Nonlinear Dynamical Analysis: Application to EEG Sleep Analysis

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Abstract

Nonlinear dynamical analysis techniques have been widely used for EEG analysis. The correlation dimension based upon the correlation is one of the most commonly used measures which quantifies the active degrees of freedom or the complexity of the dynamical system on the attractor. This article aims to provide an overview of the basic concepts of nonlinear dynamical analysis, and also to demonstrate its application in EEG sleep analysis. As one of the evidences, it is shown that there is a decrease in the correlation dimension (i.e., a loss in the complexity of the underlying dynamics of the neuronal networks in the brain) from lighter to deeper sleep stages. The use of the nonlinear dynamical analysis can be viewed in two aspects. In one aspect, the lower correlation dimension of the EEG suggests that the neuronal networks are more strongly coupled at deeper sleep stage. In another aspect, the substantial differences of the correlation dimensions of the EEG associated with various sleep stages can be used for sleep stage discrimination.

Keywords: Nonlinear dynamics, complexity, correlation dimension, electroencephalogram, sleep

1. Introduction

The electroencephalogram (EEG) or brain wave is a complex signal that quantifies the electrical activity of the brain. The EEG which results from postsynaptic potentials of cortical pyramidal cells is an important brain state indicator [1]. Temporal patterns of the EEG have been shown to provide insight into the various functional states of neuronal networks in the brain [2]. Computational analysis techniques have been applied to EEG for a number of clinical situations including sleep, coma, mental state, cognition, and epilepsy.

Sleep is essential for human's health and well-being. There are many sleep disorders (e.g., insomnia, narcolepsy, sleep apnoea) while many other disorders manifest themselves through sleep disturbances (e.g., depression, schizophrenia, Alzheimer disease) [3]. Sleep staging is one of important procedures for clinical diagnosis and treatment of sleep disorder [4]. Traditionally sleep is monitored using a polysomnography [5]. Sleep stages are mainly differentiated by features and patterns of brain wave, eye movements, and muscle tone [6].

The EEG is the major discriminating marker between waking and sleep, and between various sleep stages such as between NREM sleep and REM sleep, the two major states of sleep [7]. According to the standard guidelines for sleep classification by Rechtschaffen and Kales [8] which is a widely accepted standard, sleep recordings are divided into seven discrete stages: waking, stage 1, stage 2, stage 3, stage 4, stage REM, and movement time based on the characteristic features of EEG, in conjunction with EOG and EMG. Additional clinical information including heart rate, blood pressure, blood oxygenation, and respiration rate may be also used in sleep stage classification.

Traditional linear analysis techniques, e.g., spectral analysis, have been very valuable computational tools for EEG analysis. Furthermore, the spectral analysis, a mathematical approach that decomposes the signal (such as the EEG) into its constituting frequency components, has long served as a main computational tool especially for the study of sleep and EEG sleep analysis because the patterns of brain electrical activity, i.e., EEG, corresponding to various stages of sleep are generally defined in terms of frequency ranges or the so-called waves, e.g., delta, theta, beta, and alpha.

Recently, concepts and computational tools derived from the contemporary study of complex systems including nonlinear dynamics, also known as chaos theory, and fractals have gained increasing interest in biology and medicine [9]. One reason is that many complex and interesting phenomena in nature are due to nonlinear phenomena. The theory of nonlinear dynamics has been further developed and progressed to a point where it has been applied to examine self-organization and pattern formation in the complex neuronal networks of the brain [10].

Nonlinear dynamical analysis has been applied to various types of EEG including data obtained from both normal and abnormal clinical situations [2]. A number of clinical situations that nonlinear dynamical analysis has been used including resting state, sleep, coma, mental state, cognition, and epilepsy. In general, nonlinear dynamical analysis has been used to characterize behaviors of the underlying neuronal dynamics of the brain associated with different brain states. A number of nonlinear dynamics measures have been also developed to quantify features of brain dynamics [10].

Among the available methods of nonlinear dynamical analysis, the correlation dimension introduced by Grassberger and Procaccia [11-12] is the most commonly used measure and the algorithm

to compute the correlation dimension is relatively simple. However, the computational time required can be prohibitive. In addition, the proper computation and interpretation of the correlation dimension involves many pitfalls. The estimate of correlation dimension can be biased by autocorrelation effects, noise, and length of the time series.

One of the first applications of nonlinear dynamical analysis of the human EEG was the work by Babloyantz *et al.* [13] where the correlation dimension was computed using the Grassberger-Procaccia algorithm [11-12] and the relationship between correlation dimension and different stages of sleep was investigated. Thereafter sleep has become a major research focus in nonlinear dynamics [14]. Several similar studies (e.g. [15-19]), were carried out using the correlation dimension and Lyapunov exponents.

The main purposes of this article are 1) to provide an overview of the basic concepts of nonlinear dynamical analysis with an illustration, and 2) to demonstrate the application of the nonlinear dynamical analysis in EEG sleep analysis.

