



Engineering structures (e.g. underground excavation, dam foundation, rock slope, nuclear waste repository) are usually located in complex stress states, which are affected by several factors, such as different loading paths, lithology characters, anisotropy, environment, etc. Since Jaeger pointed out “the possibility of rock failure dependent to the loading path is worth of an argumentative problem” in 1967 [1], the effect of loading paths on mechanical behavior of rocks has been investigated during the past decades to understand and explore the fracture mechanism under different loading paths [2].

The mechanical behavior of rock is commonly analyzed either by the principle of elasticity or inelasticity. In most cases, the principle of elasticity is often used or assumed for the stress-strain analysis, because it is convenient for practical applications. Under this assumption the effect of the different loading directions and sequences (stress path) is ignored [3]. For some rocks, the assumption of inelastic behavior is more suitable than linearly elastic, especially for soft rock. The mechanical behavior of soft rock is important for the analysis and design of geological engineering structures, such as nuclear waste repository, storage caverns and underground salt mines. Chen and Zhang [3] and Shames and Cozzarelli [4] suggest that the analysis of materials in the plastic range is dependent of stress path. The influence of stress path or loading sequence and direction should therefore be considered to realistically analyze the engineering structures in soft formation.

2. Rock Samples

Surachai Komenthammasopon¹ and Kittitep Fuenkajorn²

The biaxial rock testing device is developed in order to determine the biaxial compressive strengths of the rock specimens with soft to medium strengths. This device comprises two steel cross load frames, four hydraulic load cells and two hand pumps (Figures 2).

The neoprene sheets are attached to the specimen surfaces in order to minimize the friction at the interfaces between the loading platens and the rock surfaces. The loading is exerted in a biaxial manner, providing intermediate principal stress (σ_2), maximum principal stress (σ_1) and maintaining the minimum stress equals to zero ($\sigma_3=0$). The steel platens and the three dial gauges for displacement measurements are arranged and adjusted in order to obtain the precision readings. The specimens are placed at the center of the crossed frameworks.

The biaxial compression tests are performed for three different stress paths as described before, in order to determine the compressive strength and deformation of the sandstone specimens under biaxial stress states. The measured deformation of sandstone specimens are used to determine the strains along the principal axes during loading. For the first stress path, the intermediate stress (σ_2) is varied from 0 to 70 MPa. For the other two stress paths, the mean stress (σ_m) ranges from 10 to 45 MPa. The calculations of the Poisson's ratios and tangent elastic moduli are made at 50% of the failure stress.

A 3D CAD model of a mechanical assembly. It features two vertical frames, labeled 'Frame A' and 'Frame B', which are connected by two horizontal rollers. The rollers are mounted on a central shaft. The frames are made of a dark material, and the rollers are yellow. The assembly is shown from a perspective view.

Figure 2 Two steel cross load frames (frame A and B), the main parts of the biaxial rock testing device



Figure 10 is a line graph showing the relationship between Failure stress (σ_1) on the y-axis and Intermediate principal stress (σ_2) on the x-axis. The y-axis ranges from 0 to 100 in increments of 20. The x-axis ranges from 0 to 80 in increments of 20. There are two sets of curves, labeled (I) and (II), each representing different specimens: PP (open circles), PW (open squares), and PK (open triangles). Curve (I) is generally higher than curve (II). Both curves show a peak failure stress at intermediate principal stress values between 20 and 40 MPa.

Intermediate principal stress (σ_2)	Failure stress (σ_1) - Curve (I) PP	Failure stress (σ_1) - Curve (I) PW	Failure stress (σ_1) - Curve (I) PK	Failure stress (σ_1) - Curve (II) PP	Failure stress (σ_1) - Curve (II) PW	Failure stress (σ_1) - Curve (II) PK
10	72	55	52	48	40	35
20	82	68	65	55	45	42
30	85	78	70	55	45	40
40	80	78	70	50	40	35
50	75	75	65	45	35	30
60	70	70	60	40	30	25
70	65	65	55	35	25	20

Among the three stress paths applied on different sandstone specimens, stress path (II) generally provides the highest strengths, based on the mean stress (σ_m) (Tables 1 and 2). However, PW sandstone yields slight variation of σ_m from the maximum under stress path (II), as shown in Table 3. The tables also provide the octahedral shear strength (τ_{oct}), mean stress (σ_m), and the second order of the stress deviation ($J_2^{1/2}$) at failure. These parameters can be calculated from the principal stresses at failure. This discrepancy of the test results may be partly derived from the intrinsic characteristics of sandstone. Sedimentary rocks have bedding planes and may show some transversely isotropic properties instead of isotropic in some specimens. Normally, the elastic modulus in the direction parallel to the bedding planes is greater than that normal to the bedding, thus providing directional higher strength. For the elastic parameters, the calculated elastic moduli from the tests seem to agree with the strength results while the Poisson's ratios of different rock specimens do not show any significant variation under each stress path. The calculation of the elastic moduli is made for different rock specimens under each stress path. The calculated results are of similar trend: the highest value under stress path (II), the lowest under stress path (III) and the value in between under stress path (I) (Tables 1 through 3). For example, the mean elastic moduli of PP sandstone specimens are 9.43 GPa, 9.82 GPa and 9.16 GPa under the stress path (I), (II) and (III), as shown in Table 1. It is obvious that the intermediate principal stress (σ_n) remarkably affects the strengths of rock specimens.

