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AGGREGATE BLENDING DESIGN FOR HOT-MIX ASPHALT WITH RISK ANALYSIS

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ABSTRACT

Aggregates are one of the most important elements of hot-mix asphalt. Aggregate blending is a process to find the proportion of aggregates to satisfy the gradation specification. Traditionally, in Thailand aggregating blending is a trial-and-error technique. This study proposed a new method to solve for an optimal aggregate blend where the problem is formulated as a nonlinear program. The new model has an objective function to minimize the total cost of the aggregate blend. It incorporates the Bailey ratios as constraints to ensure aggregate interlock and aggregate packing of the blend. A concept of risk analysis is introduced as a constraint. This risk is quantified by the probability that the aggregate blend will violate the gradation specification. A case study is presented. It is found that the proposed model can generate an efficient frontier, a trade-off curve between the total cost of aggregate blend and the risk of violating the gradation specification. The minimum-cost aggregate blend produces the highest risk of violating the gradation specification while the minimum-risk aggregate blend produces the highest cost of the aggregate blend. The resulting grading chart indicates that the minimum-risk aggregate blend is somewhat close to the middle points of the gradation specification whereas the minimum-cost aggregate blend is close to the boundary of the gradation specification. The proposed model is helpful for engineers to weigh the trade-off between cost and risk and to select the most suitable blend with different risk perspective.

KEYWORDS: Aggregate Blending; Optimization; Risk

1. Introduction

Aggregates are one of the most important elements of hot-mix asphalt. More than 90 percent by weight of hot-mix asphalt are aggregates. Aggregate blending is a process to find the proportion of aggregates to meet the gradation specification. Good aggregate blending brings about good pavement performance and the opposite is also true. Generally, a cost of aggregate blends is the main objective while maintaining the specified gradation. In Thailand, aggregate blending is a trial-and-error process where experienced technicians are assigned to perform. The process is based on experienced guess of aggregate proportion in the blend. Then the resulting gradation is checked against the specification. If the blend does not meet the specification, aggregate proportion is adjusted and rechecked with the specification. The process is repeated until the specification is satisfied. Currently, the Bailey method for gradation selection in hot-mix asphalt (HMA) mixture design is introduced to the Thailand Department of Highways (DOH). A trial-and-error process becomes a time-consuming procedure. Therefore, this study attempts to propose a systematic method to formulate an aggregate blending problem as a mathematical program that can be solved by any standard optimization software. The proposed formulation is based on a concept of risk analysis together with the Bailey method. Some literatures related to aggregate blending are identified and reviewed as follows.