2. Sleep

2.1 Characteristics of Sleep and Sleep Classification

In normal adults, sleep is divided into two broad types: rapid eye movement (REM) and non-rapid eye movement (NREM) [6]. These two types of sleep, i.e., REM and NREM, alternate cyclically throughout the night. NREM sleep is further divided into four stages, namely stage 1, stage 2, stage 3 and stage 4, that are roughly correlated with the depth of sleep [6]. Sleep stages 1 and 2 correspond to light sleep whereas sleep stages 3 and 4 correspond to deep sleep or also referred to as slow-wave sleep (SWS) [20]. Sleep begins in NREM

sleep and progresses from light NREM sleep stage through deeper NREM sleep stages before the first episode of REM sleep [21]. REM sleep episodes become longer as sleep progresses [6].

The standard guidelines developed by Rechtschaffen and Kales [8] have been widely accepted standard for describing the human sleep process for approximately 40 years [22]. Polysomnographic recordings are divided into 30-second epochs. The occurrence, frequency, amplitude, shape, and temporal sequence of these patterns provide information on the sleep stage assigned to a given epoch [23]. On the basis of the R&K rules, the sleep physiological recordings are divided into 7 discrete stages: waking (W), stage 1 (S1), stage 2 (S2), stage 3 (S3), stage 4 (S4), rapid eye movement (REM), and movement time (M) [22].

The R&K rules have been criticized for leaving plenty of room for subjective interpretation, which leads to a great variability in the visual evaluation of sleep stages [22],[24]. In addition, another significant drawback is that the R&K rules were developed for young healthy adults, and do not necessarily directly apply to elderly subjects and patients [22]. The American Academy of Sleep Medicine (AASM) [25] revised the standard guidelines for sleep classification by Rechtschaffen and Kales [8], and in 2007, the AASM Manual for the Scoring of Sleep and Associated Events: Rules, Terminology, and Technical Specifications was published [26].

The new sleep scoring manual addresses seven topics: digital analysis and reporting parameters, visual scoring, arousal, cardiac and respiratory events, movements and pediatric scoring [22]. One of the major changes is a change in terminology [N6]. In the AASM manual, the number of stages is reduced from seven (W, S1, S2, S3, S4, REM and M) to five (W, N1, N2, N3, R) [26]. The stage N3 reflects slow-wave sleep (SWS) and comprises the sleep stages S3 and S4 [22],[26]. The

stage REM is referred to as the stage R [22] while the movement time is not scored as a separate stage [26].

Staging of sleep is relevant to the study of sleep and crucial to the diagnosis of sleep disorders. Sleep disorders can be generally divided into two major kinds: insomnia (complaints of too little sleep) and hypersomnia (complaints of too much sleep) [23]. Both the total sleep time and the relative amount of time spent in different stages of the sleep are often affected by disease processes [20]. Sleep stage of narcoleptics goes directly into REM sleep rather than going through other sleep stages [23]. Moreover, each stage the sleep has characteristic impact on respiration [20].

2.2 Sleep Stage Classifications

According to the R&K rules, sleep stages are discriminated using three physiological recordings: EEG, EOG and EMG [23]. The characteristics and patterns of EEG change remarkably corresponding to different states of the brain including between wakefulness and sleep, and also between different levels of sleep [27]. The characteristics and patterns of the EEG and other physiological recordings associated with various sleep stages are summarized as follows [6],[8],[27],[28]

- Waking: EEG of wakefulness is low voltage and contains mixed frequency (2-7 Hz) activity. 50% of the epoch consists of alpha (8-13 Hz) activity. There is relatively high tonic EMG activity.

- Stage 1: EEG is relatively low voltage and contains mixed frequency (2-7 Hz) activity, which is similar to that of wakefulness. The EMG level is lower than in wakefulness. Alpha activity occupies less than 50% of the epoch.

- Stage 2: There is an appearance of sleep spindles (episodic generalized symmetrical complexes) and/or K-complexes (sharp transients characterized by an initial negative and then positive

component) which must last more than 0.5 second. The epoch may contain high voltage (greater than 75 μV) and low frequency 2 Hz or less) activity of EEG for more than 20%.

- Stage 3: EEG is high voltage (greater than 75 μV) and contains low frequency (2 Hz or less) activity for about 20%-50% of the epoch.

- Stage 4: The epoch consists of high voltage (greater than 75 μV) and similar low frequency (delta activity) EEG for more than 50%.

- REM: EEG is relatively low voltage and contains mixed frequency (2-7 Hz) along with episodic rapid eye movements and absent or reduced chin EMG activity.

- Movement time: If it is ambiguous to score as sleep or waking for more than a half of the epoch, then the epoch is scored as movement time.

Since the presentation of the standard guidelines for sleep classification by Rechtschaffen and Kales [8], a number of computer-assisted sleep stage classification and identification techniques have been presented [29-32].

3. Concepts of Nonlinear Dynamics

The information obtained from the nonlinear dynamical analysis is mostly reflected by the complexity parameter which is associated to the dimensionality of the underlying dynamics of the system [33].

