Development of a Taper Equation for Teak (Tectona grandis L.f.) Growing in Western Thailand

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ABSTRACT

Teak is an important and valuable tropical hardwood species. In this study, we developed and evaluated suitable taper equations for teak growing in Western Thailand using a formulation of Goodwin cubic polynomial model combined with a bark thickness model. The best taper model calibration was selected based on goodness-of-fit and leave-one-out cross validation statistical testing. In total, 12 different model calibrations were tested, with Thong Pha Phum (TPP) 2 being the most suitable for teak in Western Thailand. The mean prediction error of three validation statistics: (prediction of diameter under bark given height; prediction of height given diameter under bark; and prediction of under bark volume given log length) were within 10% and the overall validation index was 5.454, which was the lowest when compared to other calibrations. A comparison of TPP 2 with a teak taper equation developed for Northern Thailand, using a graphical analysis of the stem shape and bark thickness, indicated that the teak trees growing in the two regions have similar stem shapes, but the trees in Western Thailand tend to have a thicker bark. These results will also help in further work as they indicate that bark thickness equations are particularly important.

Keywords: Teak/ Taper equation/ Stem volume/ Bark thickness/ Western Thailand

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1. INTRODUCTION

Taper equations are used to predict the diameter under bark at all stem heights and can also estimate the volume and value of standing trees. In combination with log-bucking algorithms, they are also used to maximize the log value or optimize market constraints. The taper of a tree is a combination of natural growing processes and associated silvicultural practices. Therefore, the development of taper equations that are specific to different species, sites, and silvicultural practices can be used to accurately estimate the volume and value of a standing tree (Tasissa and Burkhart, 1998; Klos et al., 2007; Fonweban et al., 2011; Sabatia and Burkhart, 2014).

Teak is one of the most valuable tropical hardwood species in the world (Kollert and Kleine, 2017). In Thailand, the Forest Industry Organization (FIO) has around 80,000 ha under teak plantations, occurring naturally in Northern Thailand, in addition to some parts in Western Thailand (Hansen et al., 2014; FIO, 2018). The climate in the two regions is different, with Northern Thailand experiencing a tropical savanna climate, with an annual rainfall between 1,000-1,400 mm/year, while western Thailand is influenced by a tropical monsoon climate, with an annual rainfall ranging between 1,600-2,000 mm/year (Beck et al., 2018; Thai Meteorological Department, 2020). This may cause differences in the stem shape of trees growing in the two regions.

FIO generally carries out a pre-harvest inventory to estimate the volume and value of standing trees based on volume tables. However, a general volume table cannot be used to estimate the optimal value of standing trees, especially with teak log grade specifications, which are based on mid-log diameter and length classification. Warner et al. (2016) proposed that the FIO can use taper equations with log-bucking algorithms to optimize the stem crosscutting for maximum value and consequently reported the first taper model for teak plantations in Thailand. However, their equation was developed using trees sampled from teak plantations in Northern...
Thailand and was not tested on trees in plantations of Western Thailand. Consequently, this study aimed to evaluate the taper model calibration for teak growing in Western Thailand and to compare the tree shape in western and northern regions of the country.

2. METHODOLOGY

2.1 Study site

The study site is located in the Thong Pha Phum plantation in Kanchanaburi Province, Thailand (Figure 1). The teak plantation covers an area of 2,422 ha or 72% of the total area. The mean elevation of the plantation is 400 m.a.s.l. and the area was originally a mixed deciduous forest. This plantation practices selective thinning by removing all small, dead, and deformed trees. Additionally, no serious tree diseases have been reported in this area (FIO, 2016). All trees were planted at 4 m x 4 m spacing to increase profitability and management efficiency for medium and poorer quality sites (Noda and Himmapan, 2014).

![Figure 1. Planted area under teak at the Thong Pha Phum Plantation, Kanchanaburi Province](image)

2.2 Tree sampling and measurement

We sampled 60 trees from thinned stands, which included eight age classes (15, 16, 17, 18, 23, 24, 30, and 36 years). Only those trees which had a diameter at breast height (DBH) over bark of up to 15 cm and a total height of up to 10 m were sampled, as such trees were large enough to produce a commercial teak sawlog (Warner et al., 2016). Apically dominant trees were selected, as the equation was primarily used to estimate the sawlog volumes, and irregularly formed trees generally removed from thinned stands. Diseased, deformed, or dead trees were not sampled as such trees are not representative of the general population (Brooks et al., 2007; Nigh and Smith, 2012). The sample size distribution was determined based on size of the area in each age class and the sampled trees were randomly selected, with a minimum of two trees required in class (Vanclay, 1982).