Tabucanon et al [1] presented a probabilistic programming model for blending aggregates. The objective function was the sum of the total cost of the aggregate blend and the expected penalty cost of violating the gradation specification. Gradation variability was taken into consideration in the expected penalty cost. A direct research technique coupled with a linear program was proposed to solve for optimal solutions. Lee and Olsen [2] formulated an aggregate blending problem as a preemptive goal program with chance constraints on material gradation specifications. It accounted for material gradation variances. The first priority goal was to minimize the sum of both positive and negative deviations from the average percent passing of each sieve. The second priority goal was to minimize the positive deviation from the target total cost. Third priority goal was to minimize the sum of the negative deviations from the lower bounds and the sum of the positive deviations from the upper bounds of gradation specifications. All of them were expressed as chance constraints. Easa and Can [3] presented an aggregate blending model as a quadratic program where the objective function was to minimize the sum of squared deviations from the mid-points of the gradation specification. Constraints on gradation specification, plasticity index, and fineness modulus were formulated. The total cost of the aggregate blend was treated as a constraint in the proposed quadratic program. No material gradation variability was considered in their study. Easa and Can [4] was an extension of their previous paper. The objective was still to minimize the sum of squared deviation from the gradation specification. However, the concept of chance-constrained programming was used by introducing it in the lower and upper bounds of the gradation specification constraints. Contrary to their previous study, plasticity index and fineness modulus index constraints were removed from the formulation. Trade-off between unit cost of aggregate blends and mean deviation was considered. Charnes and Cooper [5] and Charnes and Cooper [6] originally presented a concept of chance-constrained programming technique to account for stochastic elements of random variables in constraints of optimization problems under uncertainty. Most research in chance-constrained programming referred to this paper. Vavrik et al [7] and [8] outlined the Bailey method for gradation selection in HMA mixture design. It accounted for the packing characteristics of aggregates. The Bailey method systematically provides practitioners an approach to aggregate blending that ensures aggregate interlock and aggregate packing while maintaining volumetric properties of the hot-mix asphalt. The coarse aggregate is for the skeleton of the mix while the proper amount of the fine aggregate provides a packed aggregate structure. Toklu [9] used genetic algorithm to solve an aggregate blending problem. Awuah-Offei and Nasab [10] and [11] proposed a linear program for aggregate blending of HMA to minimize the total cost of aggregate blend. The constraints on gradation specification and the Bailey ratios were incorporated into the formulation. However, material gradation variance was not considered in their study. Kikuchi et al [12] formulated an aggregate blending as a fuzzy optimization problem. Constraints on cost, gradation specification, fineness modulus, plasticity index, and specific gravity of the blend were introduced. This technique required many membership functions as inputs which were difficult to calibrate in practicality. Singh and Walia [13] reviewed literature on optimization methods for aggregate blending. Most objectives were either linear or quadratic functions with constraints on gradation specification. Ramu et al [14] formulated an aggregate blending problem as a linear program with an objective to minimize total cost of aggregate blend and constraints on gradation specification. Swamy et al [15] proposed a chance-constrained program to solve an aggregate blending problem for asphaltic concrete where the gradation limits were formulated as chance constraints. These accounted for gradation variability of materials. These made gradation limits become tighter when compared to the deterministic formulation. The Bailey ratios were incorporated into their model to assure the quality of mixture performance. Simulation experiments were conducted to verify that their model produced less probability of violating gradation specification than the deterministic formulation. However, the model cannot guarantee the minimum cost of aggregate for a given probability of violating the gradation specification.

Based on the identified literature, most research focused on minimizing the total cost of aggregate blend while maintaining specification constraints. Some studies considered percent passing on each sieve of each material as deterministic. Others treated percent passing on each sieve of each material as a random variable with known mean and standard deviation. They employed chance-constrained optimization to incorporate stochastic nature of material gradation. However, none proposed to find aggregate blending with minimum risk of violating the gradation specification. In fact, aggregate blending is a two-dimension problem where cost of aggregate blend and risk of violating the gradation specification should be simultaneously examined. Therefore, this study attempts to fill this gap by introducing a concept of risk analysis into an aggregate blending problem. The subsequent section provides a detail of the proposed model followed by a case study and research conclusions.

2. Model development

This section presents a framework of the aggregate blending formulation using a risk analysis concept. The model resembles optimal portfolio analysis in security investments by Markowitz [16]. He proposed a portfolio optimization model in assisting the selection of the most efficient portfolio by analyzing the expected returns and the standard deviation of various portfolio of the given securities. This concept provides an excellent paradigm of mean-variance analysis for aggregate blending problems. The following paragraphs give a detail of model development. The first part outlines risk analysis of the aggregate blending problem. The second part provides the Bailey ratio constraints. The third part presents constraints on the gradation specification together with the risk-based optimization model for aggregate blending.