3.1 Dynamical Systems

A dynamical system is a system whose state develops and evolves over the course of time. Mathematically, a dynamical system is given by a model that expresses the evolution of such system given only by a current state. That is, the next state of a dynamical system is specified by a particular function of the current state. Therefore, a dynamical system consists of two parts: a state and a dynamics. The state of a dynamical system is a set of values of all dependent variables that describe the system at a particular moment in time while the dynamics of a dynamical system is a set of rules or equations that describe how the state of the system evolves over time.

The state of a dynamical system described by m dependent variables is represented by a point in m-dimensional space. This space is a vector space called the state space or also called the phase space $\mathbb{R}^m$, and m characterizes the degrees of freedom. The state of a dynamical system is formally represented by a state vector $\mathbf{x} \in \mathbb{R}^m$

$$\mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_m(t) \end{pmatrix}. \tag{1}$$

The dynamics of a dynamical system can be described by a set of ordinary differential equations, typically the first-order differential equations, or a mapping function. For a continuous-time dynamical system, the dynamics of the system can be described by a set of ordinary differential equations

$$\frac{d\mathbf{x}(t)}{dt} = f(\mathbf{x}(t)). \tag{2}$$

On the other hand, the dynamics of a discrete-time dynamical system is represented by a mapping function

$$\mathbf{x}(t + \Delta t) = F(\mathbf{x}(t)). \tag{3}$$

A dynamical system is linear if the equations that describe the dynamics are linear; nonlinear, otherwise. A sequence of consecutive states $\mathbf{x}$ in the state space forms the trajectory of the dynamical system which corresponds to the dynamical evolution of the system.

3.2 Attractors and Characterization of Attractors

If a dissipative dynamical system is observed for a sufficiently long time (after transient behavior

has died out), its trajectory will converge to a confined subspace of the state space. This subspace is a geometrical object in the state space that is referred to as an attractor. It is called attractor because it attracts trajectories from all possible initial conditions. There are various structures of attractors can exist. Three main types of attractors are point attractors, regular attractors, and strange attractors (or chaotic attractors). Strange attractors exhibit complex geometrical objects in the state space that are called fractal geometry.

Attractors hold significant information with respect to the dynamics of dynamical systems. For a deeper understanding of behaviors of dynamical systems, knowledge of characteristics of attractors is desired. Measures derived in theory of dynamical systems are used to characterize and quantitatively describe the dynamical and geometrical properties of attractors. The more complex the attractor, the more complex the corresponding dynamics [10].

Dimensions specify how the attractor which is a geometrical object tends to be distributed spatially in the state space measure the geometry of the attractor [34]. Estimation of dimension is one approach to detect and quantify the self-organizational characteristics of complex systems [35]. The correlation dimension D_2 introduced by Grassberger and Procaccia [11-12] is the most commonly used measure [10].

4. Nonlinear Dynamical Analysis

In practice, when we analyze a nonlinear dynamical system, what we have to begin with are not the dynamics of the system (a set of differential equations, for example); but rather a set of observations that may or may not be any actual variables of the system. We therefore do not know a complete description of the underlying dynamics of the system and even the variables that involves the

state of the system. The way to obtain a more complete description of the underlying dynamics of the system with unknown properties from the observations is nonlinear dynamical analysis [10].

The process of nonlinear dynamical analysis consists of two steps: 1) reconstruction of the dynamics of the system; and 2) characterization of the reconstructed attractor. In addition, the validity of the nonlinearity of the observations may be further tested using the method of surrogate data testing.

4.1 Attractor Reconstruction

The main problem in putting the nonlinear dynamical analysis into practice is that the measurements of variables of the system. Some variables of the systems may not be known. Some variables of the system cannot be measured or accessed. The true state of the system which requires the knowledge of all variables of the system cannot thus be determined. The method of time-delay embedding allows us to obtain a more comprehensive description of the dynamics and the states of the system by unfolding the observed time series into a higher dimensional state space, called the embedding space.

Let $x[n]$ be a one-dimensional (observed) measure of the dynamical system. Note that the dynamical system of our interest is neuronal networks and a set of observations we can assess is recordings of EEGs. The m-dimensional embedding vector of the time series $x[n]$ is given by [34]

$$\mathbf{x}_n = \begin{pmatrix} x[n] \\ x[n+\tau] \\ \vdots \\ x[n+(m-1)\tau] \end{pmatrix} \tag{4}$$

