Ordering Quantity Decisions for Deteriorating Inventory Systems

Pavee Siriruk

Faculty of Engineering, Suranaree University of Technology, Muang, Nakhon Ratchasima 30000

Tel : 08-1470-7706 E-mail: pavee@g.sut.ac.th

Abstract

Due to inherent characteristics of products, items are subject to lose their value or usability overtime. A perishable inventory system has received increasing attentions in the past years. An optimal ordering policy for deteriorating inventory system has become more and more important. This paper has focused on the fixed life time and continuous review (Q, r) perishable inventory systems with a positive lead time. A method to determine the optimal order quantity that minimizes the total expected cost for perishable items is developed. The results indicate that the expected cost function is convex in ordering quantity. The solution methodologies to obtain the optimal policy are presented. Moreover, the results of sensitivity analysis show that the relationships between the holding costs, outdating costs, and ordering costs, and the demand with the optimal ordering quantity are varied.

Keywords: Continuous review inventory systems; perishable products.

1. Introduction

The ever increasing attention in determining an optimal ordering policy for perishable inventory systems has added a new level of complexity to the task of managing inventory. The traditional inventory management problem works well under deterministic demand assumption. However, when the demand is assumed to be a random variable and items are assumed to be perishable to reflect the real life situations, the classical approach leads to a poor performance and unsatisfactory management.

Nahmias [1] has done a comprehensive review of previous research on perishable inventory systems. His review has focused on perishable inventory ordering policies. Most of the literatures examined deterministic and stochastic demand models for fixed-lifetime perishables. In the deterministic case, ordering policies are developed in such a way that items will never perish. For the stochastic demand, most of the research is based on a single product. The optimal policies are developed both for two periods of lifetime problem and the one with a general period of lifetime 'm'. Nahmias also mentioned in his review that the first analysis of optimal policies for a fixed life perishable commodity was begun by Van Zyl. This topic was extended later by Fries [2] and Nahmias [3-5].

Fries [2] has developed an optimal policy for a perishable commodity with a fixed lifetime in a finite horizon problem with continuous demand where the quantity of goods expiring in any period. He has focused on no backlogging case where the demand distribution is stationary over time. Nahmias [3] has developed a bound on the outdating cost that is a function of the total inventory on hand. He also approximated the optimal ordering policy myopically for the new problem. Results obtained from the numerical analysis suggest that the approximation is effective to the optimal policy. In another paper, Nahmias [4] generalizes the case with the 'm' period product to extend his model in which ordering, holding, stock out, and outdating costs are charged. In order to analyze the dynamics of the perishable process, Nahmias has used a multi-echelon structure where the i^{th} echelon corresponding to the amount of product would outdate exactly 'i' periods of time after its receipt on order in the future. The solution of the multi periods dynamic model for the single ordering decision problem has lead to the analysis of the perishable nature of the inventory. When an outdating occurs each time, the cost of inventory is incurred. Nevertheless, the total cost function to be minimized is determined as pseudo convex. Nahmias and Wang [5] have developed an approximation of the fixed life continuous review problem. They have considered the effect of the lead time in the decay problem and have developed and tested a heuristic (Q, r) model, which allows for a random demand, an exponential decay and a positive lead time for ordering.

Most of the previous studies such as those of Nahmias [6], Cohen [7], and Chazan and Gal [8] have concentrated on the periodic review and multi-period lifetime problem with zero lead time.

In recent works, Sivakumar, Anbazhagan, and Arivarignan [9] have developed a two-commodity perishable stochastic inventory system under continuous review at a service facility with a finite waiting room. A joint reordering policy is created with a random lead time for orders. In addition, Ignaciuk and Bartoszewicz [10] proposed linear-quadratic optimal control for periodic–review perishable inventory systems. Unlike other papers that mainly deploy heuristics and static optimization, they have applied formal methodology of discrete-time dynamical optimization. This methodology can solve the optimal control problem analytically.