2.1 Risk of violating the gradation specification

The resulting gradation of the aggregate blend at any sieve is expressed as the sum of the product of proportion of each aggregate source and gradation of aggregate source at that specific sieve. When percent passing of materials at each sieve is stochastic, the probability that the resulting aggregate blend is within the specification limit for a given sieve is defined as:

$$P[LB_j \le \sum_{i=1}^{N} a_{ij} x_i \le UB_j] \tag{1}$$

Where LB_j is the lower limit of the specification at sieve "j"

UB_j is the upper limit of the specification at sieve "j"

a_{ij} is the percent passing of material "i" at sieve "j"

x_i is the proportion of material "i" in the aggregate blend

This study assumes that the percent passing of material "i" at sieve "j" is normally distributed with parameters $a_{ij} \sim N(\mu_{ij}, \sigma_{ij}^2)$ where μ_{ij} is the mean and σ_{ij}^2 is the variance of the percent passing of material "i" at sieve "j". Then, $\sum_{i=1}^{N} a_{ij} x_i$ is also a random variable with a normal distribution with parameters, $\sum_{i=1}^{N} a_{ij} x_i \sim N(\sum_{i=1}^{N} \mu_{ij} x_i, \sum_{i=1}^{N} \sigma_{ij}^2 x_i^2)$. The probability that the resulting aggregate blend is within the specification limit at the control sieve "j" can be expressed as:

$$P[LB_j \le \sum_{i=1}^{N} a_{ij} x_i \le UB_j] = \Phi\left[\frac{UB_j - \sum_{i=1}^{N} \mu_{ij} x_i}{(\sum_{i=1}^{N} \sigma_{ij}^2 x_i^2)^{\frac{1}{2}}}\right] - \Phi\left[\frac{LB_j - \sum_{i=1}^{N} \mu_{ij} x_i}{(\sum_{i=1}^{N} \sigma_{ij}^2 x_i^2)^{\frac{1}{2}}}\right] \tag{2}$$

Where $\Phi[..]$ is the standard normal cumulative distribution function.

If there are K control sieves in the gradation specification, the probability of violating the specification at least one sieve is expressed as:

$$1 - \prod_{j=1}^{K} P\left[LB_j \le \sum_{i=1}^{N} a_{ij} x_i \le UB_j\right] = 1 - \prod_{j=1}^{K} \left\{\Phi\left[\frac{UB_j - \sum_{i=1}^{N} \mu_{ij} x_i}{(\sum_{i=1}^{N} \sigma_{ij}^2 x_i^2)^{\frac{1}{2}}}\right] - \Phi\left[\frac{LB_j - \sum_{i=1}^{N} \mu_{ij} x_i}{(\sum_{i=1}^{N} \sigma_{ij}^2 x_i^2)^{\frac{1}{2}}}\right]\right\} \tag{3}$$

This probability is considered as a risk of violating the gradation specification. One can set a maximum allowable risk to ensure that the resulting aggregate blend will not go off the specification with a certain confidence. Equation (3) will be later included in the proposed optimization model as a constraint.

2.2 The Bailey ratio constraints

To guarantee pavement mixture performance, the Bailey ratios are also encompassed in the proposed model. The Bailey method defines Nominal Maximum Particle Size (NMPS) as one sieve larger than the first sieve that retains more than 10 percent. Half Sieve (HS), Primary Control Sieve (PCS), Secondary Control Sieve (SCS), and Tertiary Control Sieve (TCS) are dependent on the NMPS as shown in Table 1.

Table 1 Asphalt mix control sieves

	NMPS: 37.5 mm	NMPS: 25.0 mm	NMPS: 19.0 mm	NMPS: 12.5 mm	NMPS: 9.5 mm	NMPS: 4.75 mm
Half-sieve	19.0	12.5	9.5	4.75	4.75	2.36
PCS	9.5	4.75	4.75	2.36	2.36	1.18
SCS	2.36	1.18	1.18	0.60	0.60	0.30
TCS	0.60	0.30	0.30	0.15	0.15	0.075

The Bailey method defines three ratios, 1) coarse aggregate ratio (CA), 2) fine aggregate coarse fraction ratio (FA^c), and 3) fine aggregate fine fraction ratio (FA^f) with recommended ranges a shown in Table 2. The CA ratio is used to evaluate packing of the coarse portion of the aggregate gradation and to analyze the resulting void structure. As the CA ratio decreases, compaction of the fine aggregate fraction increases. The CA ratio below the suggested ranges in Table 2 could indicate a blend that may be prone to segregation. However, the CA ratio above the suggested ranges in Table 2 could make the coarse aggregate fraction become unbalanced. For the FA^c ratio, as it increases, the fine aggregate packs together tighter. However, it is preferrable to have the FA^c ratio less than 0.50. The value above 0.50 generally indicates an excessive amount of sand. The FA^f shows how the fine portion of the fine aggregate packs together. The void in the mixture will increase with a decrease in this ratio.