where m and τ are the embedding parameters denoting the embedding dimension and the delay time, respectively. A sequence of embedding vector $\mathbf{x}_n$ forms the reconstructed attractor. It is
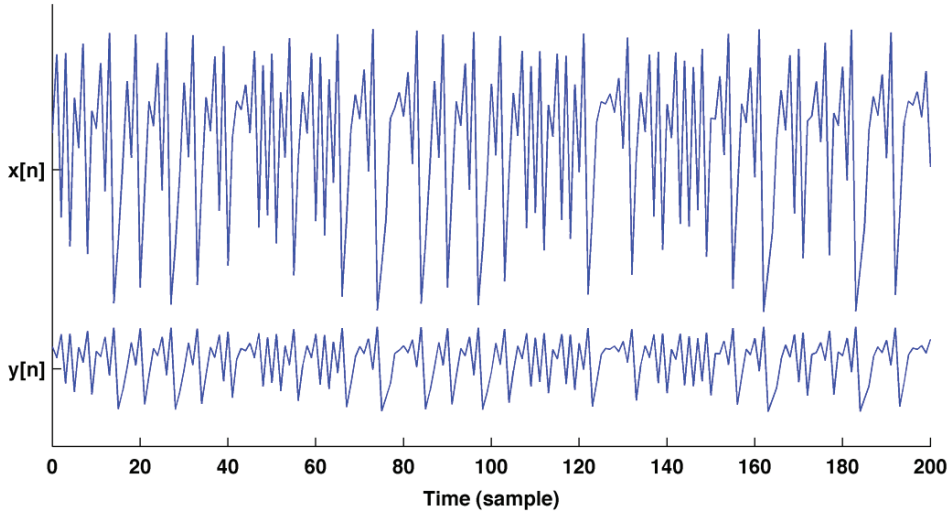
Fig. 1 Time series for the state variables $x[n]$ and $y[n]$ of the Hénon map.

proved that the reconstructed attractor has the same dynamical properties as the actual attractor by Takens [36].

The choice of the embedding dimension m and the delay time τ has an effect on accuracy of estimation of the correlation dimension. The important parameter for time-delay embedding is neither the embedding dimension m nor the delay time τ separately but the embedding window [37]. There are a number of methods for determining the time delay τ such as the autocorrelation function [37], mutual information [38], average displacement [39], etc. A sufficient embedding dimension m can be determined by using the false nearest neighbor technique [40], for example.

4.2 Correlation Integral and Dimension

The correlation dimension D_2 computed using the Grassberger-Procaccia algorithm is the easiest dimension to compute [41], although the computational time required can be prohibitive. The correlation dimension is based upon the correlation integral. The correlation integral $C(r)$ computed from the reconstructed attractor $\mathbf{x}_n$ is defined by [11-12]

$$C(r) = \lim_{N \to \infty} \frac{2}{N_c} \sum_{i=0}^{N-1} \sum_{j=i+1}^{N-1} \Theta\left(r - \|\mathbf{x}_i - \mathbf{x}_j\|\right) \qquad (5)$$

where N denotes the length of the reconstructed attractor, $N_c = N(N-1)$ and the Heaviside function $\Theta(n) = 1$ if $n \geq 0$; 0 otherwise. The correlation integral is thus a measure of the probability that pairwise distances of points on the attractor in the state space is less than or equal to a specific distance r. A revised algorithm was introduced by Theiler [35],[42] to correct for autocorrelation effects in the time series by adding a new parameter called the Theiler window w.

According to Grassberger and Procaccia [11-12], the correlation integral $C(r)$ behaves as a power of v for small distances r, that is,

$$C(r) \propto r^v. \qquad (6)$$

The exponent v is defined as the correlation dimension D_2. The correlation dimension can be estimated from the local slope of the $\log$-$\log$ plot, i.e.,

