

Formation Control of Mobile Robots: Survey

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Abstract

Robot formation control has drawn significant attention for many years, and now it is well understood and mature in its field. The goal of this survey is twofold. First, we update the current literature of formation control for ground mobile robots. Second, we classify publications on formation control by using a problem-oriented criterion. This classification includes formation shape generation, formation reconfiguration and selection, formation tracking, and role assignment in formation. We further categorize the common combination of the solutions into the following three components: system architectures, robot models, and formation control strategies. We also conclude by identifying some future research topics in formation control: further steps towards real-world applications, hybrid control frameworks, and networked systems and information consensus.

Keywords: Mobile robots, formation control, multi-robot systems

1. Introduction

Formation control has been one of the most important issues in Multi-Robot Systems (MRS) for decades. It is defined as a coordination of a group of robots to get into and to maintain a formation with a certain shape. The motivations that draw much attention of researchers to this problem can be summarized as follows:

(a) *Biological inspirations*: researchers have observed the remarkable group-level

characteristics that are exhibited as emergent properties from individual-level behaviors, such as flocking and schooling.

- (b) *Challenging control problems*: the design of control schemes for decentralized systems presents a number of challenges not present in single robots or centralized systems, such as complex interactions, inherent parallelism, high system dimensionality, incomplete information, and uncertainties.
- (c) *Demands of multi-robot systems*: a given task may be too complex to be achieved by a single robot working alone, or a given task cannot be physically executable by a single robot at all, or multiple robots can achieve the same task of a single robot while reducing execution time and increasing performance.

Previous reviews on MRS (e.g., [1-3]) have taken a broad view while a previous survey on formation control was given by Chen and Wang [4] in 2005. Different from these MRS reviews [1-3], this article has a narrower span and limits itself to the recent literature on formation control of ground mobile robots. Compared to [4], we give a broader view of formation control by using a problem-oriented criterion. We divide formation control problems into four groups, i.e., formation shape generation, formation reconfiguration and selection, formation tracking, and role assignment in formation. The advantages of using this criterion are as follows. First, we can measure progress and delineate unresolved difficulties on each formation control

subproblem. This may encourage researchers to contribute more to the subproblem that shows less progress. Second, most publications on formation control classify formation control approaches into three dominating strategies, i.e., leader-following, virtual-structure, and behavior-based. However, using this classification may not be suitable because many novel solutions cannot fit into them. Thus, using our problem-oriented criterion can give a wider and clearer view of formation control problems.

Furthermore, to solve these four formation control problems mentioned above, we present three common components: (i) system architectures, i.e., what infrastructure is behind formation achievement, (ii) robot models that describe systems' nature and behavior, and (iii) formation control strategies, i.e., how a group of robots can be controlled to get into and to maintain a desired formation. These three components are described in Section 2, Section 3, and Section 4, respectively. We also suggest some future directions for research in Section 6 and finally, we close our review with some conclusions in Section 7.

2. System Architectures

The system architectures provide the infrastructure upon which formation control is implemented. They furthermore determine the capabilities and limitations of the system [1], [3], [5].

2.1 Heterogeneity vs. Homogeneity

Homogeneous teams are composed of team members that have exactly the same hardware and control software, while in heterogeneous teams the robots differ either in the hardware or in the control software.

Using homogeneous robots makes the system robust because no single robot is critical to the mission, while using heterogeneous robots in formation tasks may be necessary in some

applications. For instance, the formation can involve different kinds of robots equipped with different sensors. Only few robots may possess all the sensors and thus can serve as the leader of the whole team, providing higher level information, such as mapping or exploration.

In addition, robots can be considered to be anonymous, i.e., they are not distinguishable by their appearance, and they do not have any kind of identifiers that can be used during the operation. In this case, the number of robots participating in formation control tasks can change dynamically.

2.2 Communication Structures

Communication configuration can be categorized as follows (see [5] for details).

- (a) *Communication range*: the maximum distance between two team members such that communication is still possible. We list three key classes: (i) no direct communication: robots cannot communicate with other robots directly, but indirect communication by observing the behavior of other robots is possible, (ii) local communication: robots can only communicate with other robots which are sufficiently nearby and (iii) global communication: robots can communicate with any other robots.
- (b) *Communication topology*: it captures physical interconnections among team members. The topology can be either static, if the topology is fixed, or dynamic, if the relationship of team members can change arbitrarily. The interconnection structure can also be either bidirectional or unidirectional.
- (c) *Communication bandwidth*: it indicates the amount of data that a communication link can transmit in a given period of time.

