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VERIFICATIONS OF EMPIRICAL METHOD AND NUMERICAL SIMULATION USING PHYSICAL MODEL FOR SUBSIDENCE PREDICTION OF MAHA SARAKHAM FORMATION

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ABSTRACT

Physical model tests are performed to verify the representativeness and accuracy of the hyperbolic, exponential and trigonometric profile functions that have been widely used to define the ground surface subsidence under sub-critical to critical conditions, induced by potash and salt mine openings. Numerical simulations using FLAC program are also performed to confirm the correctness and reliability of the physical model test results. Synthetic gel mixed with paraffin additive is used to simulate the overburden for the physical model test. Based on the similarity theory the gel properties, and the opening models can be defined as the prototype that is equivalent to the Maha Sarakham formation. The results obtained from the physical model test agree well with those for the numerical analyses, suggesting that the laboratory test simulations are reliable and correct. The angle of draw and maximum subsidence exponentially increase with opening width. Both decrease with increasing opening depth. The hyperbolic and exponential functions overestimate the slope of the surface profile, as compared to those of the laboratory measurements for all opening depths. The trigonometric function overestimates the volume of the subsidence trough. Application of the hyperbolic function would be a conservative approach for prediction the surface subsidence size and slope for sub-critical to critical conditions.

KEYWORDS: Surface Subsidence, Physical Model, Trough, Profile Function

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1. Introduction

A variety of methods has been developed to study the surface subsidence induced by underground mines [1-3]. One of them is the empirical method which is sets of equations derived based on observations and actual field measurements which can be presented in several forms of mathematical functions. Such relations are intended to describe the profile of the surface subsidence, and hence sometimes called "profile function". Somes that are notably mentioned include hyperbolic, exponential and trigonometric functions. There functions are widely accepted for sub-critical to critical conditions because they are quick and simple to use, and yield fairly satisfactory results [4-5].

Numerical methods have also been widely employed for the subsidence analysis, primarily to predict the maximum subsidence, and the sizes and shape of the subsidence trough. The extent of subsidence area is predominantly controlled by geological conditions of the overburden strata. A variety of numerical codes have been used ranging from linearly elastic, plastic, to visco-elastic plastic models [6-10]. The main drawback of the numerical approaches is that they require representative material parameters and accurate boundary and loading conditions of the simulated domains. This means that extensive laboratory and field testing and measurements are required to obtain the accuracy input data.

Physical modelling has long been a research tool for understanding of the subsidence mechanisms [11-13]. Several modeling techniques has been developed worldwide to study the ground responses to the underground excavations. These techniques range from two-dimensional trap door tests to miniature tunnel boring machines that can simulate the process of tunnel excavation and lining installation in a centrifuge [14-16]. The primary advantage of the physical or scaled-down model test is that the boundaries and loading conditions and material properties can be well controlled, and hence provides the results that are isolated for the effects of material inhomogeneity, complex shape and boundary loadings.

Even though the profile function and numerical simulations have widely been accepted to describe the surface subsidence characteristics, verification of their predictability has rarely been attempted. To ensure that these simple methods are adequately reliable and can be representative of the actual field phenomena, comparison of their results with the laboratory test (physical) model under identical boundary conditions is needed.

The objective of this study is to verify some widely used profile functions and numerical code by using laboratory (physical) model test under identical and well-controlled parameters. The investigation is focused on the sub-critical to critical subsidence conditions. The angle of draw, maximum subsidence, maximum slope and volume of the trough are determined under a variety of opening depths and widths. Synthetic gel mixed with paraffin additive is used to simulate the overburden in the physical model. Based on the similarity theory (scale law), the mechanical and physical properties of the tested gel and the corresponding model opening width and depth can be correlated with the overburden of the Maha Sarakham formation in the northeast of Thailand. The physical model results are compared with those obtained for the profile function and numerical simulation. The similarity and discrepancies are addressed and discussed.

2. Physical Model Tests

2.1 Overburden Simulator

Synthetic gel mixed with paraffin additive under 60 $^\circ$C is used to simulate the overburden in the physical model. After cooling down for 48 hrs the gel becomes semi-solid. Figure 1 shows the elastic modulus and Poisson's ratios of the gels under various paraffin contents from 0 to 50% by weight. These parameters are obtained by performing uniaxial compression test based on the ASTM standard practice [17]. It is found that the elasticity of the gels (E_m) increases exponentially with the paraffin additive while the Poisson's ratios (ν_m) slightly increase with the paraffin content.