Table 2 Recommended Bailey Ratios

Ratio	NMPS: 37.5 mm	NMPS: 25.0 mm	NMPS: 19.0 mm	NMPS: 12.5 mm	NMPS: 9.5 mm	NMPS: 4.75 mm
CA	0.80 – 0.95	0.70 – 0.85	0.60 – 0.75	0.50 – 0.65	0.40 – 0.55	0.30 – 0.45
FA^c	0.35 – 0.50	0.35 – 0.50	0.35 – 0.50	0.35 – 0.50	0.35 – 0.50	0.35 – 0.50
FA^f	0.35 – 0.50	0.35 – 0.50	0.35 – 0.50	0.35 – 0.50	0.35 – 0.50	0.35 – 0.50

The Bailey Ratio constraints are expressed as functions of the aggregate blend proportion as follows.

$$CA_{Lower} \leq \frac{\sum_{i=1}^{N} \mu_{i,HS}x_i - \sum_{i=1}^{N} \mu_{i,PCS}x_i}{100 - \sum_{i=1}^{N} \mu_{i,HS}x_i} \leq CA_{Upper} \quad (4)$$

$$FA^{C}_{Lower} \leq \frac{\sum_{i=1}^{N} \mu_{i,SCS}x_i}{\sum_{i=1}^{N} \mu_{i,PCS}x_i} \leq FA^{C}_{Upper} \quad (5)$$

$$FA^{f}_{Lower} \leq \frac{\sum_{i=1}^{N} \mu_{i,TCS}x_i}{\sum_{i=1}^{N} \mu_{i,SCS}x_i} \leq FA^{f}_{Upper} \quad (6)$$

Where CA_{Lower} and CA_{Upper} are the lower and upper bounds of the coarse aggregate ratios in Table 2.

FA^{C}_{Lower} and FA^{C}_{Upper} are the lower and upper bounds of the fine aggregate coarse fraction ratios in Table 2.

FA^{f}_{Lower} and FA^{f}_{Upper} are the lower and upper bounds of the fine aggregate fine fraction ratios in Table 2.

$\mu_{i,HS}$ is the mean percent passing of material "i" at the Half Sieve (HS).

$\mu_{i,PCS}$ is the mean percent passing of material "i" at the Primary Control Sieve (PCS).

$\mu_{i,SCS}$ is the mean percent passing of material "i" at the Secondary Control Sieve (SCS).

$\mu_{i,TCS}$ is the mean percent passing of material "i" at the Tertiary Control Sieve (TCS).

Equations (4), (5), and (6) will be included in the optimization model as constraints to ensure the conformity with the Bailey method.

2.3 Gradation specification constraints

The resulting of the aggregate blend must be within the specification limits of the control sieves. These can be expressed as:

$$LB_j \leq \sum_{i=1}^{N} \mu_{ij}x_i \leq UB_j \; ; \; \forall j = control\ sieves \quad (7)$$