Generally, goods having finite lifetimes are subject to the perishables. Hence, a perishable inventory, such as fashion garments, blood, and drugs, is one in which all the units of one material item in stock will be outdated, if not being used before the expiration date, resulting in an additional outdating cost of perished items. Therefore, it is required that the outdating issue is taken into account to reflect the real-life situations. In this work, the focus is placed on the fixed life time and continuous review (Q, r) perishable inventory systems with a positive lead time. The objective of this research is to determine the optimal ordering quantity that minimizes the expected cost, for a perishable item over a finite horizon.

The traditional model is extended in which the outdating costs are considered. An approximate expected outdating of the current order size from [11-12] is used to obtain an optimal ordering policy under positive order lead time, which minimizes the total expected average cost. A solution methodology to find the optimal order quantity is presented. In addition, the behavior of the expected cost function that is composed of ordering cost, holding cost and outdating cost, is analyzed and shown that it is convex in order quantity.

The paper is organized as follows. In section two, a mathematical model for the problem is presented. The solution methodology is described in section three. In section four, the numerical examples with preliminary results are shown. The sensitivity analysis is carried out in section five. In section six, primary contributions of this paper and suggestions for the potential future research are summarized.

2. Mathematical Model

In this section, we will introduce assumptions of the model as well as model development.

2.1 Assumptions

The following assumptions and notations will be used throughout this paper:

1. One perishable item is considered. It is assumed that each unit of the item has a fixed lifetime equal to m and no loss or decrease in utility occurring before m time units.

2. Inventory levels are reviewed continuously. When the inventory level reaches the reorder point r an order size Q where $Q > 0$ is placed.

3. All units of a replenishment order arrive in fresh condition.

4. A positive order lead time L for replenishment; L is less than the lifetime m.

5. The demand in unit time t, d_t is a nonnegative random variable and normally distributed. It is also assumed that if $\Phi(t)$ is cumulative normal demand by time t then $\Phi(t)$ is a stochastic process with stationary independent increments.

6. d_{m+L} is a random variable $\Phi(m+L)$ has normal density $f_{m+L}(d_{m+L})$ and mean $(m+L)d$.

7. No shortage is allowed (all demands are met).

8. Units are always depleted according to an FIFO (First in first out) issuing policy.

9. If each unit has not been used to meet a demand before the expiration date, it must be discarded and an outdate cost equals to W is charged.

10. The cost which spent to keep and maintain each unit of goods per unit time is called holding cost, h.

11. Given that X is a random variable during lead time demand. A parameter D stands for the total demand in a year and K is the fixed ordering cost per order.

2.2 Model Development

The traditional model is extended in which the outdating costs are considered. In this paper, we assume that the total expected cost function consists of ordering cost, holding cost, and outdating cost as shown below.

$$EC(Q,r) = E[\,Ordering\ cost + Holding\ cost + Outdatting\ cost\,] \tag{1}$$

Based on the assumption that the demand is normally distributed, reorder point can be calculated by using the safety factor of normal distribution. Therefore, the following relationship can be used to express the lead time demand.

$$\mu_X = E[\,X\,] = L \times E[\,D\,] = DL \tag{2}$$

$\sigma_X^2 = Var[\,X\,] = L \times Var[\,D\,]$, so that.

$$\sigma_X = \sigma\sqrt{L} \tag{3}$$

$Pr\{\,X \geq r\,\}$ is the probability of stockout during the lead time, then choosing r such that $Pr\{\,X \geq r\,\} = q$, where q is the allowable stockout probability. Due to the normal distributed probability, the $Pr\{\,X \geq r\,\} = q$ becomes $Pr\{\,Z \geq k\,\} = q$, where $k = \frac{r - \mu_X}{\sigma_X}$ is the safety factor. Thus, $r = \mu_X + k\sigma_X$

From (2) and (3), the reorder point using the safety factor becomes,

$$r = DL + k\sigma\sqrt{L} \tag{4}$$

After approximating the reorder point by the safety factor, equation (1) can be recognized as

$$EC(Q) = E[\,Ordering\ cost + Holding\ cost + Outdatting\ cost\,] \tag{5}$$

Ordering Cost

$$Ordering\ cost = K(number\ of\ cycles) = \frac{KD}{Q} \tag{6}$$

Holding Cost

The expected inventory level can be obtained by

$$E[Inventory\ level] = \frac{1}{2}[Inventory\ level\ at\ the\ beginning\ of\ a\ cycle + Inventory\ level\ at\ the\ end\ of\ a\ cycle\,]$$