We followed the tree measurement procedures used for sampling teak trees in Northern Thailand (Warner et al., 2016). Briefly, a diameter tape was used as a representative (no obvious defect or exceptional bumps in the diameter) measurement of the diameter above ground level at 0.3 m, 0.5 m, and 0.8 m, where pronounced buttressing is often presented and finally at the breast height (1.3 m above ground on the uphill side of a tree). As noted in other
studies, pronounced buttressing may increase the model variability, resulting in a poor prediction of the lower bole (Fonweban et al., 2011; Sumida et al., 2013; Westfall and Scott, 2010). Therefore, the trees were felled and cut at 0.3 m, 0.5 m, and 0.8 m and a digital photograph of the cross section was taken alongside a steel ruler as a standard for metric scale measure, to provide for any corrections to the sectional area of any pronounced buttressing. Any diameter data affected by pronounced buttressing were adjusted using a cross sectional area analysis using the digital images as proposed by Warner et al. (2017). Further, the measurement of diameter over bark and bark thickness was done usually at an interval of 2 m above the breast height at a representative point until the main stem was no longer apparent. The ground diameter under bark was estimated assuming a convex equation and the sectional under bark volumes for each tree were calculated using Smalians formula. Summary statistics are provided in Table 1.

### Table 1. Summary statistics of 60 sampled trees

<table>
<thead>
<tr>
<th>Tree or stand variable</th>
<th>Minimum</th>
<th>Mean (± SD)</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>DBH over bark (cm)</td>
<td>15.3</td>
<td>29.5±9.0</td>
<td>52.7</td>
</tr>
<tr>
<td>DBH under bark (cm)</td>
<td>13.4</td>
<td>26.7±8.7</td>
<td>48.6</td>
</tr>
<tr>
<td>Total height (m)</td>
<td>12.6</td>
<td>22.4±4.4</td>
<td>32.0</td>
</tr>
<tr>
<td>Double bark thickness (mm)</td>
<td>12.0</td>
<td>28±8.0</td>
<td>52.0</td>
</tr>
<tr>
<td>Age (year)</td>
<td>15.0</td>
<td>24±4.3</td>
<td>36.0</td>
</tr>
<tr>
<td>Numbers of sample points per tree</td>
<td>8.0</td>
<td>11±1.8</td>
<td>15.0</td>
</tr>
</tbody>
</table>

#### 2.3 Taper modeling

The Goodwin (2009) taper model was selected for calibration as it has been successfully used for teak growing in Northern Thailand (Warner et al., 2016). The model is described by a cubic function comprising of hyperbolic and parabolic terms (Goodwin, 2009) (Equation 1); it is algebraically invertible and integrable and can accommodate one or two diameter constraints, neither of which needs to be at the breast height. Only one diameter constraint was used in this work. The primary has three parameters ($\beta_1$, $\beta_2$ and $\beta_3$), which are modelled as secondary functions of tree, stand, and regional variables. Second stage models in Goodwin (2009) did not include DBH under bark and therefore the diameter constraints in that model could be at any height. However, DBH was an important term in the second stage models for the present work, and so it was sensible to constrain the primary model with DBH under bark.

\[
d_{ub} = (H-h)\left[\frac{\beta_1}{(1+\beta_2H_{BH})(1+\beta_3H)} + \beta_3(h-BH) + \frac{D_{ub}}{H-BH}\right] + f_i
\]

Where; $d_{ub}$ is the diameter under bark (cm) at height $h$ (m), $D_{ub}$ is diameter under bark (cm) at breast height (BH) (m), $H$ is the total height of the tree (m), and $c_1$, $d_i$, and $f_i$ are second stage candidate coefficients for the terms which have been reported to be significant for other species and regions (Warner et al., 2016; Goodwin, 2009; Wang and Baker, 2007).

The Goodwin model uses $D_{ub}$, $H$, and $h$ to predict $d_{ub}$ and this approach is suitable when applied at two heights to estimate the under bark volume of the section between the two heights. As such, a separate bark thickness model is required to utilize $D_{ub}$ as an input parameter, which is measured at the time of inventory. Therefore, the sample tree data were used to develop a bark thickness model using a power law in equation 2 and combined with equation 3 to convert the measured $D_{ub}$ to $d_{ub}$ to be used in the taper model as:

\[
BT2 = a (d_{ub})^b, \quad (2)
\]

\[
d_{ub} = d_{ub} - \frac{BT2}{10^n}. \quad (3)
\]

Where; $BT2$ is the double bark thickness (mm), $a$ and $b$ are model coefficients, and $d_{ub}$ is the diameter over bark (cm).