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In this study the elastic modulus of 5 MPa is selected, primarily because it can be appropriately correlated with the Maha Sarakham overburden. The selected value corresponds to the Poisson's ratio of 0.36 and the density of 990 kg/m³. The correlation between the properties of the test model material (synthetic gel) and the prototype (Maha Sarakham overburden) is based on the similarity theory, explained in the following section.

2.2 Apparatus and Method

A trap door apparatus [16] is used to simulate the surface subsidence and to assess the effects of the opening geometry and depth. Figure 2 shows the test parameters and variables defined in the simulations. The opening widths (W) are from 100 mm to 250 mm with an increment of 50 mm. The overburden thickness or opening depth (Z) is varied from 40 to 100 mm with the outer boundary of 0.5 m from the openings. The opening length and height are 200 and 10 mm.

Plastic blocks are used to simulate the openings by placing them on the surface of the material container (Figure 2). The synthetic gel pre-heated up to 60 °C is then filled into the container up to a pre-defined thickness. After the
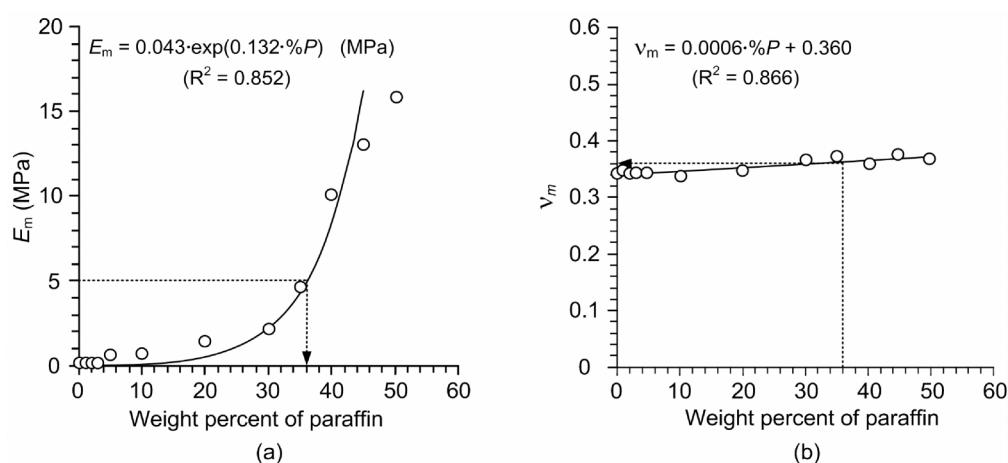

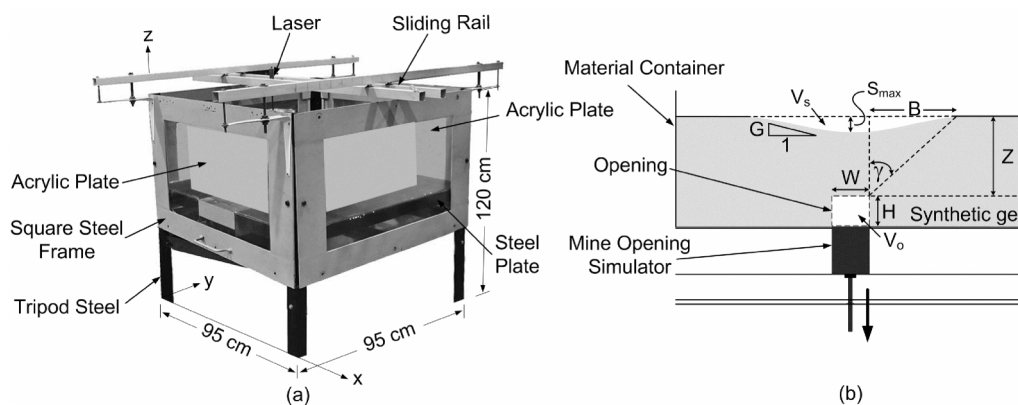**Figure 1** Elastic modulus (a) and Poisson's ratio (b) of synthetic gel as a function of paraffin additive content

Figure 2 Trap door apparatus used for physical model testing [16]

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gel is set and cooled down for 48 hrs, the underground opening is simulated by lowering the plastic blocks to below the material container (Figure 2(b)). The gel surface is measured using laser scanner sliding on x-y axes. The resolution of the measurements is 0.01 mm. The results are recorded and plotted in two-dimensional profiles. The profiles are used to calculate the angle of draw, the maximum surface subsidence, maximum slope, width and volume of subsidence trough.