Studies that require global information may suffer from lack of scalability but allow more accurate forming of a wider range of formations. On

the other hand, studies using only local communication and sensor data tend to be more scalable, more robust, and easier to build; but they are also limited in variety and precision of formations.

2.3 Centralization vs. Decentralization

Centralized controllers deal with systems in that a single controller processes all the information needed to achieve the desired control objectives. Therefore, they can ideally yield superior performance and optimal decisions for both the individual members and the formation as a whole. However, they require high computational power, massive information flow, and are not robust due to heavy dependence on a single controller. On the other hand, in decentralized control, each team member has its own controller and is completely autonomous in the decision process. This can significantly reduce the number of signals being communicated, is more flexible and robust, requires less computational effort, and is more scalable.

Nevertheless, there is also the need to provide some degree of centralization with an interface to human operators for programming, tasking, and monitoring of the system. There are also some hybrid centralized/decentralized architectures wherein there is a central planner that applies high-level control over autonomous robots.

3. Robot Models

For robot models, point robots with simple (single or double integrator) dynamics (e.g., [6]) or fully actuated robots are usually investigated in many papers. Robots with nonholonomic constraints are also controlled (e.g., [7-9]) through either a kinematic model (e.g., [9-10]) or a dynamic model (e.g., [11-12]). A common technique used to simplify the dynamics is feedback linearization of a point off the center of the wheel axis [13]. This technique

reduces the equations to double integrator dynamics. Since the presence of inaccurate parameters or the complexity of the resulting model may limit applying the first principle approaches (*white box*) of modeling, some sufficiently general *black-box* structures can be employed to approximate the system. However, a severe drawback of this technique is that the structure and parameters of these models usually do not have any physical significance. Thus, there are some approaches that attempt to combine the advantages of the *white-box* and *black-box*, such that the known parts of the systems are modeled using physical knowledge, and the unknown or less certain parts are approximated using a *black box* approach. These methods are often called *hybrid* or *gray-box* modeling.

4. Formation Control Strategies

Most publications in formation control classify formation control approaches into three basic strategies, i.e., behavior-based, virtual-structure, and leader-following. Each strategy has its own advantage and disadvantage, as discussed in the following subsections.

4.1 Behavior-based Approach

Behavior-based approaches start by designing simple behaviors or motion primitives for each individual robot, e.g., formation keeping, trajectory tracking, goal seeking, and obstacle avoidance. Then, more complex motion patterns can be generated by using a weighted sum of the relative importance of these primitives and the interaction of several robots (for an example, see Figure 1). The main drawback of this approach is that the mathematical analysis of this approach is difficult and consequently the convergence of the formation to a desired configuration cannot be guaranteed.

Nevertheless, the advantage of behavior-based schemes is that formation feedback is implicitly

integrated by coupling the weights of the actions that depend on the relative coordinates of neighboring robots. Behavior-based approaches are also useful in guiding a multi-robot system in an unknown or dynamically changing environment using local sensory information only.

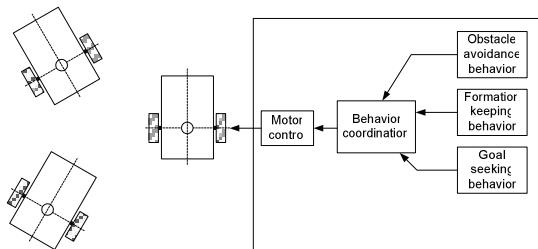


Figure 1. The behavior-based approach

Balch and Arkin [14] followed the motor scheme control. All the behaviors are summed together through suitable weight coefficients which set the relative priority between them. They also introduced three different robot position strategies, i.e., leader-referenced, neighbor-referenced and unit-center. Balch and Hybinette [15] employed social potential fields to snap robots into predefined attachment sites positioned around each robot with respect to formation shapes. Antonelli *et al.* [16] solved a flocking problem by using the Null-Space-based Behavior (NSB) control. Simple behaviors are defined for each team member, and these behaviors are properly arranged in priority. Monteiro *et al.* [17] used nonlinear attractor dynamics to design a dynamic control architecture, where behaviors are generated as a time series of asymptotically stable states, which contribute to the asymptotic stability of the overall control system. Michaud *et al.* [18] employed a hybrid control architecture that combines a behavioral level with global level deliberation. All behaviors run in parallel and their resulting commands are prioritized and managed through a finite state machine to generate the control actions of the robot.