Please note that the chance constraints are not used in the gradation specification constraints. They are formulated as deterministic constraints as shown in Equation (7). However, the variability of material gradation is already accounted for in Equation (3) as the risk of violating the gradation specification. Upon assembling all relevant constraints, the resulting mathematical program of the aggregate blending problem can be formulated as follows:

$$min\ Z = \frac{1}{100} \sum_{i=1}^{N} c_i x_i \quad (8)$$

Subject to:

$$1 - \prod_{j=1}^{K} \left\{ \Phi\left[\frac{UB_j - \sum_{i=1}^{N} \mu_{ij}x_i}{(\sum_{i=1}^{N} \sigma_{ij}^2 x_i^2)^{\frac{1}{2}}}\right] - \Phi\left[\frac{LB_j - \sum_{i=1}^{N} \mu_{ij}x_i}{(\sum_{i=1}^{N} \sigma_{ij}^2 x_i^2)^{\frac{1}{2}}}\right] \right\} \leq Acceptable\ Risk\ Threshold \quad (9)$$

$$\sum_{i=1}^{N}\mu_{ij}x_i \geq LB_j;\ \forall j = 1,2,..,K \quad (10)$$

$$\sum_{i=1}^{N}\mu_{ij}x_i \leq UB_j;\ \forall j = 1,2,..,K \quad (11)$$

$$\sum_{i=1}^{N}x_i = 100 \quad (12)$$

$$CA_{Lower} \leq \frac{\sum_{i=1}^{N}\mu_{i,HS}x_i - \sum_{i=1}^{N}\mu_{i,PCS}x_i}{100-\sum_{i=1}^{N}\mu_{i,HS}x_i} \leq CA_{Upper} \quad (13)$$

$$FA_{Lower}^{C} \leq \frac{\sum_{i=1}^{N}\mu_{i,SCS}x_i}{\sum_{i=1}^{N}\mu_{i,PCS}x_i} \leq FA_{Upper}^{C} \quad (14)$$

$$FA_{Lower}^{f} \leq \frac{\sum_{i=1}^{N}\mu_{i,TCS}x_i}{\sum_{i=1}^{N}\mu_{i,SCS}x_i} \leq FA_{Upper}^{f} \quad (15)$$

$$x_i \geq 0\ ;\ \forall\ i \quad (16)$$

Equation (8) is the objective function to minimize the total cost of the aggregate blend. Equation (9) is the risk threshold constraint where the acceptable risk of violating the gradation specification threshold is specified. Equations (10) and (11) are the lower and the upper bound constraints on the gradation specification limits. Equation (12) is the sum of proportion constraint. Equations (13) to (15) are the Bailey ratio constraints to ensure the mixture performance. Equation (16) is the non-negativity constraint. Please note that if the risk of violating the gradation specification is the most critical point of concern, the probability of violating the gradation specification, as specified in Equation (3), can be used as the objective function instead of the total cost of the aggregate blend. It is a common practice at the DOH to avoid violating the gradation specification with maximum caution. The proposed formulation is then solved by standard optimization software. In this study, Microsoft Excel Solver is used to do the computation.

3. Case Study

To illustrate an application of the proposed aggregate blending model, a sample problem is presented with the real gradation data as shown in Table 3. Table 3 provides the mean and standard deviation of percent passing of each material at each sieve. There are six stockpiles of materials with costs 370, 500, 170, 220, 270, and 310 bahts per ton respectively. The aggregate blend must be obtained by mixing these materials in compliance with the DOH gradation specification in Table 4 where the lower and upper bound of the percent passing at each control sieve is specified.

Table 3 Mean and standard deviation of percent passing of each material stockpile.