2.3 Similarity Theory

According to the similarity theory (sometimes called "scale law" [18]), the elastic modulus and density of overburden, and opening dimension of the laboratory test model can be correlated with the Maha Sarakham overburden (prototype) by [13]:

$$\frac{C_E}{C_{ñ} \cdot C_L} = 1 \tag{1}$$

where C_E, C_ρ and C_L are the constants for the elastic modulus, density and dimension similarity or ratios of the prototype to the model. These constants can be calculated as:

$$C_E = \frac{E_p}{E_m}, \ \ C_{ñ} = \frac{\rho_p}{\rho_m}, \ \ C_L = \frac{L_p}{L_m} \tag{2}$$

where E_p and E_m are the elastic moduli of prototype and model, ρ_p and ρ_m are the density of prototype and model, and L_p and L_m are the dimension of prototype and model.

The Maha Sarakham overburden above salt or potash openings is initially referred to in this study for surface subsidence. The similarity constants based on the elasticity, density and dimension of the synthetic gel correlating with the Maha Sarakham formation can be calculated as: C_L = 1,000, C_ρ = 2.2 and C_E = 2,200, where the E_p and ρ_p of the Maha Sarakham formation are 11 GPa and 2,185 kg/m^3 [19-20], and the E_m and ρ_m of gel are 5 MPa and 990 kg/m^3 (Table 1). This leads to the L_p equal to 10 m (mine width or height) and L_m equal to 0.01 m (model opening width or height). Table 2 shows methods to estimate the E_p and ρ_p for the Maha Sarakham formation at Na Chueak district [21].

2.4 Results of Physical Model Test

A total of 16 model tests have been performed in the laboratory. The two-dimensional surface subsidence profile obtained from the laser scanning images is used to calculate the angle of draw (γ) and the maximum subsidence (S_{max}). The γ is a parameter used for defining the position of the limit of subsidence on the surface, which is the angle between the inflection line and the point of zero settlement at the edge of the trough. The location of S_{max} is at the center of the trough. Figure 3(a) shows γ values as a function of the opening width. The angles increase with increasing opening width. They are however not that much sensitive to the opening depth. The results suggest that γ tends to approach a certain value when the opening width increases beyond 300 mm. The maximum subsidence increases rapidly when the opening becomes wider for each depth (Figure 3(b)). The deeper openings lead to the smaller subsidence.

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Table 1 Similarity theory constants [19-20]

	Elastic modulus (MPa)	Density (kg/m^3)	Mine Opening Geometry (m)
Physical model	5	990	0.01
Prototype for Maha Sarakham overburden	11,000	2,185	10
Similarity constants	C_E = 2,200	C_ρ = 2.2	C_L = 1,000

Table 2 Estimation of average density (ρ_p) and average elasticity (E_p) of the Maha Sarakham formation as prototype for the study [21]

No. (i)	Rock units	Thickness, T_i (m)	Density, ρ_i (kg/m^3)	Elastic modulus, E_i (GPa)	ρ_p* (kg/m^3)	E_p* (GPa)
1	Middle Clastic	136	2,150	0.7		
2	Middle Salt	94	2,140	4.0	2,185	11.0
3	Lower Clastic	21	2,180	0.7		
4	Lower Salt	585	2,200	14.8		

Note: * $\rho_p = \dfrac{\sum\limits_{i=1}^{n} \rho_i T_i}{\sum\limits_{i=1}^{n} T_i}$, $E_p = \dfrac{\sum\limits_{i=1}^{n} E_i T_i}{\sum\limits_{i=1}^{n} T_i}$; i = no. of rock unit

Figure 3 Angle of draw (a) and maximum subsidence (b) as a function of the opening width