$$v = \lim_{r \to 0} \frac{\log(C(r))}{\log(r)}. \qquad (7)$$
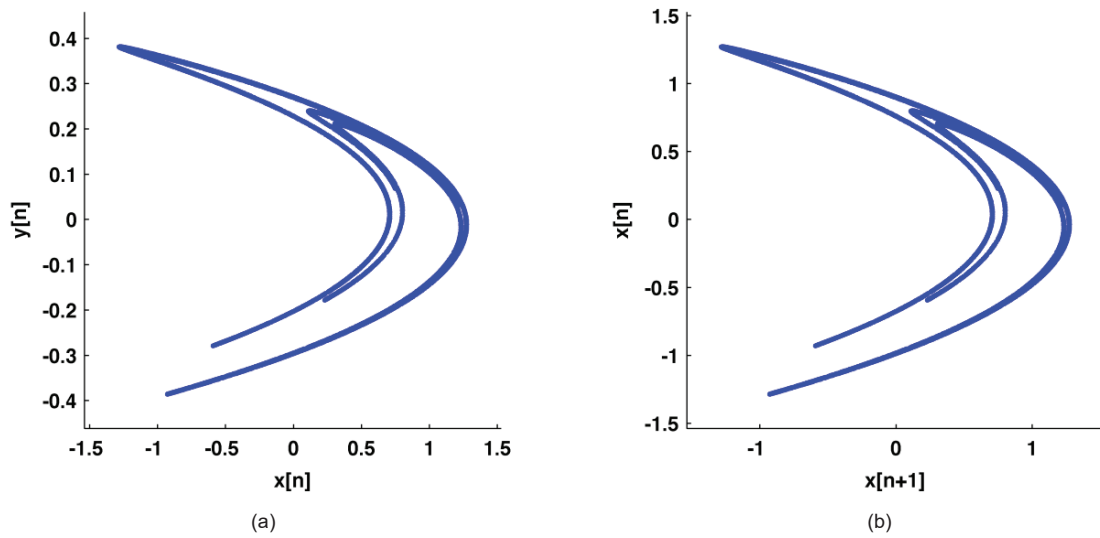
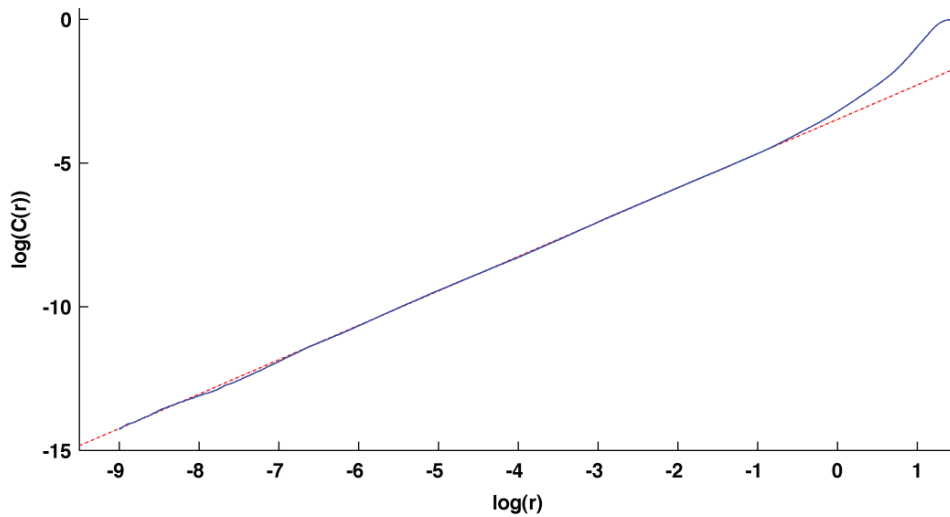Fig. 2 Hénon map: (a) the actual attractor and (b) the reconstructed attractor.

Fig. 3 The plot of the logarithm of correlation integral $C(r)$ versus the logarithm of distance r.

The correlation dimension quantifies the active degrees of freedom or the complexity of the dynamical system on the attractor.

4.3 Example of Nonlinear Dynamical Analysis

Consider a discrete nonlinear dynamical system known as Hénon map [43]:

$$\begin{aligned} x[n] &= 1 - \alpha x^2[n-1] + y[n-1] \\ y[n] &= \beta x[n-1] \end{aligned} \qquad (8)$$

where $\alpha = 1.4$ and $\beta = 0.3$. Fig. 1 illustrates the time series for the state variables of the dynamical system, i.e., $x[n]$ and $y[n]$. The Hénon attractor which is formed by a trajectory of state vector $\left(x[n], y[n]\right)^{\mathrm{T}}$ is shown in Fig. 2(a). This is a strange attractor where its dynamics never repeats the same state.

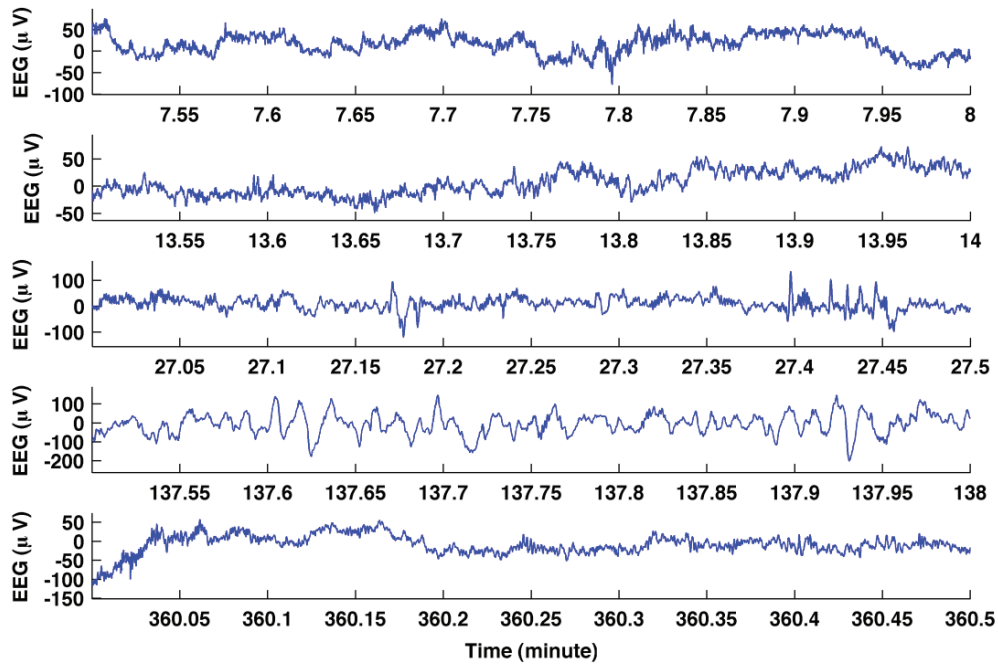If only the state variable x of the system is observed, a more comprehensive description of the

Fig. 4   Exemplary EEG signal associated with waking, stage 1, stage 2, slow wave sleep (stage 4), and REM (from top to bottom).

dynamics and the states of the system can be obtained by unfolding a time series of $x[n]$ into a higher dimensional state space. Fig. 2(b) illustrates the reconstructed attractor that is obtained using the method of time-delay embedding with the embedding dimension $m = 2$ and the delay time $\tau = 1$. Clearly, the reconstructed attractor shown in Fig. 2(b) provides the same dynamical properties on the state space as the Hénon attractor shown in Fig. 2(a).