mm	SIEVE SIZE	Mean of Percent Passing (Standard deviation) of Percent Passing					
		HB 1 Stockpile 1	**HB 2 Stockpile 2**	**HB 3 Stockpile 3**	**HB 4 Stockpile 4**	**HB5 Stockpile 5**	**HB6 Stockpile 6**
37.50	**1-1/2"**	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)
25.00	**1"**	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)
19.00	**3/4"**	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)
12.50	**1/2"**	95.00 (1.730)	96.00 (1.550)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)	100.00 (0.000)
9.50	**3/8"**	44.30 (4.350)	44.00 (1.350)	100.00 (0.000)	100.00 (0.140)	100.00 (0.120)	100.00 (0.050)
4.75	**#4**	2.50 (0.080)	3.80 (1.540)	99.70 (0.300)	92.80 (2.950)	93.70 (3.210)	99.20 (0.153)
2.36	**#8**	1.55 (0.150)	2.05 (0.850)	94.90 (0.360)	64.20 (4.100)	68.00 (9.100)	91.17 (0.420)
1.18	**#16**	1.33 (0.145)	1.54 (0.405)	73.20 (1.700)	40.06 (3.240)	48.30 (7.320)	75.05 (1.833)
0.60	**#30**	1.08 (0.135)	1.43 (0.285)	45.85 (2.440)	25.90 (3.100)	36.30 (6.010)	51.90 (4.490)
0.30	**#50**	0.97 (0.129)	1.33 (0.240)	19.72 (0.170)	14.60 (2.050)	24.80 (3.750)	27.30 (4.327)
0.15	**#100**	0.83 (0.118)	0.97 (0.190)	5.30 (0.500)	6.90 (1.190)	15.80 (2.190)	6.10 (1.010)
0.075	**#200**	0.54 (0.070)	0.82 (0.070)	1.18 (0.070)	2.90 (0.595)	8.90 (1.367)	0.75 (0.037)
Cost (Bahts/ton)		370.0	500.0	170.0	220.0	270.0	310.0

Table 4 The DOH gradation specification for wearing course with thickness from 40 to 70 mm

mm	SIEVE SIZE	Percent Passing for Wearing Course	
		Lower Bound	**Upper Bound**
37.50	**1-1/2"**		
25.00	**1"**		
19.00	**3/4"**	100	
12.50	**1/2"**	80	100
9.50	**3/8"**		
4.75	**#4**	44	74
2.36	**#8**	28	58
1.18	**#16**		
0.60	**#30**		
0.30	**#50**	5	21
0.15	**#100**		
0.075	**#200**	2	10

The proposed model is then applied to calculate optimal aggregate blends by varying the risk or the probability of violating the DOH gradation specification from a very small number to the value of one. The resulting proportion, cost, Bailey ratios of the final blends are summarized as shown in Table 5.

Table 5 Optimal aggregate blends with varying risk of violating the gradation specification.

RISK	Aggregate Proportion from each stockpile (%)						TOTAL COST (bahts)	Bailey Ratios		
	x_HB1	**x_HB2**	**x_HB3**	**x_HB4**	**x_HB5**	**x_HB6**		**CA**	**FAc**	**FAf**
0.00007238	16.751	16.919	-	36.373	29.957	-	307.4769	0.500	0.467	0.364
0.00010000	17.667	13.778	-	42.284	26.270	-	298.2155	0.547	0.458	0.352
0.00050000	23.365	5.892	-	44.159	26.584	-	284.8371	0.594	0.457	0.350
0.00100000	25.268	3.381	-	44.456	26.895	-	280.8165	0.608	0.457	0.350

Table 5 Optimal aggregate blends with varying risk of violating the gradation specification. (continued)

RISK	Aggregate Proportion from each stockpile (%)						TOTAL COST (bahts)	Bailey Ratios		
	x_HB1	x_HB2	x_HB3	x_HB4	x_HB5	x_HB6		CA	FAc	FAf
0.00150000	26.471	1.864	-	44.609	27.056	-	278.4538	0.615	0.457	0.350
0.01000000	26.302	-	2.423	39.247	32.028	-	274.2552	0.650	0.467	0.350
0.01500000	26.090	-	3.206	37.294	33.410	-	274.2370	0.650	0.470	0.350
0.02000000	25.939	-	3.762	35.906	34.393	-	274.2240	0.650	0.472	0.350
0.03000000	25.721	-	4.566	33.899	35.813	-	274.2053	0.650	0.475	0.350
0.04000000	25.560	-	5.161	32.415	36.864	-	274.1914	0.650	0.477	0.350
0.05000000	25.430	-	5.642	31.216	37.713	-	274.1802	0.650	0.479	0.350
0.06000000	25.319	-	6.049	30.200	38.432	-	274.1707	0.650	0.480	0.350
0.07000000	25.223	-	6.404	29.312	39.061	-	274.1624	0.650	0.482	0.350
0.08000000	25.137	-	6.723	28.518	39.622	-	274.1549	0.650	0.483	0.350
0.09000000	25.058	-	7.011	27.798	40.133	-	274.1482	0.650	0.484	0.350
0.10000000	24.986	-	7.277	27.135	40.602	-	274.1420	0.650	0.485	0.350
0.15000000	24.689	-	8.373	24.399	42.538	-	274.1164	0.650	0.489	0.350
0.20000000	24.454	-	9.242	22.232	44.073	-	274.0962	0.650	0.492	0.350