4.2 Virtual-Structure Approach

Virtual structures consider the entire formation as a rigid body, see Figure 2. The control law for a single vehicle is derived by defining the dynamics of the virtual structure and then translates the motion of the virtual structure into the desired motion of each vehicle. The main advantages of the virtual structure approach are that it is easy to prescribe the coordinated behavior for the group, and that the formation can be maintained well during maneuvers, i.e., the virtual structure evolves as a whole in a given direction with given orientation. However, if the formation has to maintain the same virtual structure all the times, the possible applications are limited, especially when the formation shape needs to be frequently reconfigured.

One of pioneering work was proposed by Lewis and Tan [19]. Their algorithm iteratively fits the virtual structure to the robots' positions, displaces the virtual structure in some desired direction and updates the robots' positions. Their method includes formation feedback, but they cannot guarantee that a formation converges to a final configuration.

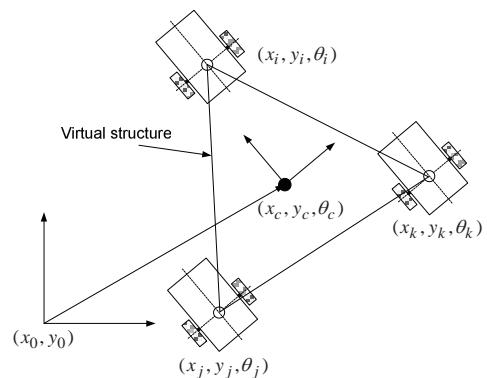


Figure 2. The virtual-structure approach

4.3 Leader-following Approach

With the leader-following strategy, some robots are considered as leaders, while others act as followers. The primary advantage of using such a strategy is this approach reduces to a tracking

problem where stability of the tracking error is shown through standard control-theoretic techniques: The leader pursues some group objectives, while the following robots track transformed coordinates of the leader with some prescribed offsets. The internal formation stability is induced by the individual robot's control laws (the input-to-state stability of the leader-following formation was studied by Tanner in [20]). However, the disadvantages are that the chain structure leads to a poor disturbance rejection property and the leader's motion is independent of the followers, i.e., there exists no explicit feedback from the followers to the leader. In addition, the formation does not tolerate leader faults.

One of the most popular control techniques for the leader-following strategy was presented by Das *et al.* [10] using a feedback linearization control method. They proposed two controllers: Separation-bearing control and separation-separation control. In separation-bearing control (see Figure 3), robot j follows robot i at a desired separation l_{ij}^d and desired relative bearing ψ_{ij}^d , while in separation-separation control (see Figure 4), robot k follows two leaders, robot i and robot j , at desired separations l_{ik}^d and l_{jk}^d , respectively. Other control techniques found in the literature include dynamic feedback linearization [21], backstepping [22], model predictive control [23-24], first-order sliding mode control [25], and second-order sliding mode control [12].

In this approach, the linear velocity and angular velocity of the leader robot and relative orientation are required by the formation tracking controller of the follower robot. In the absence of communication, this becomes quite challenging from a sensing viewpoint, because the motion of multiple moving objects needs to be estimated simultaneously. To solve these problems, most of the leader-follower approaches rely on nonlinear observers and image information, e.g., on an extended Kalman filter [10],

[26], an unscented Kalman filter [21], or a high gain observer [27]. To use image information, either position-based visual servoing [10], [21], [26-27] or image-based visual servoing [28] is integrated within a motion control loop. Likewise, to eliminate the need for measurement or estimation of the absolute velocity of the leader, Defoort *et al.* [12] used a second-order sliding mode formation controller which is only based on the relative motion states. Dierks and Jagannathan [29] developed a neural network tracking controller that considers the dynamics of the leader and the followers using backstepping with the Robust Integral of Sign of the Error (RISE) feedback.

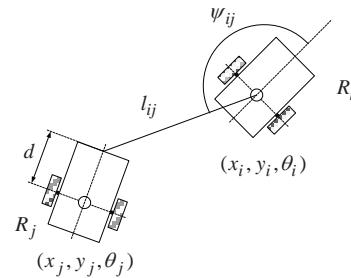


Figure 3. Notation for separation-bearing control (note that l_{ij} is the actual separation and ψ_{ij} is the actual relative bearing.)