A plot of the logarithm of correlation integral $C(r)$ computed using the Grassberger-Procaccia algorithm versus the logarithm of distance r is shown as a solid line in Fig. 3. The $\log$-$\log$ plot shown in Fig. 3 manifests a linear relationship between $\log(C(r))$ and $\log(r)$ for small distances r. The correlation dimension is thus estimated from the local slope of the $\log$-$\log$ plot. A dashed line plotted in Fig. 3 shows a straight line with a slope as the correlation dimension. The

correlation dimension of the Hénon attractor is about 1.2 which specifies the active degrees of freedom or the complexity of the dynamical system on the reconstructed attractor.

5. Nonlinear Dynamical Analysis of EEG Sleep
5.1 EEG Sleep Data

In this review, the EEG signal of the Fpz-Cz channel is analyzed and used for demonstration. The EEG signal is a part of the EEG data obtained from The Sleep-EDF Database available online at http://www.physionet.org/physiobank/database/sleep-edf/. Electrophysiological data, including Fpz-Cz EEG, Pz-Oz EEG, horizontal EOG, submental EMG and event marker, were obtained from 4 subjects who had mild difficulty falling asleep but were otherwise healthy during a night in the hospital. Subjects and recordings are more extensively described in [44].

The recordings were obtained using a digital telemetric system [45-46] with frequency response
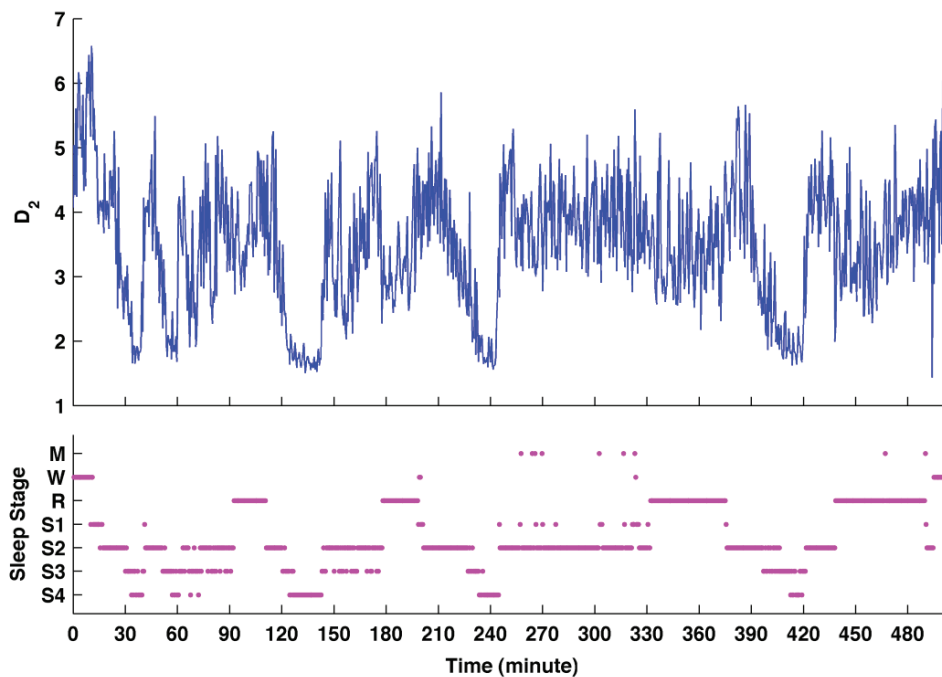
Fig. 5 Correlation dimension of the EEG sleep data corresponding to various sleep stages.

range (3-dB points) 0.03-1,000 Hz. The EEG data were digitized with a sampling rate of 100 Hz and a 14-bit A/D converter. The event marker was sampled at 1 Hz. Hypnograms are manually scored according to Rechtschaffen and Kales [8] based on the Fpz-Cz and Pz-Oz EEGs for every 30-second epoch of the EEG data, and classified into the following stages: waking (W), stage 1 (S1), stage 2 (S2), stage 3 (S3), stage 4 (S4), REM (R), and movement time (M).