5. Strength Criteria

5.1 Modified Wiebols and Cook Criterion

The modified Wiebols and Cook criterion proposed by Zhou [6] is used here to describe rock strengths as affected by stress path. The criterion is originally developed by Wiebols and Cook [7] based on the additional energy stored around Griffith cracks due to the sliding of crack surfaces over each other. The modified version defined $J_{2\frac{1}{2}}$ at failure in terms of J_1 as:

$$J_2^{1/2} = A + BJ_1 + CJ_1^2 \quad (1)$$

The constant J_1 is in terms of σ_1 , σ_2 and σ_3 as defined by:

$$J_1 = (1/3) \times (\sigma_1 + \sigma_2 + \sigma_3) \quad (2)$$

The constant A , B and C depend on rock material and the minimum principal stresses (σ_3) (Table 4). They can be determined under the conditions where $\sigma_3 = 0$, as follows [8]:

$$C = [27^{1/2} / (2C_1 + (q-1)\sigma_3 - C_0) \times \{[(C_1 + (q-1)\sigma_3 - C_0)/(2C_1 + (2q+1)\sigma - C_0)] - [(q-1)/(q+2)]\}] \quad (3)$$

where $C_1 = (1 + 0.6\mu) C_0$, C_0 = uniaxial compressive strength of the rock, $\mu_1 = \tan\phi$, $q = [(\mu^2 + 1)^{1/2} + \mu]^2 = \tan^2(\pi/4 + \phi/2)$, μ = coefficient of internal friction of the material, ϕ = angle of internal friction. The friction angles for different sandstone specimens on each condition, biaxial stress: $\phi = 50^\circ$ for PP, $\phi = 46^\circ$ for PK, $\phi = 58^\circ$ for PW; stress path: $\phi = 35^\circ$ for PP, $\phi = 35^\circ$ for PK, $\phi = 33^\circ$ for PW, and

$$A = C_0/3^{1/2} - BC_0/3 - CC_0^2/9 \quad (4)$$

$$B = 3^{1/2} (q-1)/(q+2) - C/3(2C_0 + (q+2)\sigma_3) \quad (5)$$

Table 1 Compressive Strengths and Elastic Parameters of PP Sandstone Specimens

Specimen Number	Stress path	Failure Stresses		σ_m (MPa)	τ_{oct} (MPa)	$J_2^{1/2}$ (MPa)	Elastic Modulus (GPa)	Poisson's Ratio (ν)
		σ_2 (MPa)	σ_1 (MPa)					
PP-Bai-01	I	0.00	29.46	9.82	13.89	17.01	9.51	0.21
PP-Bai-02		10.00	72.00	27.33	31.85	39.00	9.32	0.22
PP-Bai-03		30.00	83.03	37.68	34.33	42.04	9.30	0.25
PP-Bai-04		50.00	77.67	42.56	32.14	39.37	9.61	0.18
Mean $\pm$ SD							9.43 $\pm$ 0.15	0.21 $\pm$ 0.03
PP-Bai-05	II	66.96	66.96	44.64	31.57	38.66	9.82	0.19
PP-path-06	III	0.00	49.98	16.66	23.56	28.86	8.74	0.25
PP-path-07		5.50	55.53	20.34	24.98	30.60	9.73	0.21
PP-path-08		44.40	61.09	35.16	25.78	31.58	8.31	0.22
PP-path-09		16.66	64.64	27.10	27.40	33.56	9.82	0.23
PP-path-10		33.32	65.25	32.86	26.64	32.63	9.20	0.20
Mean $\pm$ SD							9.16 $\pm$ 0.64	0.22 $\pm$ 0.02