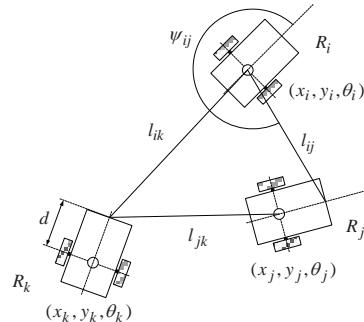


Figure 4. Notation for separation-separation control (note that l_{ij} , l_{ik} and l_{jk} are the actual separation and ψ_{ij} is the actual relative bearing.)

There also exist some algorithms employing a variant of the leader-follower strategy. For instance, Consolini *et al.* [30] proposed a method that the follower position is not rigidly fixed with respect to the leader but varies in proper circle arcs centered in

the leader reference frame. Gamage *et al.* [31] developed high-level supervisory control of discrete event systems to take care of the coordination of low level continuous feedback-linearized controllers including formation keeping, obstacle avoidance, wall following, and goal navigation. Bai *et al.* [32] studied a problem where the reference velocity is available to only one robot while the others estimate this information with a passivity-based adaptive design.

4.4 Other Control Strategies

One way to solve the formation control problem is to formulate it as an optimization problem. Receding Horizon Control (RHC), also recognized as Model Predictive Control (MPC) is a well-known control strategy in which the current control action is computed by solving a finite horizon optimal control problem online. In general, the centralized implementation is not practical due to high computation requirements. Thus, the research directs at decomposing the centralized system into smaller subsystems, which are independently controlled in the RHC framework. Some approaches based on this strategy were proposed by Dunbar and Murray [33], and Kanjanawanishkul and Zell [34], and references therein.

Besides the optimization based approach, a considerable amount of attention has focused on the problem of coordinated motion based on graph-theory consensus protocols over the past few years. We intentionally do not collect all published contributions using this approach because of a large number of publications. A non-exhaustive list of relevant research includes formation stability, e.g., [35] and decentralized formation tracking, e.g., [8], [36-38]. Furthermore, Chung and Slotine [39] used nonlinear contraction theory to study synchronization, which is related to the consensus problem.

5. Formation Control Subproblems

In this section, the following four major subproblems based on using a problem-oriented criterion are described in details.

5.1 Formation Shape Generation

Although many studies assume that the desired formation shapes are given and that these shapes can be arbitrary, this is not always the case. Therefore, this subsection focuses on how to form and to maintain a formation.

Formation-control specifications can be encoded in a formation constraint function for example, Egerstedt and Hu [9] defined a mathematical constraint function for a virtual structure. Another approach is to consider some artificial potential functions that usually play the role of Lyapunov function candidates to shape the dynamics of the formation. Leonard and Fiorelli [6] considered two types of potential functions: an interaction function between neighboring vehicles and a potential generated by virtual leaders. However, the drawback in [6] is that as the number of vehicles increases, many local minima appear. DeGennaro and Jadbabaie [40] used a decentralized navigation function to drive each robot of a group towards a final configuration which is expressed in terms of distances between the connected robots. Zhang [41] modeled formation dynamics as controlled Lagrangian systems on Jacobi shape space. The formation shape is invariant under translation and rotation, and is also independent of the coordinate system.

Olfati-Saber and Murray [42] provided a unified graph-theoretic framework that formally defines formations of multiple vehicles and their stabilization issues. They clarified the important role of graph rigidity and minimally rigid graphs in construction of structural potential functions and manipulation of multiple formations. For more information on rigidity

and persistence of the graph-theoretic framework, we refer the reader to [43].

Some strategies have appeared to control the exact shape of the formation swarm, in which a large number of robots is spread, but not to specify their exact positions, see Figure 5 for an example. Belta and Kumar [44] proposed a control method for a large group of robots to move along a specified path. They expressed the configuration space Q of a robot swarm as $Q = G \times S$, where S is the shape space of the swarm and G is the Lie group representing the space of positions and orientations for a given shape of the swarm. Using this formal abstraction enables controllers for the group motion to be decoupled from formation configuration (shape variables). Recently, Michael and Kumar [45] extended this concept to control the position and orientation of a formation and to adapt the shape of a formation to the environment. Hou and Cheah [46] proposed a region based shape control method, using both multiplicative and additive potential energy functions, to form a certain shape. Freeman *et al.* [47] developed a distributed estimation algorithm that allows robots in a communication network to maintain estimates of summary statistics describing the shape of the swarm formation, i.e., the first and second-order inertial moments.