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3. Profile Functions

Singh [22] proposes several profile functions to represent the subsidence characteristics above underground openings. Hyperbolic, exponential and trigonometric functions are among of those widely used, and hence are applied here to describe the subsidence profile obtained for the laboratory test models. They are briefly defined below [22]:

hyperbolic:
$$S(x) = 0.5 \cdot S_{max} \left[1 - \tanh\left(\frac{cx}{B}\right) \right] \tag{3}$$

exponential:
$$S(x) = S_{max} \cdot \exp\left[-0.5 \frac{(x+B)^2}{B^2} \right] \tag{4}$$

trigonometric:
$$S(x) = S_{max} \cdot \sin^2\left[\left(\frac{\pi}{4}\right) \cdot \left(\frac{x}{B} - 1\right) \right] \tag{5}$$

where S_{max} is the maximum subsidence obtained from the physical model results, x is the horizontal distance from the point of inflection, c is a arbitrary constant ($c = 1.4$ for subcritical widths [22]) and B is the width of trough which can be calculated as:

$$B = Z \cdot \tan \cdot \gamma \tag{6}$$

The slope, $G(x)$, is determined by taking the first derivative of $S(x)$ in equations (3), (4) and (5) with respect to x. The three profile functions can therefore be written as:

hyperbolic:
$$\frac{\partial S(x)}{\partial x} = G(x) = 0.5 \cdot S_{max} \cdot \frac{c}{B} \cdot \operatorname{sech}^2\left(\frac{cx}{B}\right) \tag{7}$$

exponential:
$$\frac{\partial S(x)}{\partial x} = G(x) = \frac{(x+B) \cdot S_{max} \cdot \exp\left[-0.5 \frac{(x+B)^2}{B^2} \right]}{B^2} \tag{8}$$

trigonometric:
$$\frac{\partial S(x)}{\partial x} = G(x) = S_{max} \frac{\pi \cdot \cos\left(\left(\frac{\pi}{4}\right) \cdot \left(\frac{x}{B} - 1\right)\right) \cdot \sin\left(\left(\frac{\pi}{4}\right) \cdot \left(\frac{x}{B} - 1\right)\right)}{2B} \tag{9}$$

The numerical results obtained from these profile functions will be later compared with the physical model results.

4. FLAC Simulations

The finite difference program ─ FLAC [23] is used to simulate the subsidence of the physical model. This is because FLAC has been widely used in the simulations of geological and engineering structures, particularly for underground work [7-9]. To cover the entire range of the opening dimensions, over 4,000 meshes have been constructed to obtain accurate simulation results. The variables include opening depths from 40 to 100 mm and opening widths of 100, 150, 200 and 250 mm. The analyses are performed in plane strain condition. The distance between the left and right boundary edges and the center are 0.5 m. The left and right boundaries are fixed in the x-axis, and the bottom boundary is fixed in the

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y-axis. The upper boundary can move freely in both directions. The smallest mesh used around the opening is 1×2 mm^2 because the stress and strain gradients are high at this zone. The mesh far from the opening is gradually larger. Figure 4 gives an example of finite difference mesh for opening depth = 40 mm and opening width = 250 mm. The material properties are used for the synthetic gel are defined in section 2.1. To simulate the opening, the meshes inside the defined opening are deleted. The overburden is then deformed. The processing cycles of about 10,000 is used. The vertical displacement on the top surface is calculated to obtain the subsidence profile, angle of draw, maximum slope, and volume of trough. The simulated results are compared with the physical model results in the next section.

5. Comparisons

There are close agreements of the results obtained between the physical model testing and the FLAC simulations. The numerical simulations slightly underestimate the laboratory testing. The discrepancies are less than 2% for the S_{max} (Figure 5(a)), and 3% for the γ values (Figure 5(b)). This holds true for the entire ranges of the simulated opening widths and depths. The discrepancies are probably due to the sizes and number of the elements used in the mesh model. The smaller elements and greater number of the mesh would provide even closer of the numerical solution to the physical model test result. Both shows the exponential decrease of S_{max} and γ with the increase of opening depth. The decrease is more rapid for the wider openings, as compared to the narrower ones.