Table 5 Optimal aggregate blends with varying risk of violating the gradation specification. (continued)

RISK	Aggregate Proportion from each stockpile (%)						TOTAL COST (bahts)	Bailey Ratios		
	x_HB1	x_HB2	x_HB3	x_HB4	x_HB5	x_HB6		CA	FAc	FAf
0.25000000	24.252	-	9.986	20.374	45.388	-	274.0788	0.650	0.494	0.350
0.30000000	24.071	-	10.655	18.704	46.570	-	274.0632	0.650	0.497	0.350
0.35000000	23.902	-	11.277	17.153	47.668	-	274.0487	0.650	0.499	0.350
0.38508524	23.789	-	11.695	16.109	48.407	-	274.0389	0.650	0.500	0.350

When the risk or the probability of violating the gradation specification is smaller than 0.01, the optimal blends consist of materials from stockpiles 1, 2, 4, and 5. All blends satisfy the Bailey ratios. The minimum risk that can be achieved is 0.00007238 with the maximum total of cost of aggregate blend of 307.4769 bahts/ton. However, when the risk of violating the gradation specification is 0.01 or higher, the optimal blends consist of materials from stockpiles 1, 3, 4, 5. All blends satisfy the Bailey ratios. The minimum total cost of the aggregate blend is 274.0389 bahts/ton with the maximum risk of violating the gradation specification of 0.38508524. Risk of violating the gradation specification and total cost of the aggregate blends is plotted as a dotted line in Figure 1. The total cost of aggregate blend is decreasing with the increase of the risk of violating the gradation specification. The dotted line represents an efficient frontier of the aggregate blending problem. Any blend above the efficient frontier is an inferior solution. This is because at the same level of risk it will produce a higher cost of blend than the efficient frontier. Any blend below the efficient frontier is an infeasible solution. In this sample problem, the 0.01 risk of violating the gradation specification seems to be preferable as it appears to be the point that the total cost of aggregate blend changes slightly with the change of risk. The resulting aggregate blends with three different risk levels, 1) minimum risk, 2) 0.01 risk of violating the gradation specification, and 3) maximum risk, are plotted with the mean percent passing on the grading chart as shown in Figure 2.