The inventory level at the beginning of a cycle can be computed by

$$r - E[\,X\,] + Q \tag{7}$$

The inventory level at the end of a cycle can be calculated by

$$r - E[\,X\,] \tag{8}$$

From equation (7) and (8), the expected inventory level can be written as

$$E[Inventory\ level] = \frac{1}{2}(r - E[X] + Q + r - E[X]) = \frac{Q}{2} + r - E[X] \quad (9)$$

The holding cost function is known as

$Holding\ Cost = h \times E[Inventory\ level]$

From equation (9), holding cost function can be expressed as

$$Holding\ Cost = h\{\frac{Q}{2} + r - E[X]\}$$

From (2) and (4), the holding cost function can be rewritten as

$$Holding\ Cost. = h\{\frac{Q}{2} + k\sigma\sqrt{L}\} \quad (10)$$

Outdating Cost

The expected outdating approximation is borrowed from [11, 12]. The approximate expected outdating of the current order size with a positive lead time can be obtained by

$$Expected\ Outdating\ Quantity = \int_0^{r+Q}(r + Q - d_{m+L})f_{m+L}(d_{m+L})dd_{m+L} - \int_0^{r}(r - d_{m+L})f_{m+L}(d_{m+L})dd_{m+L} \quad (11)$$

Hence, the expected outdating cost can be expressed as

$$Expected\ Outdating\ Cost = W\left[\int_0^{r+Q}(r + Q - d_{m+L})f_{m+L}(d_{m+L})dd_{m+L} - \int_0^{r}(r - d_{m+L})f_{m+L}(d_{m+L})dd_{m+L}\right] \quad (12)$$

The total Expected Cost Function

From equations (5), (6), (10), and (12), the total expected cost function can be written as

$$EC(Q) = \frac{KD}{Q} + h\left\{\frac{Q}{2} + k\sigma\sqrt{L}\right\} + W\left[\int_0^{r+Q}(r + Q - d_{m+L})f_{m+L}(d_{m+L})dd_{m+L} - \int_0^{r}(r - d_{m+L})f_{m+L}(d_{m+L})dd_{m+L}\right] \quad (13)$$

By using Leibniz's rule, it can be shown that the first derivatives of equation (13) is

$$\frac{\partial EC(Q)}{\partial Q} = -\frac{KD}{Q^2} + \frac{h}{2} + W\left[\int_0^{r+Q} f_{m+L}(d_{m+L})dd_{m+L}\right] \quad (14)$$

From (14), the second derivative of the total expected cost function using Leibniz's rule can be written as

$$\frac{\partial^2 EC(Q)}{\partial Q^2} = \frac{2KD}{Q^3} + W\left[\int_0^{r+Q} f_{m+L}(r+Q)d(r+Q)\right] \quad (15)$$

Since equation (15) is greater than 0, $\forall Q > 0, EC(Q)$ is a convex function.

As the total cost function is a convex function, equating equation (14) to zero and solving yields the optimal Q^*.

$$\Phi\left(r + Q^*\right) = \frac{KD}{WQ^{*2}} - \frac{h}{2W}, \quad or$$

$$\Phi\left(r + Q^*\right) - \frac{KD}{WQ^{*2}} + \frac{h}{2W} = 0 \quad (16)$$

Finally, the proposed model (16) is compared with the conventional EOQ model to show that the results of proposed model are more realistic than the conventional EOQ.

The conventional EOQ equation is known as

$$Q^2 = \frac{2KD}{h} \quad (17)$$

After simplifying proposed model, equation (16) can be written as

$$Q^2 = \frac{2KD}{[2W\Phi(r+Q) + h]} \quad (18)$$

It can be shown that equation (17) ≥ (18) by the following proof

$$\frac{2KD}{h} \geq \frac{2KD}{[2W\Phi(r+Q) + h]}$$

$$= \frac{1}{h} \geq \frac{1}{[2W\Phi(r+Q) + h]}$$

$$= h \leq 2W\Phi(r+Q) + h$$

We know that $2W\Phi(r+Q) \geq 0$, since $W \geq 0$ and $0 \leq \Phi(r+Q) \leq 1$.