Figure 6 compares the subsidence profiles obtained from the three functions with those measured from the physical model test for the case of Z = 40 mm and W = 250 mm. It is obvious the three functions give different subsidence characteristics in terms of slope, curvature and trough volume even though they have the same S_{max} and trough width which are fixed in the calculation. It seems that hyperbolic function provides closest agreement with the test model.
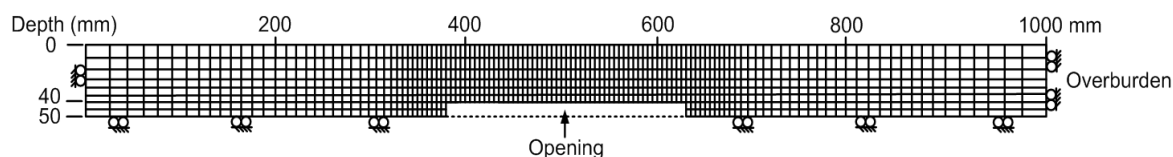

Figure 4 Example of finite difference mesh developed for FLAC simulation, for opening depth = 40 mm and opening width = 250 mm

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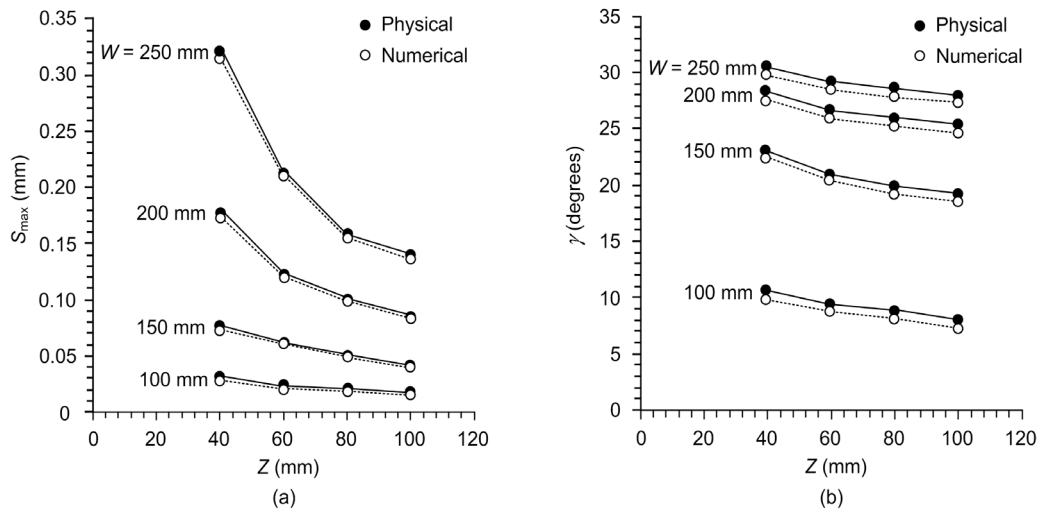
Figure 5 Maximum subsidence (a) and angle of draw (b) as a function of the opening depth

The discrepancies among the three functions and the test model are shown in Figure 7 in terms of the maximum surface slope (G_{max}) and trough-to-opening volume ratio (V_s/V_o). All solutions show the rapid decrease of the surfaces slope and troughs volume when the opening depths increase from 40 to 100 mm. This is true for all opening widths, which implies that both laboratory test results and profile solutions are correct. Nevertheless some discrepancies remain. The hyperbolic and exponential functions overestimate the surface slope for all opening widths and depths. The trigonometric function greatly underestimates the measured slope. For the trough volume prediction, the hyperbolic function gives the closest prediction to the test results.
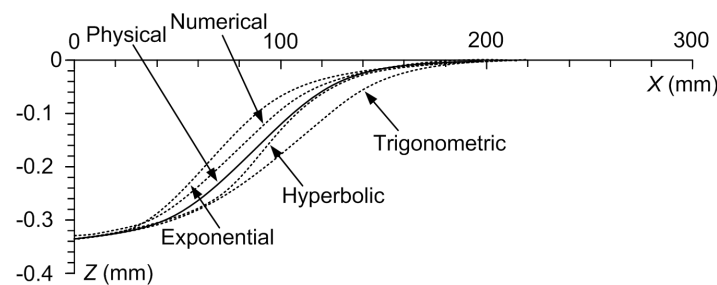