Specimen Number	Stress path	Failure Stresses		σ_m (MPa)	τ_{oct} (MPa)	$J_2^{1/2}$ (MPa)	Elastic Modulus (GPa)	Poisson's Ratio (ν)
		σ_2 (MPa)	σ_1 (MPa)					
PK-Bai-01	I	0.00	48.00	16.00	22.63	27.71	7.82	0.21
PK-Bai-02		20.00	69.00	29.67	28.99	35.50	8.12	0.22
PK-Bai-03		30.00	78.00	36.00	32.12	39.34	7.98	0.20
PK-Bai-04		40.00	78.00	39.33	31.85	39.00	8.54	0.23
PK-Bai-05		50.00	76.00	42.00	31.54	38.63	8.11	0.22
Mean $\pm$ SD							8.11 $\pm$ 0.27	0.22 $\pm$ 0.01
PK-Bai-06	II	68.00	68.00	45.33	32.06	39.26	8.23	0.21
PK-path-07	III	0.00	38.80	12.93	18.29	22.40	8.18	0.23
PK-path-08		2.27	47.20	16.49	21.74	26.62	8.32	0.24
PK-path-09		5.00	53.51	19.50	24.13	29.56	7.22	0.22
PK-path-10		13.88	63.86	25.91	27.42	33.59	7.44	0.21
PK-path-11		55.33	65.44	40.26	28.76	35.23	7.86	0.23
PK-path-12		22.21	66.64	29.62	27.71	33.93	8.23	0.22
PK-path-13		40.23	70.33	36.85	28.81	35.29	7.55	0.21
Mean $\pm$ SD							7.83 $\pm$ 0.43	0.22 $\pm$ 0.01

Specimen Number	Stress path	Failure Stresses		σ_m (MPa)	τ_{oct} (MPa)	$J_2^{1/2}$ (MPa)	Elastic Modulus (GPa)	Poisson's Ratio (ν)
		σ_2 (MPa)	σ_1 (MPa)					
PW-Bai-01	I	0.00	39.00	13.00	18.38	22.52	8.30	0.23
PW-Bai-02		10.00	49.00	19.67	21.14	25.89	7.90	0.22
PW-Bai-03		21.00	55.00	25.33	22.66	27.75	8.43	0.20
PW-Bai-04		31.00	55.00	28.67	22.51	27.57	7.32	0.21
PW-Bai-05		40.00	48.97	29.66	21.29	26.07	9.43	0.25
Mean $\pm$ SD							8.28 $\pm$ 0.77	0.22 $\pm$ 0.02
PW-Bai-06	II	42.00	42.00	28.00	19.80	24.25	8.77	0.19
PW-path-07	III	0.00	24.00	8.00	11.31	13.86	8.45	0.21
PW-path-08		5.50	36.00	13.83	15.83	19.39	8.67	0.23
PW-path-09		8.30	41.00	16.43	17.70	21.68	8.37	0.22
PW-path-10		13.00	46.50	19.83	19.59	23.99	7.98	0.23
PW-path-11		20.00	46.00	22.00	18.83	23.07	7.93	0.22
PW-path-12		30.00	45.00	25.00	18.71	22.91	8.11	0.24
Mean $\pm$ SD							8.25 $\pm$ 0.29	0.23 $\pm$ 0.01

Rock Types	Calibrated Parameters								R ²
	Stress paths (I) and (II)				Stress path (III)				
	<i>A</i> (MPa)	<i>B</i>	<i>C</i> (MPa ⁻¹)	ϕ (°)	<i>A</i> (MPa)	<i>B</i>	<i>C</i> (MPa ⁻¹)	ϕ (°)	
PPSS	-1.62	2.44	-0.034	50	5.50	1.96	-0.034	35	0.95
PKSS	10.38	1.31	-0.015	46	3.92	1.78	-0.025	35	0.96
PWSS	6.74	1.64	-0.033	58	0.03	2.08	-0.046	33	0.93

$$J_2^{1/2} = [1/6((\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2)]^{1/2} = (3/2)^{1/2} \tau_{oct} \quad (6)$$
$$\tau_{opt} = 1/3 [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \quad (7)$$

$$\sigma_m = (\sigma_1 + \sigma_a + \sigma_2)/3 \quad (8)$$

Figure 10 consists of three subplots, each showing the relationship between the square root of the second invariant of the deviatoric stress, $J_2^{1/2}$ (MPa), on the y-axis, and the mean stress, σ_m (MPa), on the x-axis. The subplots are for PPSS, PKSS, and PWSS, respectively. Each plot includes experimental data points for three different stress paths (I, II, and III) and a fitted curve for the modified Wiebols and Cook criterion. The R-squared values for the fits are 0.95 for PPSS, 0.96 for PKSS, and 0.93 for PWSS.