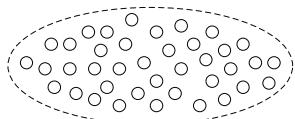


Figure 5. A team of robots converge and stay inside a desired ellipse, represented by dashed lines. Small circles denote robots.

Boundary coverage for a robotic swarm has also been studied over a decade; see Figure 6 for an example. Recently, Hsieh *et al.* [48] proposed a gradient-based decentralized controller that allows a large team of robots to converge to some desired two-dimensional boundary curve while maintaining

inter-robot constraints via local interactions. More details concerning this problem can be found in [48] and references therein.

Another research direction in pattern generation is that how a set of *very weak* mobile robots can achieve a given spatial pattern in a decentralized fashion. Different settings arise from different assumptions that are made on the robots' capabilities and on the amount of information that they share and use during the accomplishment of the assigned task. The reader is referred to [49] and related references for details on this research area.

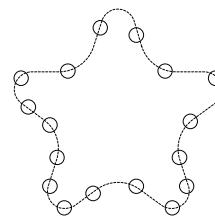


Figure 6. A team of robots converge and travel along a desired star-shaped boundary, represented by dashed lines. Small circles denote robots.

5.2 Formation Reconfiguration and Selection

It is sometimes necessary to change/split/join the formation due to either a change in coordinated task specifications or a change in environmental conditions, e.g., the presence of uncertainty, adversarial vehicles, and narrow corridors (see Figure 7 for an example).

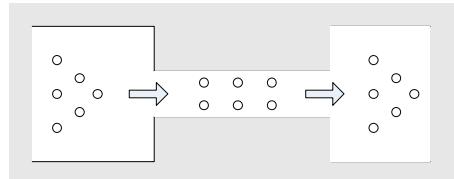


Figure 7. Robots arrange themselves in a two-column formation while moving through a narrow corridor. Small circles denote robots.

Switching between different rigid formations was studied by Das *et al.* [10], where the switching between simple decentralized controllers on follower

robots allows formation switching while follower robots are following a leader. Desai [50] proposed a graph-theoretic approach for coordinating transitions between two formations. Fierro *et al.* [51] split the problem using a hybrid approach. They designed the continuous-state control algorithms based on input-output feedback linearization, while discrete-state formation control is used to achieve a desired formation by sequential composition of basic maneuvers. However, the problem with the control strategies in [10], [50-51] is that the control problems get more complicated as the number of robots in the formation increases.

The behavior-based formation switching proposed by Michaud *et al.* [18] involves the assignation of a new leader, which is influenced by the situation experienced by the robot, while in a method proposed by Fredslund and Mataric [52], special cases must be programmed to keep robots organized according to their ID numbers and switching must preserve the ordering of the robots based on their ID.

McClintock and Fierro [53] investigated when formation changes should occur. The robots change between formations by choosing the one which can operate in the current environment with minimum formation error.

The problem of selecting a particular formation shape possibly depending on a dynamical context (e.g., environment modifications and dynamical tasks) has not been deeply investigated in the literature. Haque and Egerstedt [54] modeled the bottlenose dolphins' behavior: Agents in hunting phase have to choose their formation between small or large circles using a hybrid control strategy and decentralized networked control in order to capture a prey. Recently, Di Rocco *et al.* [55] proposed an approach to select an optimal formation shape using an online selection of the optimal shape of the

formation that maximizes some performance indices related to the task and to the environment.

5.3 Formation Tracking

Formation tracking is the largest portion of formation control research. The goal of formation tracking is that a group of robots has to maintain a desired formation, while tracking or following a reference. This task may also include path planning, trajectory generation and motion feasibility [56] for robot formation.

Formation tracking in the literature can be classified into two groups, i.e., trajectory tracking and path following. In particular, Aguiar *et al.* [57] highlighted a fundamental difference between the path following and the standard trajectory tracking by demonstrating that performance limitations due to unstable zero-dynamics can be removed in the path following problem. Typically, in path following, smoother convergence to the path is achieved and control signals are less likely pushed into saturation when compared to trajectory tracking.