Exemplary epochs of the EEG sleep data associated with waking, stage 1, stage 2, slow wave sleep (i.e., stage 4) and REM are illustrated in Fig. 4 (from top to bottom). In general, the EEG of wakefulness and the EEG of stage 1 sleep are low voltage and contain fast activity. The EEG of wakefulness however contains more fast activity contents. Similarly, REM sleep is also characterized by fast and low-voltage EEG. The EEG of stage 2 sleep clearly contains sleep spindle and K complexes. On the other hand, the EEG associated

slow wave sleep (e.g., sleep stage 4) is high voltage and contains slow wave activity, as the name suggested.

5.2 Characteristics of Dynamical Complexity of EEG Sleep

The correlation dimension of the EEG sleep data compared to the corresponding hypnogram is shown in Fig. 5. Evidently, the correlation dimension of the EEG sleep varies corresponding to different sleep stage. It is also observed that the correlation dimension tends to decrease with deeper sleep stages from stage 1 to stage 4. Furthermore, a box plot shown in Fig. 6 compares the distribution of correlation dimension of the EEG sleep data associated with various sleep stages.

Table 1 summarizes the statistical values including the mean and standard deviation values of the correlation dimension of the EEG sleep data associated with various sleep stages. The computational results show that the correlation dimension of the EEG during sleep tends to be
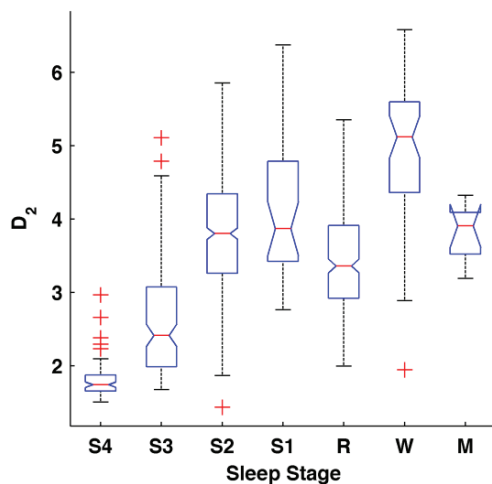
Fig. 6 Comparison of the correlation dimension of the EEG sleep data corresponding to different sleep stages.

Table 1 The statistical values of the correlation dimension of the EEG sleep data corresponding to various sleep stages.

Sleep Stage	Mean D_2	Std D_2
Stage 4	1.7969	0.2353
Stage 3	2.6554	0.8245
Stage 2	3.8042	0.7620
Stage 1	4.0635	0.8850
REM	3.4404	0.6684
Waking	4.9265	0.9844
Movement time	3.7987	0.3922

lower than that during waking, and the correlation dimension of the EEG associated with a deeper sleep stage tends to be lower than that associated with a lighter sleep stage. This therefore implies that the underlying neuronal networks of the brain during a deeper sleep stage are associated with a less complex dynamics.

To determine whether the EEG data associated with various sleep stages are actually different in the correlation dimension, the analysis of variance (ANOVA) is performed. From the ANOVA of the correlation dimensions of the EEG data associated with various sleep stages, the mean square, F-statistic and p-values are 44.5627, 87.6688 and $\ll 10^{-16}$, respectively. The ANOVA results thus imply that the means of the correlation dimension of the EEG data associated with various sleep stages are significantly different. Furthermore, the statistical analysis results suggest that there are significant differences between the correlation dimensions of the EEG data during waking and during sleep. The correlation dimension of the EEG data associated with slow wave sleep (sleep stages 3 and 4) is significantly different from that of the EEG data associated with lighter sleep stages, i.e., sleep stages 1 and 2.

6. Discussion

Temporal patterns of the EEG have been shown to provide insight into the various functional states of neuronal networks in the brain [2]. Traditional linear analysis techniques, both in time and frequency domains, have been very valuable computational tools for EEG analysis and provide specific information along with their own assumptions and limitations. For example, the spectral analysis that is a fundamental analysis tool specifies the amount of constituent components corresponding to different frequencies of the EEG. Accordingly, the sharp spectral peaks are one of the features used to classify the EEG and discriminate the state of the brain. Nonlinear dynamical analysis can provide complementary information to traditional linear analysis techniques leading to a deeper understanding of the brain dynamics associated with different functional/physiological states [47-48]. The nonlinear dynamical systems need to be analyzed as a global system rather than in a modularized way for the linear dynamical systems.

For the nonlinear dynamical analysis, the EEG signal, which is an observation of the dynamical system (i.e., neuronal networks), is first embedded

into a higher dimensional state space to reconstruct an attractor representing a trajectory of the state of the neuronal networks. Knowledge of the attractor as a geometrical object in the state space provides a deeper understanding of the behavior of the underlying dynamics. The properties of the neuronal networks, which are usually expressed in terms of the complexity of the dynamical system, are then characterized from the reconstructed attractor. The correlation dimension, one of the most commonly used measures in nonlinear dynamical analysis, quantifies the active degrees of freedom or the complexity of the dynamics of the system.