Nonlinear fixed effects analysis was used in the taper modeling. Model calibrations were started with 12 second stage candidate coefficients and insignificant terms (p-value>0.05) were neglected, resulting in different candidate calibrations having different terms or number of second stage candidate coefficients. Each calibration was named as TPP,
which was the abbreviated form of Thong Pha Phum. The best calibrations were parsimonious models and were those with the best combined goodness-of-fit (GOF) statistics, namely a low residual standard error (RSE), a high adjusted coefficient of determination (adjusted R²), and a low Bayesian information criterion (BIC) (Fonweban et al., 2011; Hastie et al., 2013; Warner et al., 2016). Calibrations were compared using a leave-one-out (LOO) cross validation to assess the prediction accuracy (Kozak and Kozak, 2003; Miguel et al., 2012; Kuželka and Marušák, 2014; Yang and Huang, 2014; Warner et al., 2016). As the residuals for the records of BH (1.3 m above ground) were already constrained to zero through the Goodwin model formulation, such records were omitted during the validation process.

Estimates were tested using the percentage error (E %) (Equation 4) as a measure of the overall prediction accuracy, which also indicates any overestimation (negative values) or underestimation (positive values) and the relative prediction error (RE %) (Equation 5) to indicate the precision of the model estimates, which is always a positive value. Values close to zero indicate that the model is accurate and precise for the measured data (Kozak and Smith, 1993; Fonweban et al., 2011; Huang et al., 2003). The mathematical formulations of the two indicators are as follows:

\[ E\% = 100 \times \frac{\sum_{i=1}^{n} \left( y_i - \hat{y}_i \right)}{n \cdot \bar{y}}, \]  
\[ RE\% = 100 \times \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n \cdot \bar{y}}, \]  

Where; \( y_i \) is the observed value, \( \hat{y}_i \) is the respective predicted value, \( n \) are the number of observations, and \( \bar{y} \) is the mean of the observed values.

It was noted that both the equations are more likely to be biased towards the larger trees compared to the smaller ones. However, the majority of trees sampled were either older or larger trees, with a majority of commercial harvesting often occurring in this tree size class. Therefore, both the equations were appropriate for use in this study.

LOO cross validation was used to investigate 3 different aspects of the taper models combined with the bark thickness as follows: 1) prediction of \( d_{ab} \) given \( h \); 2) prediction of \( h \) given \( d_{ab} \); and 3) prediction of the under bark volume given log lengths. The sample tree measurements were binned into classes with approximately equal size and the validation was appraised for different diameter and relative height ranges in the sampled trees. The absolute values for each calibration in each class and each statistic were summarized using a single model index evaluated by taking the mean of the scaled statistics, based on a simple, unweighted scaling of each statistic. Using this approach results in each statistic having an equal contribution to the index, with a perfect calibration having an index value close to zero (Kozak and Kozak, 2003; Goodwin, 2009; Fonweban et al., 2011; Miguel et al., 2012; Warner et al., 2016).

### 3. RESULTS AND DISCUSSION

#### 3.1 Best candidate calibrations

Twelve calibrations were fitted using an unweighted nonlinear regression and evaluated based on their GOF statistics. The GOF results for the selected candidate calibrations are summarized in Table 2. High adjusted R² values (0.98279-0.98428) indicate that these calibrations are a good fit for the present data. Generally, the calibration variants can be reduced to include only 4-6 terms without any noticeable reduction in the value of GOF statistics and these calibrations form a parsimonious model which is selected for validation (the coefficients for these calibrations are shown in Table 1). As shown for an example calibration in Figure 2, a plot of the standardized residuals did not indicate any heteroscedasticity trend. The standardized residuals are normally distributed (p-value>0.05; tested by Shapiro-Wilk test).