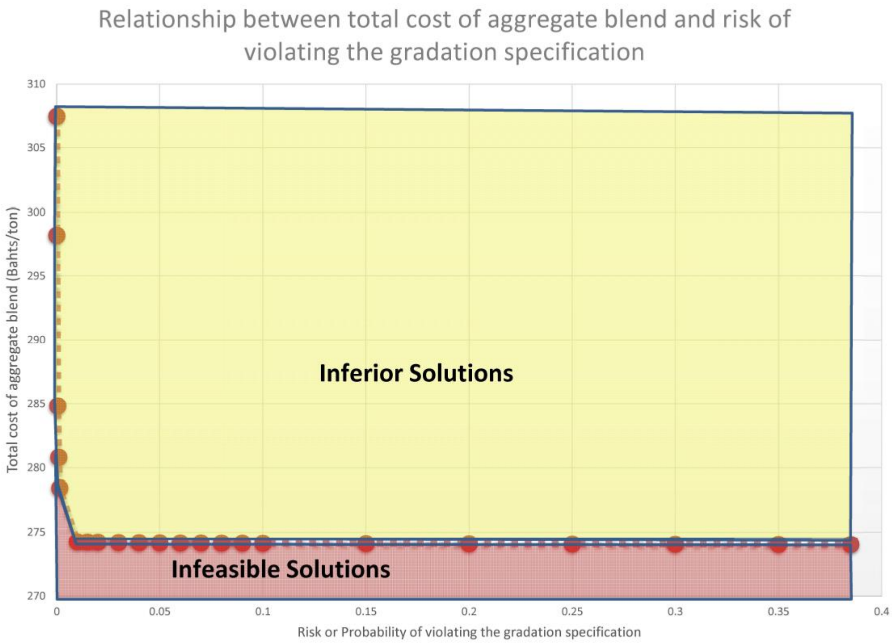

Figure 1 Relationship between total cost of aggregate blend and risk of violating the gradation specification

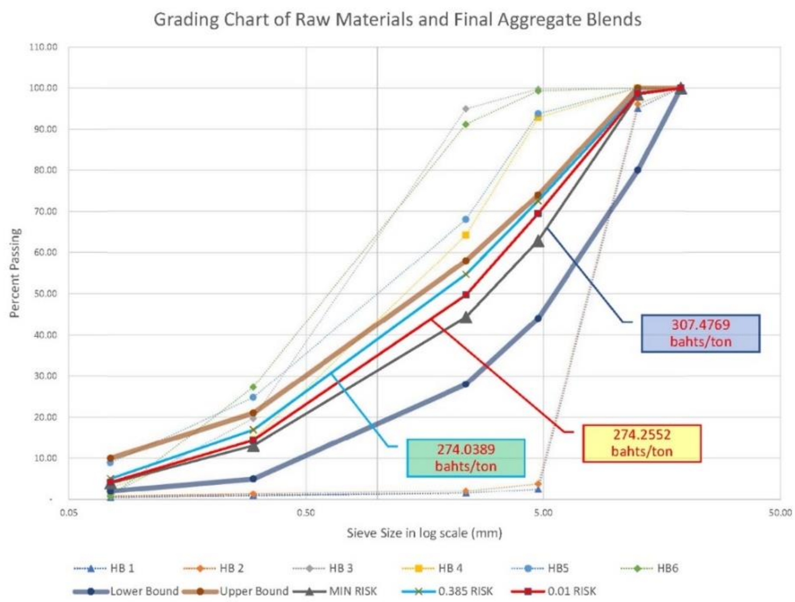

Figure 2 Grading chart of the raw materials and the final aggregate blends

The grading chart exhibits that the minimum risk aggregate blend is somewhat in the middle of the lower and the upper bounds of the gradation specification limits. The maximum risk with the minimum total cost of blend is somewhat close to the upper bound of the gradation specification while the 0.01 risk blend lies between the two scenarios. It is up to an engineer to select the most suitable blend with different risk perspective.

4. Conclusion

This study presents a new method to an aggregate blending problem for HMA using a concept of risk analysis. Risk is defined as the probability that the resulting blend is violating the gradation specification limits. The proposed model is formulated as a nonlinear program whose objective function is to minimize the total cost of the aggregate blend. There are constraints on risk, the Bailey ratios, the lower and upper bounds of the resulting blend gradation, and the sum of proportion. The Bailey ratios are enforced to ensure aggregate interlock and aggregate packing while maintaining volumetric properties of the HMA. A case study is presented. It is found that the proposed model can provide an efficient solution between the total cost of aggregate blend and the risk of violating the gradation specification. The minimum cost of aggregate blend will produce the highest risk of violating the gradation specification while the minimum risk aggregate blend will produce the highest cost of the aggregate blend. The grading chart also shows that the minimum risk aggregate blend is somewhat close the middle of the gradation specification limit whereas the minimum cost aggregate blend is somewhat close to the boundary of the gradation specification limit.

Conflict of interest

The authors declared that this article has no conflict of interest.

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