Material	Stress Path	σ_m (MPa)	$J_2^{1/2}$ (MPa)	
PPSS	I and II	18	30	
		28	40	
		38	43	
	III	18	29	
		28	35	
		38	33	
	PKSS	I and II	15	23
			25	35
			35	40
III		15	22	
		25	34	
		35	36	
PWSS		I and II	10	15
			20	28
			28	29
	III	10	14	
		20	25	
		28	22	

The empirical Mogi criterion used here defines $\tau_{oct,f}$ in terms of the effective mean stress ($\sigma_{m,2}$) using a power equation relation [10]:

$$\tau_{act} = A' (\sigma_{m_2})^{B'} \quad (9)$$

$$\sigma_{m_2} = (\sigma_1 + \sigma_2)/2 \quad (10)$$

Surachai Komenthammasopon¹ and Kittitep Fuenkajorn²

The modified Wiebols and Cook and the empirical Mogi can predict the biaxial compressive strengths well (Figures 5 and 6). The former criterion provides regression analysis $r^2 = 0.95$, 0.96 and 0.93 while the latter yields 0.92, 0.93 and 0.95 for PP, PK and PW.

Tables 1 through 3 suggest that the intermediate principal stress (σ_2) affects the biaxial strength or the maximum principal stress (σ_1). The σ_1 reaches its maximum at an increased value of σ_2 , and then becomes lower with higher σ_2 . Figure 4 clearly suggests that the intermediate principal stress notably affect the rock strengths. Due to the influence of σ_2 on failure stress, this Figure also shows that the compressive strengths of the specimens of each rock type under constant σ_3 is higher than the ones with reduced σ_3 . Such effect is not linear.

It is postulated that the effects of the intermediate principal stress σ_2 are caused by two mechanisms working simultaneously but having opposite effects on the rock strengths: 1) the mechanism that strengthens the rock matrix in the direction normal to $\sigma_1 - \sigma_3$ plane, and 2) the mechanism that induces tensile strains in the directions of σ_1 and σ_3 . The intermediate principal stress can strengthen the rock matrix on the plane normal to its direction, and hence a higher differential stress is required to induce failure. This is the same effect obtained when applying a confining pressure to a cylindrical specimen in the conventional triaxial compression testing. Considering this effect alone, the higher the magnitude of σ_2 applied, the higher σ_1 is required to fail the specimen. However, the relationship between σ_2 magnitudes and the degree of strengthening is non-linear. This relation should depend on rock types, their textures and compositions (e.g. grain sizes, distribution of grain sizes, cementation, pore spaces, fissures and micro-cracks, types of rock forming minerals).

Rock Types	Calibrated Parameters		R ²
	Stress paths (I) and (II)	Stress path (III)	
PPSS	$A' = 1.27 \text{ MPa}$, $B' = 0.89$	$A' = 4.63 \text{ MPa}$, $B' = 0.50$	0.92
PKSS	$A' = 0.44 \text{ MPa}$, $B' = 1.27$	$A' = 1.83 \text{ MPa}$, $B' = 0.77$	0.93
PWSS	$A' = 3.49 \text{ MPa}$, $B' = 0.56$	$A' = 1.53 \text{ MPa}$, $B' = 0.80$	0.95



This study is funded by Suranaree University of Technology and by the Higher Education Promotion and National



References

- [1] Jaeger J. C. Brittle fracture of rocks. Proceedings of Eighth Symposium on Rock Mechanics, Port City, Baltimore, 1967, 3–57.
- [2] Cai, M. (2008). Influence of stress path on tunnel excavation response—numerical tool selection and modeling strategy. *Tunnelling and Underground Space Technology* 23, 618–628.
- [3] Chen W. F. and Zhang H. Structural Plasticity, Theory, Problems and CAE Software. New York–Berlin, Springer, 1991.
- [4] Shames I. H. and Cozzarelli F. A. Elastic and Inelastic Stress Analysis. Philadelphia, Taylor & Francis, 1997.
- [5] Boonsener M. and Sonpirom K. Correlation of tertiary rocks in northeast, Thailand. International Conference on Stratigraphy and Tectonic Evolution of Southeast Asia and the South Pacific, Bangkok, Thailand, 19–24 August, 1997.
- [6] Zhou, S. A. (1994). Program to mode the initial shape and extent of borehole break out. *Computers and Geosciences* 20 (7–8), 1143–60.
- [7] Wiebols, G. A. & Cook, N. G. W. (1968). An energy criterion for the strength of rock in polyaxial compression. *International Journal of Rock Mechanics and Mining Sciences* 5, 529–549.
- [8] Colmenares L. B. & Zoback M. D. (2002). A statistical evaluation of intact rock failure criteria constrained by polyaxial test data for five different rocks. *International Journal of Rock Mechanics and Mining Sciences* 39, 695–729.
- [9] Jaeger J. C., Cook N. G. W. and Zimmerman R. W. (ed.) Fundamentals of Rock Mechanics, 4th edn. Chapman and Hall, London, 2007.
- [10] You, M. (2008). True-triaxial strength criteria for rock. *International Journal of Rock Mechanics and Mining Sciences* 46, 115–127.