed formula as follows:

i) In case of shallow beams ($a/d \geq 2$), following equation is to be used:

$$V_{uc,s} = f_{vc,s} b_w d \quad (2)$$

where $V_{uc,s}$ is the shear capacity provided by shallow beam in diagonal tensile shear failure mode

$$f_{vc,s} = 0.9\,\beta_d \beta_p \beta_a f_c^{'1/3}$$

$\beta_d = (100/d)^{1/4}$ d in cm, if $\beta_d > 1.5$ then $\beta_d = 1.5$

$\beta_p = (100\rho_w)^{1/3}$, $\rho_w = A_s / (b_w d)$, $\beta_p \leq 1.5$

$\beta_a = (0.75 + \frac{1.4}{a/d})$, (for $a = a_1$ and a_2)

d : effective depth (cm)

b_w : web width (cm)

A_s : area of tensile reinforcement (cm^2)

a : equivalent shar span (cm), i.e., a_1: distance from the nearest support to inflexure point, or a_2: distance from the maximum moment point to inflexure point (Fig. 9)

$f_c^{'}$: compressive strength of concrete (ksc)

ii) In case of deep beams ($a/d < 2$), the following equation is to be used:

$$V_{uc,d} = f_{vc,d} b_w d \quad (3)$$

where, $V_{uc,d}$ is the shear capacity provided by deep beam in the shear failure mode,

$$f_{vc,d} = 0.6\,\beta_p \beta_d \beta_a \sqrt{f_c^{'}}$$

$$\beta_a = \frac{5}{1+(\frac{a}{d})^2}, \text{ (for } a = a_1 \text{ and } a_2\text{)}$$

β_d and β_p are same as above

For the calculation of V_{uc} in Eq.(1) by using Eqs. (2), (3), first the equivalent shear span a setting to be a_1 or a_2 is substituted into Eqs. (2), (3) to compute $V_{uc,s}$ and $V_{uc,d}$ respectively, then the maximum value between these two values is taken as the V_{uc} for the respective case of a_1 and a_2. Finally, the value of V_{uc} is determined to be the minimum value between the results of two cases of a_1 and a_2.

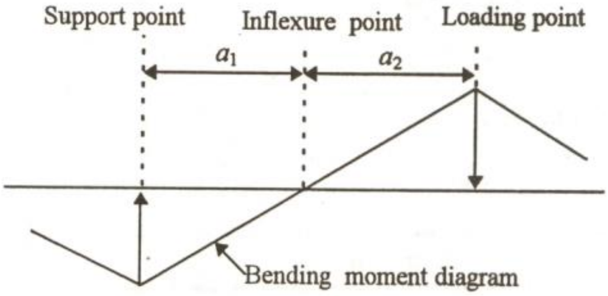


Figure 9 Definition for equivalent shear span

V_{us} = the ultimate shear force carried by stirrups which is expressed by employing the truss analogy as follows:

$$V_{us} = \frac{A_{sw} f_{wy} z}{s} (\cot\theta + \cot\alpha) \sin\alpha \quad (4)$$

where,

A_{sw} : area of the stirrup within spacing s

f_{wy} : yield strength of shear reinforcement

α : angle between stirrup and beam axis

θ : angle between concrete compression strut and beam axis in truss analogy (for the definitions of α and θ, see Ref. [5]-[7])

z = d/1.15, d : effective depth of beam

In Fig. 10, ultimate shear capacities based on ACI code [5], Australian Standard code [6], JSCE code [7] and the above proposed formula for all cases of different a/d ratios were compared with experimental and FEM results. From this comparison, it can be seen that the proposed formula gives better prediction of the ultimate shear capacities than other current design codes. In table 2, the ratios between the ultimate shear capacity obtained from experiment and that by the proposed formula are tabulated, and the mean value is about 1.34, and the standard deviation is 0.118. The difference is due to the fact that the yielding load of the shear reinforcement was taken as the ultimate load in the proposed formula, but from the experimental results shown in Fig. 8, it was found that the beams were able to sustain some extra loads even after the yieldind of the stirrups. The quantitative evaluation of this residual shear capacity needs further investigations.

Table 2 Ratio between the ultimate shear capacities obtained from experiment and by the proposed formula

Beam No.	a/d	Ratio
T-1-1	0.00	1.15
T-1-2	0.00	1.18
T-2-1	0.50	1.52
T-2-2	0.75	1.52
T-2-3	0.50	1.43
T-3-1	1.00	1.36
T-3-2	1.00	1.17
T-4-1	2.00	1.34
T-4-2	2.00	1.31
T-5-1	3.00	1.38
T-5-2	3.00	1.39
T-6	4.00	1.32

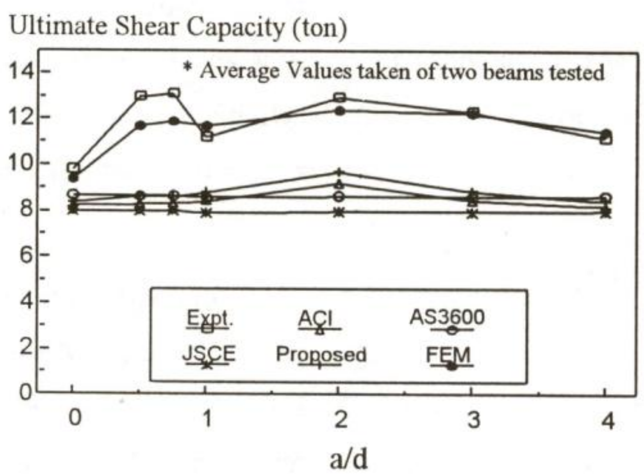


Figure 10 Comparison of ultimate shear capacity by various formula

CONCLUDING REMARKS

From the present study, following concluding remarks have been obtained.

1. The position of inflexure point has a considerable effect on the shear capacity of reinforced concrete beams with stirrups. The increment in the ultimate shear capacity with respect to the beams without inflexure points is as much as 34% when the equivalent shear span-to-depth ratio is 0.5-4.0.
2. The FEM analysis based on WCOMR gave close resemblance of the results with experimental ones, i.e. ultimate shear capacity, load-defection response and crack patterns. Hence the present analytical study can be ultimate shear capacity, load-deflection response and crack patterns. Hence the present analytical study can be used to study shear behavior of RC beams with shear reinforcements with an acceptable accuracy.
3. The proposed formula which takes the position of inflexure points on the shear span into account gives the best fit to the experimental values in computing the ultimate shear capacity of RC beams in comparison to the current codes of practices. However, more investigations on the reserved capacity after stirrup yielding are needed to improve the accuracy of the formula.

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