5.3.1 Trajectory Tracking

Typically, tracking problems for mobile robots are solved by designing control laws that make the robots track predetermined feasible trajectories, i.e., trajectories that specify the time evolution of the position, orientation (spatial dimension), as well as the linear and angular velocities (temporal dimension) [57]. The most common strategy for trajectory tracking is leader-following (e.g., [10], [21], [24], [51]). Other strategies include optimization-based approaches (e.g., [33]) and graph-theory based approaches (e.g., [8], [36-38]).

5.3.2 Path Following

Path following problems [57] are primarily concerned with the design of control laws that steer a robot to reach and to follow a geometric path, i.e., a manifold parameterized by a continuous scalar s , while a secondary goal is to force the robot moving

along the path to satisfy some additional dynamic specifications, e.g., time, speed, or acceleration assignments. This setting is more general than the common trajectory tracking problem, in which the path's parameter s is left as an extra degree of freedom for the secondary goal.

In the literature, there are two general techniques, i.e., the coordinated path following approach and the virtual-structure approach. In the former approach, each team member requires an individual parameterized reference path so that when all paths' parameters are synchronized, each member will be in formation; see Figure 8 for an example. In the latter approach, the path for a virtual leader is computed as a reference point for the real robots to follow.

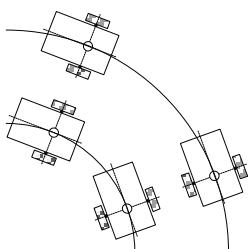


Figure 8. Four robots move along the path in such a way as to maintain a desired formation pattern compatible with those paths.

The main advantage of using the formation control of path following is that the entire formation will slow down if the robots get out of formation and it moves towards its goal if the robots are maintaining formation. However, practical constraints arise from the characteristics of the supporting inter-robot communication network. They have to exchange the path's parameter to each other via communication. Thus, the quality of communication channels becomes a crucial part.

Ghabcheloo *et al.* [7] presented a solution to the problem of steering a group of wheeled mobile robots along given spatial paths, while holding a

desired inter-vehicle formation pattern with bidirectional communication constraints. Ihle *et al.* [58] showed a passivity property for the path following control and then combined this with a passivity-based synchronization algorithm. Recently, Xiang *et al.* [59] addressed the problem of simultaneous path following control, obstacle avoidance and collision free for coordinated multiple nonholonomic autonomous vehicles under formation constraints.

In case of the virtual structure technique, Egerstedt and Hu [9] proposed formation constraint functions to decouple the coordination and following problems, while maintaining the stability of the formation. The path for a virtual leader, including formation feedback is computed as a reference point for the real robots to follow. Similarly, Young *et al.* [60] included a specific form of formation feedback in the coordination variable evolution. Ghomman *et al.* [61] used the derivative of the path parameter as an additional control input to synchronize the formation motion. However, they assumed that each robot has to broadcast its state and reference to the rest of the team and it has to receive states and references from the other robots of the team.

5.4 Role Assignment in Formation

In general, an explicit assignment of the robots in a formation might be desired, however, for indistinguishable robots, this is not always easy. Figure 9 shows an example that a team of robots starting from an initial configuration can reach a desired formation configuration, i.e., the letter "R" without any explicit assignment. This problem can be seen as combinatorial optimization, where n persons are optimally dividing among n objects. It is also referred to as the *assignment problem*, or the minimum weight perfect matching problem in bipartite graphs. In 1955, Kuhn [62] developed the Hungarian method – the first polynomial solution for

the assignment problem. Another approach is the *auction algorithm*. This problem might also be related to the Multi-Robot Task Allocation (MRTA) architecture [63], in which the question is encountered: "Which robot should execute which task?"

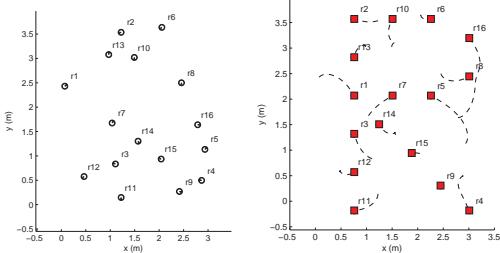


Figure 9. Role assignment in formation: (a) the initial configuration and (b) the robot trajectories (dash lines) and the final configuration.