Therefore, it can be concluded that the conventional EOQ model is the upper bound of the proposed model.

3. Solution Methodology

The optimal order quantity that minimizes total expected cost can be calculated by using equation (16). However, equation (16) cannot be solved directly due to the computational complexity of the normal density function, which represents the demand distribution. When all fixed values of parameters are given, except for the order quantity, one method that can solve equation (16) is to find a value of Q^* that makes equation (16) equal to 0. Some heuristic algorithms can be applied. In this paper, we choose to use a function in Microsoft Excel called Goal Seek to find an optimal solution. This function keeps randomizing the value of Q^* until the value, which makes equation (16) equal to 0, is found. After solving equation (16), the optimal order quantity that minimizes the total expected cost is obtained.

4. Numerical Examples

In this section, we illustrate a numerical example. Assume that $h = 1$, $L = 1$, $K = 10$, $W = 5$, and demand is normally distributed with mean 10 and variance 10, and the stockout probability is 0.10. Based on the safety factor, the value of k is determined as 1.28 from the unit normal table. After the value of k is calculated, the reorder point is obtained by using equation (4). In this paper, we decided to find an optimal solution of (16) by using the Goal Seek function in Microsoft Excel. Fig. 1 demonstrates how to set up the numerical example in Microsoft Excel. All parameters in this example are inputs of the Goal Seek. The output of this function is the optimal order quantity. Goal Seek finds the optimal order quantity, Q^*, in equation (16) equal to 4.26. Table 1 demonstrates the results.

Table 1. Results of the numerical example

Parameters	Values
k	10
W	5
L	1
h	1
D	10
σ^2	10
k	1.28
r	14.05
Variable	Values
Q^*	4.26

5. Sensitivity Analysis

The sensitivity analysis is implemented to study how the optimal order quantity behaves when the selected parameters are changed. The parameters used in the analysis are the holding cost per unit, outdating cost per unit, ordering cost, demand and lead time. The results of sensitivity analysis when changing holding cost per unit is shown in Table 2. Table 3 shows the effects of changing outdating costs on the order quantity. When the ordering costs are altered, the results of sensitivity analysis are displayed in Table 4. The results of sensitivity analysis when changing the demand is shown in Table 5. Table 6 shows the effects of lead time on the order quantity.

The results of sensitivity analysis show that the relationships between the holding costs, outdating costs, and ordering costs, and the demand with the optimal ordering quantity are varied. For instance, while the holding cost increases, the optimal ordering quantity decreases. On the other hand, while the ordering cost increases, the optimal ordering quantity increases. The increase in outdating cost leads to the decrease of the optimal ordering quantity. Additionally, the increase in demand leads to the increase of the optimal ordering quantity. While the lead time increases to 1.2, the normal cumulative distribution function equals to one indicating that the optimal ordering quantity would not change after this point.

The values of optimal ordering quantity obtained from the proposed model are also compared with the values obtained from the classic EOQ. The solutions indicate that the classic EOQ is the upper bound of the proposed model. The results of the classic EOQ are included in the tables 2 to 6.

From a managerial perspective, it can be concluded that the ordering size obtained from the solution of the proposed model, is better than the conventional EOQ, since it considers outdating of the perishable products. Additionally, the model can also help to determine the optimal ordering quantity that will minimize the total cost incurred during a planning horizon. From a strategic perspective, the results of this research imply that the implementation of the solution methodology given in this work will provide benefits in determining the optimal ordering quantity by considering the outdating of the perishables.

6. Conclusions

This paper presented a solution methodology for the fixed life time perishable inventory systems with a positive lead time to determine the optimal ordering quantity. It was shown that the cost function to be minimized is convex in ordering quantity. Direct computation of an optimal ordering policy for a perishable product has been shown. Finally, the sensitivity analysis is implemented to study how the optimal order quantity behaves as the selected parameters are modified.