Figure 6 Comparisons of the subsidence trough predicted by different functions for opening depth = 40 mm and opening width = 250 mm

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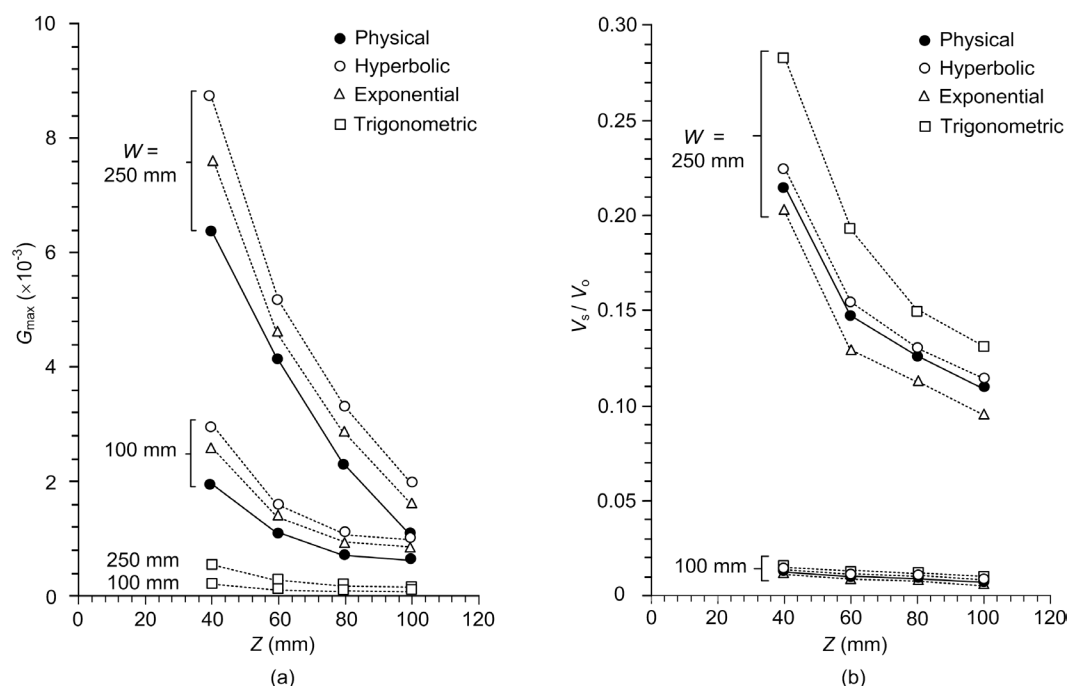
Figure 7   Maximum slope (a) and volumetric ratio of trough to excavation ratio (b) as a function of the opening depth for various prediction methods

6.   Discussions and Conclusions

The results obtained from this study meet all objectives and requirements. The conclusions can be drawn as follows:

- The close agreement between the numerical simulations and the physical model tests suggests that the concept, procedure and results in the physical model are appropriate and correct. Both methods indicate that S_{max} and γ decrease with increasing opening depths. Such reduction is more rapid, particularly for wide openings as compared to the narrower one.

- It is interesting to note that the angles of draw (γ) are less sensitive to the opening depth than to the opening width (see Figures 3 and 5). The maximum subsidence magnitudes however are highly sensitive to both opening depth and width.

- A new finding is obtained in terms of the accuracy and representativeness of the commonly used profile functions for the subsidence predictions under sub–critical to critical conditions. The exponential and hyperbolic functions provide similar $G_{max} - Z$ and $V_s/V_o - Z$ relations with the laboratory measurements. They however tend to overestimate the slope of the test model by about 20–40% while the trigonometric function greatly underestimates the results by more than 90%.

- The largest subsidence trough volume is predicted by the trigonometric function while the smallest by the exponential function. The hyperbolic function gives the trough volume closest to the laboratory results of the physical model with the deviation of less than 5% (Figure 6).

- A conservative approach for the actual subsidence profile prediction under sub–critical to critical conditions would be obtained by applying the hyperbolic function for both maximum slope and trough volume predictions.

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- • Applications of the results obtained here to other different sequences and thickness of the Maha Sarakham formation are possible by using the scale law (section 2.3). Different opening widths can also be incorporated to obtain the desired subsidence profiles.

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