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Adjusted R²</th>
<th>RSE</th>
<th>BIC</th>
<th>Number of coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPP 1</td>
<td>0.98425</td>
<td>1.071</td>
<td>1889</td>
<td>6</td>
</tr>
<tr>
<td>TPP 2</td>
<td>0.98423</td>
<td>1.071</td>
<td>1883</td>
<td>5</td>
</tr>
<tr>
<td>TPP 3</td>
<td>0.98389</td>
<td>1.083</td>
<td>1903</td>
<td>5</td>
</tr>
<tr>
<td>TPP 4</td>
<td>0.98430</td>
<td>1.069</td>
<td>1977</td>
<td>7</td>
</tr>
<tr>
<td>TPP 5</td>
<td>0.98434</td>
<td>1.067</td>
<td>1978</td>
<td>8</td>
</tr>
<tr>
<td>TPP 6</td>
<td>0.98436</td>
<td>1.064</td>
<td>1987</td>
<td>9</td>
</tr>
<tr>
<td>TPP 7</td>
<td>0.98437</td>
<td>1.065</td>
<td>2047</td>
<td>12</td>
</tr>
<tr>
<td>TPP 8</td>
<td>0.97886</td>
<td>1.145</td>
<td>2213</td>
<td>3</td>
</tr>
<tr>
<td>TPP 9</td>
<td>0.97564</td>
<td>1.361</td>
<td>2352</td>
<td>3</td>
</tr>
<tr>
<td>TPP 10</td>
<td>0.98279</td>
<td>1.095</td>
<td>1959</td>
<td>4</td>
</tr>
<tr>
<td>TPP 11</td>
<td>0.98387</td>
<td>1.084</td>
<td>1903</td>
<td>5</td>
</tr>
<tr>
<td>TPP 12</td>
<td>0.98428</td>
<td>1.070</td>
<td>1888</td>
<td>6</td>
</tr>
</tbody>
</table>
3.2 Bark thickness model

A bark thickness model was constructed using all the measured double bark thickness data, which excluded the buttressed records to avoid any issues resulting from the complexity of bark thickness in the heavily buttressed lower parts of a bole (the regression ANOVA table is shown in Table 2). The standardized residuals of the double bark thickness prediction were evenly distributed without any heteroscedasticity (Figure 3). This indicated that the model was suitable for use with the taper models selected for validation. The bark thickness model equation is shown in equation 6 (adjusted R²=0.68943).

\[ BT_2 = 4.9736 \times (d_{ob})^{0.5052} \]  

3.3 Validation of calibrations

The three aspects of validation statistics that were evaluated are summarized in Table 3. Generally, the prediction of diameter given the height resulted in the most consistent predictions followed by the model used to determine the volume given diameter, and lastly, the model used to predict height given the diameter. Based on an even weighting of the three validation statistics, it was observed that no single statistic was suitable to determine the best calibration and no calibration performed the best on all the tests, supporting the deliberate inclusion of more than one test and instead using an overall best fit ranking (Kozak and Kozak, 2003).

The results showed that TPP 2 was the best taper model calibration for predicting the current dataset,
Table 3. Validation indices and overall statistic (numbers in **bold** indicate the best calibration for the respective statistic)

<table>
<thead>
<tr>
<th>Calibration</th>
<th>d_{ab} at any given h</th>
<th>h at any given d_{ab}</th>
<th>Log volume at any given log length</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPP 1</td>
<td>3.417</td>
<td><strong>8.982</strong></td>
<td>3.986</td>
<td>5.462</td>
</tr>
<tr>
<td>TPP 2</td>
<td>3.271</td>
<td>9.346</td>
<td><strong>3.744</strong></td>
<td><strong>5.454</strong></td>
</tr>
<tr>
<td>TPP 3</td>
<td>3.460</td>
<td>9.137</td>
<td>4.123</td>
<td>5.573</td>
</tr>
<tr>
<td>TPP 10</td>
<td>3.538</td>
<td>9.865</td>
<td>4.269</td>
<td>5.891</td>
</tr>
<tr>
<td>TPP 11</td>
<td>3.484</td>
<td>10.231</td>
<td>4.147</td>
<td>5.954</td>
</tr>
<tr>
<td>TPP 12</td>
<td><strong>3.179</strong></td>
<td>9.942</td>
<td>3.781</td>
<td>5.634</td>
</tr>
</tbody>
</table>

as it resulted in the lowest overall validation index in addition to the residual histogram being normal distributed. The d_{ab} and log volume were slightly over-estimated while the model tended to slightly underestimate h (Figure 4). However, the mean prediction errors of all the three validation statistics were less than 10%, which indicated that the model prediction was sufficiently accurate, or as noted by Huang et al. (2003), was in a range that was realistic and reasonable.

![Figure 4. TPP 2 validation histograms: (a) diameter under bark (d_{ab}); (b) height (h) at d_{ab}; (c) log volume](image)

An investigation of each class for each validation statistic of TPP 2 indicated that the mean prediction error of d_{ab} prediction at any given h, h prediction at any given d_{ab}, and log volume prediction at any given log length was within 0.8-2.3%, 1.5-4.8%, and 1.8-2.5%, respectively (Table 4). Importantly, the model predicted all 3 values within 1.8% in the lower bole, due to more sample points, resulting in a relatively higher accurate prediction in the lower bole, which is a more valued section of a tree (Fonweban et al., 2011; Warner et al., 2016; Zheng et al., 2017; López-Martínez et al., 2019). Overall, there was reduction in prediction accuracy for samples collected toward the top of the tree and of log volume prediction in taller trees. The log volume was generally overestimated except for a tendency to underestimation in the lower 6 m (the maximum teak log length sold to processors (FIO, 2013)).
Table 5. Model coefficient comparison (**): p<0.001