Different continuous and discrete demand distributions for perishables require further study. Furthermore, the reorder point should be considered as a variable to approximate the optimal ordering quantity more precisely. An extension of this study can be conducted to formulate more accurate estimates. Various optimal ordering policies can be compared with the proposed quantity to evaluate the performance of the solution method presented in this work. A comparison of the proposed ordering policy with simulated policies can also be carried out in order to further analyze the model. Finally, upper and lower bounds for the expected outdating can be obtained to determine a confidence interval for the outdating quantity.

References

[1] Nahmias, S. 1982. Perishable Inventory Theory: A Review. Operations Research, 30:680-708.

[2] Fries, B. E. 1975. Optimal Ordering Policy for a Perishable Commodity with Fixed Lifetime. Operations Research, 23:46-61.

[3] Nahmias, S. 1976. Myopic Approximations for the Perishable Inventory Problem. Management Science, 22:1002-1008.

[4] Nahmias, S. 1975. Optimal Ordering Policies for Perishable Inventory-II. Operations Research, 23: 735-749.

[5] Nahmias, S. and Wang, S. 1979. A Heuristic Lot Size Reorder Point Model for Decaying Inventories. Management Science, 25:90-97.

[6] Nahmias, S. 1977. Higher Order Approximations for the Perishable Inventory Problem. Operations Research. 25:630-640.

[7] Cohen, M. A. 1976. Analysis of Single Critical Number Ordering Policies for Perishable Inventories. Operations Research, 24:726-741.

[8] Chazan, D. and Gal, S. 1977. A Markovian Model for a Perishable Product Inventory. Management Science, 23:512-521.

[9] Sivakumar, B., Anbazhagan, N. and Arivarignan, G. 2005. A Two-Commodity Perishable Inventory System. ORiON, 21:157-172.

[10] Ignaciuk, P. and Bartoszewicz, A. 2012. Linear-Quadratic Optimal Control of Periodic-Review Perishable Inventory Systems. IEEE Transactions on Control Systems Technology, 20:1400-1407.

[11] Chiu, H. N. 1995. An Approximation to the Continuous Review Inventory Model with Perishable Items and Lead Times. European Journal of Operations Research, 87:93-108.

[12] Chiu, H. N. 1999. A Good Approximation of the Inventory Level in a *(Q, r)* Perishable Inventory System. Rairi Operations Research, 33:29-45.