In a centralized fashion, a reasonable strategy would be to minimize the sum of the distances traveled by each robot to arrive at its target. However, with distributed decision-making and limited communication, the problem of deploying robots to form arbitrary target configurations is still open.

In [64], each robot exchanges a visibility table and then assigns itself as being the conductor of the desired formation. It searches the tree to find the best assignment of positions for other robots in the group. The best result obtained by each robot is broadcasted to the others, and the one with the minimum cost is selected as the conductor of the formation, with positions assigned accordingly to the other robots. Smith and Bullo [65] proposed an assignment-based algorithm using greedy rules and circular ordering of the targets. Lee *et al.* [66] used a coupled combinatorial and continuous optimization framework, in which the inner loop consists of computing the costs associated with a particular assignment by using a discrete optimal control method. In the outer optimization loop, combinatorial techniques are used to determine the optimal

assignments based on the costs computed in the inner loop.

Ji *et al.* [67] showed how the simultaneous rotation, translation, and assignment optimization problem can be casted as a parameterized assignment problem. Derenick and Spletzer [68] employed a formal definition from shape analysis for formation representation and used second-order cone programming techniques to find an optimal solution by minimizing either the total distance or the minimax distance the robots must travel. A distributed auction-based approach for a rotational and translational invariant configuration was proposed by Zavlanos and Pappas [69]. Michael *et al.* [70] extended this solution to the dynamic task allocation problem where the assignment of robots to tasks may need to be continuously adjusted depending on changes in the task environment or group performance.

6. Future Research Perspectives

The topics listed here are not intended to be exhaustive, but rather to be indicative of the classes of problems which we are interested in.

6.1 Towards Real-world Applications

The application of multi-robot systems to real-world scenarios requires the consideration of many challenging details that increase the complexity of the implementation. The ability to interact with a dynamic, changing environment is of key importance. Robots must be able to handle various real-world events that can disrupt the formation, thus requiring obstacle avoidance, formation repair, and changes in the formation. Higher levels of decision making become vital since many autonomous systems must make decisions for which an underlying set of system variables may not provide. Techniques from artificial intelligence that allow identification of strategies and tactics may be

needed [2]. Furthermore, robots themselves must satisfy dynamic constraints, such as velocity and acceleration bounds.

Recent experimental results [71-72] provide verification of formation control and show its usefulness in real-world applications. The aerial platform can control a team of ground robots without any knowledge of the specifics of individual vehicles [71] and the team of robots forms patterns that trap the object to be manipulated and drags the object to the goal configuration [72].

6.2 Hybrid Control Frameworks

Since formation control problems may consist of several sub-tasks, the traditional control theory may fail due to its fixed single mode of operation. Therefore, the need of a higher level coordination protocol to handle the switching of the single modes of control-theoretic operations should be highlighted. Research in hybrid systems, in which continuous controllers and discrete protocols are integrated, are a step in the right direction but these techniques often ignore issues associated with distributed computing and communication channels which are very vital in formation control. We also need to address a number of problems, e.g., stability and reachability analysis, hybrid control design algorithms, and state estimation [73].

6.3 Networked Systems and Information

Consensus

Information flow via network communication has been the center of much attention lately since technological advances in computation and communication over the past few years have provided efficient and inexpensive ways to share and compute information. Researchers are currently uncovering rich connections between information flow and motion coordination/formation control (see [35], [74-75] and a myriad of references for details on consensus algorithms). Control-theoretic

consensus algorithms have proven to be effective tools for performing network-wide distributed computation tasks.

Formation control together with the framework of networked systems enables us to realize a variety of useful tasks including distributed robotic surveillance, exploration, and mobile sensor networks. Compared to conventional systems, one of unique technical challenges is how to analyze the effects of information flow topologies among the robots, i.e., to determine how the local action of each individual robot propagates throughout the group. However, at the same time, only certain information topologies would be feasible other than the fully-connected one, especially when the number of robots is large. Another important challenge is time delays in the inter-robot information flows. Analyzing the effects of the information delays on the group behaviors is very important because they can lead to unstable group behaviors.

7. Conclusions

Although formation control has been particularly well studied and is quite mature, as seen in our survey, much work remains to be done to develop strategies capable of yielding robust performance of autonomous mobile robots in the presence of real-world complications, complex robot dynamics, and severe communication constraints. Nevertheless, we hope that we will witness significant progress in real-world applications of formation control in the near future.

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