If a nonlinear dynamical system has a low-dimensional chaotic attractor [49], the correlation dimension D_2 is defined as the exponent v at small distances r such that the correlation integral $C(r)$ has a power-law characteristic. This exponent can be estimated as the slope of the $\log$-$\log$ plot between $C(r)$ and r [11-12]. Therefore the working hypothesis when using the nonlinear dynamical analysis is that the neuronal networks in the brain that generate spontaneous EEG have a low-dimensional attractor. Subsequently, the correlation dimension D_2 specifies the complexity of the underlying dynamics of the neuronal networks.

There is a controversy regarding the use of nonlinear dynamical analysis in the EEG analysis. The questions that emerge are related to whether or not the EEG time series contains nonlinear features and the underlying neuronal networks of the brain are nonlinear dynamical systems? The surrogate data testing introduced in [50-51] and refined in many subsequent works (e.g. [52-53]) is the method for verifying that a time series of interest is originated from a nonlinear dynamical system through statistical tests. If there is a statistically significant difference between the computational results of the original time series and the surrogate data time series, the *null hypothesis* that the original time series can be described by a stochastic linear model can be rejected [55].

From several studies (e.g. [15-16],[18-19],[54-55]), the consistent results were reported that the correlation dimension and the largest Lyapunov exponent decreased with sleep stages in adults from stage 1 to stage 4. Moreover, there were studies reporting that the correlation dimension of neonatal EEG sleep during quiet sleep stage was lower than during active sleep stage [48],[56]. There were however inconsistent findings regarding to the surrogate data testing. Using the surrogate data testing, evidence of nonlinear features in the EEG time series was found for a large number of subjects [51],[57-58]. On the other hand, some studies reported that it was not possible to distinguish the EEG time series from a linear stochastic time series (e.g. [59-60]).

As evidenced in the computational results, the brain, as a complex dynamical system, appears to contain both high-dimensional processes (high correlation dimension) and low-dimensional processes (low correlation dimension). During sleep the underlying dynamics of neuronal networks associated with lower-dimensional processes is less complex while the underlying dynamics of neuronal networks during waking associated with higher-dimensional processes is more complex. In addition, a decrease in the correlation dimension from lighter sleep stages to deeper sleep stages further indicates that the underlying dynamics of neuronal networks during a deeper sleep stage is less complex. There is thus a loss in complexity or dimensionality of the underlying dynamics of neuronal networks corresponding to deeper sleep stages. A loss in complexity or dimensionality of the underlying dynamics of neuronal networks is resulted from stronger coupled oscillations (stronger synchronization) of neuronal networks at deeper

sleep stages. This leads to larger amplitude of EEG during a deeper sleep stage.

The use of the nonlinear dynamical analysis can be viewed in two aspects. In one aspect, the concepts of nonlinear dynamical analysis and the computational tools for nonlinear dynamical analysis are used to explain the behaviors and the characteristics of the dynamical system, which in this presentation the dynamical system of interest is the neuronal networks in the brain. This provides more understandings on the underlying dynamics of the corresponding dynamical systems and also provides complementary information to traditional linear analyses. For instance, from the computational results presented in this article, the difference in the correlation dimension corresponding to various sleep stages suggests that the underlying dynamics of neuronal networks in the brain associated with different sleep stages are different. Furthermore, it is revealed that there is a stronger synchronization of neuronal networks at deeper sleep stages which results a loss in complexity or dimensionality of the underlying dynamics of neuronal networks in the brain. In another aspect, the measures obtained from the computational tools for the nonlinear dynamical analysis, which generally quantify the complexity of the dynamical system, are used for classifying and identifying the states of the dynamical system. The computational results show that the correlation dimensions of the EEG associated with various sleep stages are substantially different from each other. Accordingly, the correlation dimension has been used as a measure for sleep stage discrimination.

This article presents the application of the nonlinear dynamical analysis, using the correlation dimension, in particular, as a measure, in EEG sleep analysis. However, the nonlinear dynamical analysis techniques have been applied to analyze EEG in a number of clinical situations. Epilepsy is another important application for nonlinear dynamical analysis. It is observed that the correlation dimension of the EEG during epileptic seizure activity is substantially less than that during non-seizure activity. This might be due to a pathological loss of complexity [10]. In epilepsy research, the nonlinear dynamical analysis has been used for localization of the epileptogenic zone, and detection and prediction of epileptic seizures [10]. Recently, as prediction or anticipation of epileptic seizures using the nonlinear dynamical analysis has become a hot topic [10], there are a number of reviews in this topic (for example, [61-63]) have appeared.

In conclusion, in this article the basic concepts of nonlinear dynamical analysis are introduced, and also the application of nonlinear dynamical analysis in EEG sleep analysis is demonstrated. The nonlinear dynamical analysis techniques have been applied for various applications. However, caution and care must be used in both computation and interpretation of the results because the proper computation and interpretation of the correlation dimension involves some pitfalls, caution and care must be used in both computation and interpretation of the results. In practice, the estimate of correlation dimension may not exactly quantify the active degrees of freedom of the underlying dynamics of the brain. The nonlinear dynamical measures obtained from the nonlinear dynamical analysis such as the correlation dimension can however be used for characterizing the physiological or pathological states of the brain.

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