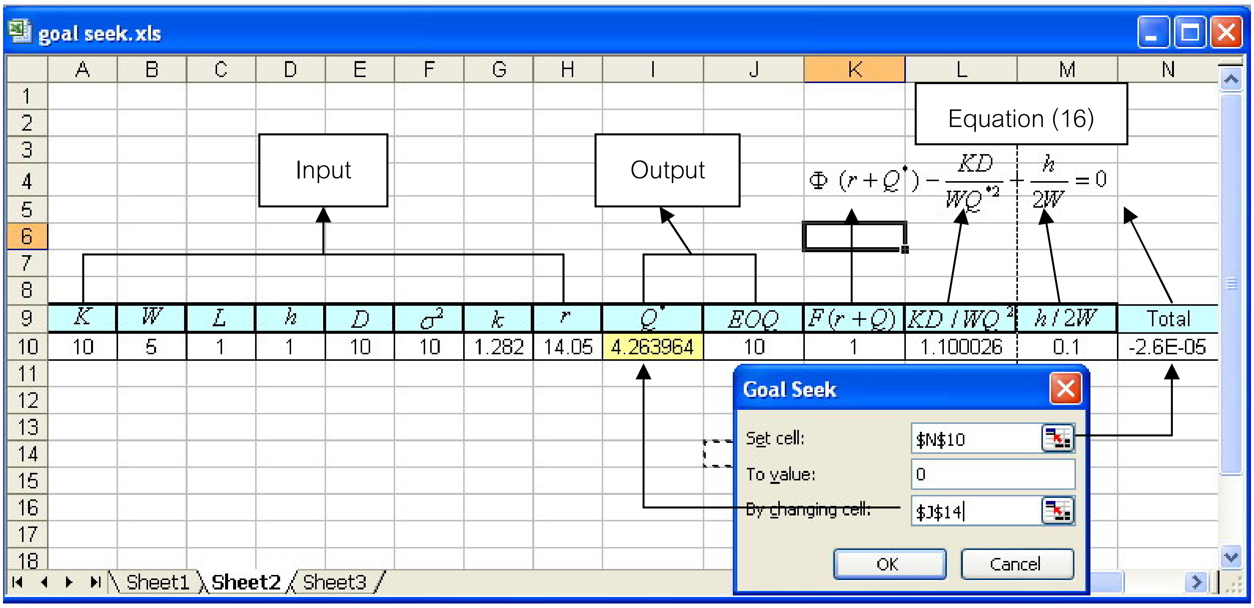

Fig.1 The input and output of the numerical example using Goal Seek function

Table 2. The results of sensitivity analysis of changing the holding cost

K	W	L	h	D	σ^2	k	r	Q^*	EOQ
10	5	1	1	10	10	1.2815	14.05246	4.270556	14.14214
10	5	1	2	10	10	1.2815	14.05246	4.09093	10
10	5	1	2.5	10	10	1.2815	14.05246	4.009358	8.944272
10	5	1	2.51	10	10	1.2815	14.05246	4.007761	8.926437
10	5	1	3	10	10	1.2815	14.05246	3.93059	8.164966
10	5	1	3.5	10	10	1.2815	14.05246	3.857792	7.559289
10	5	1	100	10	10	1.2815	14.05246	1.351085	1.414214

Table 3. The results of sensitivity analysis of changing the outdating cost

K	W	L	h	D	σ^2	k	r	Q^*	EOQ
10	5	1	1	10	10	1.2815	14.05246	4.272207	14.14214
10	6.5	1	1	10	10	1.2815	14.05246	3.791122	14.14214
10	8	1	1	10	10	1.2815	14.05246	3.445038	14.14214
10	8.1	1	1	10	10	1.2815	14.05246	3.425209	14.14214
10	9	1	1	10	10	1.2815	14.05246	3.259957	14.14214
10	9.5	1	1	10	10	1.2815	14.05246	3.178177	14.14214
10	10	1	1	10	10	1.2815	14.05246	3.103544	14.14214

Table 4. The results of sensitivity analysis of changing the ordering cost

K	W	L	h	D	σ^2	k	r	Q^*	EOQ
10	5	1	1	10	10	1.2815	14.05246	4.273002	14.14214
20	5	1	1	10	10	1.2815	14.05246	6.032123	20
25	5	1	1	10	10	1.2815	14.05246	6.742922	22.36068
30	5	1	1	10	10	1.2815	14.05246	7.385484	24.4949
40	5	1	1	10	10	1.2815	14.05246	8.526783	28.28427
50	5	1	1	10	10	1.2815	14.05246	9.534013	31.62278
55	5	1	1	10	10	1.2815	14.05246	9.999313	33.16625

Table 5. The results of sensitivity analysis of changing the demand

K	W	L	h	D	σ^2	k	r	Q^*	EOQ
10	5	1	1	5	10	1.2815	9.052459	3.031799	10
10	5	1	1	10	10	1.2815	14.05246	4.272147	14.14214
10	5	1	1	15	10	1.2815	19.05246	5.227161	17.32051
10	5	1	1	25	10	1.2815	29.05246	6.74404	22.36068
10	5	1	1	35	10	1.2815	39.05246	7.977387	26.45751
10	5	1	1	50	10	1.2815	54.05246	9.534013	31.62278
10	5	1	1	55	10	1.2815	59.05246	9.999312	33.16625

Table 6. The results of sensitivity analysis of changing the lead time

K	W	L	h	D	σ^2	k	r	Q^*	EOQ
10	5	1	1	10	10	1.2815	14.05246	4.271766	14.14214
10	5	1.2	1	10	10	1.2815	16.43925	4.264797	14.14214
10	5	1.5	1	10	10	1.2815	19.96323	4.264843	14.14214
10	5	1.8	1	10	10	1.2815	23.43694	4.263975	14.14214
10	5	2	1	10	10	1.2815	25.73104	4.264147	14.14214
10	5	2.2	1	10	10	1.2815	28.01077	4.263911	14.14214
10	5	2.5	1	10	10	1.2815	31.4075	4.263912	14.14214