<table>
<thead>
<tr>
<th>Model</th>
<th>Coefficient (±SE)</th>
<th>FIO-teak1 (Warner et al., 2016)</th>
<th>TPP 2 (This study)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bark thickness model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>3.035±0.445***</td>
<td>4.976±0.368***</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>0.629±0.038***</td>
<td>0.505±0.023***</td>
<td></td>
</tr>
<tr>
<td>Taper model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c1</td>
<td>0.593±0.012***</td>
<td>0.697±0.017***</td>
<td></td>
</tr>
<tr>
<td>d0</td>
<td>0.633±0.025***</td>
<td>0.511±0.021***</td>
<td></td>
</tr>
<tr>
<td>f2</td>
<td>0.777±0.031***</td>
<td>0.692±0.034***</td>
<td></td>
</tr>
<tr>
<td>f3</td>
<td>0.013±0.001***</td>
<td>0.015±0.001***</td>
<td></td>
</tr>
<tr>
<td>f4</td>
<td>-0.003±0.000***</td>
<td>-0.003±0.000***</td>
<td></td>
</tr>
</tbody>
</table>

The effects of different coefficient values used in the bark thickness models and taper model calibrations on the predicted values cannot be easily ascertained through visual inspection of the coefficients. Therefore, a scatter plot of double bark thickness at breast height versus diameter at breast height over bark was plotted to determine the differences between the different bark thicknesses models used to predict the bark thickness at breast height (Figure 5). For a comparison of taper models, D_pb was chosen as the model input to remove any confounding effects resulting from bark thickness and a plot of height above ground versus diameter under bark was drawn to assist with the comparison between different taper model calibrations based on the stem shape of a tree (Figure 6). A comparison of the bark thickness models indicated that the predicted bark thickness at breast height in TPP 2 was higher than for similar values using the FIO-teak1 bark thickness model, especially for small trees. A comparison of the taper model calibrations indicated that decreasing trends in the predicted diameter under bark along the stem from both taper models were similar for all representative tree sizes.

Figure 6 indicates that the stem shapes of teak trees grown in the Thong Pha Phum plantation and those in the teak plantations of Northern Thailand are similar. This could be because the seedlings planted in each FIO teak plantation are clones originating from the same genetic base as the ones in Northern Thailand. This observation is consistent with previous studies that reported that cloned trees had no distinct variations in stem shape even when they were grown under different conditions (Gomat et al., 2011; Morley...
However, Figure 5 indicates that the teak trees sampled from Thong Pha Phum plantation had a relatively thicker bark compared to the ones sampled from Northern Thailand. This may be because teak trees in Western Thailand grow in moist environments experiencing a tropical monsoon climate (Am), with an annual rainfall between 1,600-2,000 mm/year, compared to the north where the environment is of a dry tropical savanna (Aw) type with an annual rainfall measured in the range of 1,000-1,400 mm/year. According to a study related to the structure and function of tree barks done by Rosell (2016), it was reported that for trees growing under moist conditions in a tropical climate, the investment in a thicker bark occurs because transpiration and photosynthesis can be activated when the tree gets
enough water. As such, the secondary phloem, which is the main structure of the inner tree bark, is produced by the vascular cambium. Moreover, a secondary phloem can produce phloem parenchyma to increase the storage of water and photosynthates and also transport food to other parts of a tree (Ryan and Asao, 2014). Therefore, such a cell division can result in a thicker tree bark.

4. CONCLUSIONS

Thong Pha Phum plantation, Kanchanaburi Province, Thailand was selected as a representative area for the development of suitable taper model calibrations for the volume estimation of teak growing in Western Thailand. The plantation is the largest teak plantation in the region and is also intensively managed. From the 12 taper model calibrations developed, TPP 2 was selected as the most suitable taper model calibration for teak growing in Western Thailand as it had the best overall performance (validation index=5.454 and adjusted $R^2=0.98423$). This calibration can predict the diameter under bark, tree height, and log volume with sufficient accuracy, especially in the lower bole, which is a more valued section of a teak tree. Comparison of TPP 2 with teak taper model (FIO-teak1) and bark equations, developed previously for teak trees in Northern Thailand, indicated that the trees growing in the two regions have similar stem shapes, but the teak trees grown in Western Thailand tend to have a thicker bark. For general usage, TPP 2 will be encoded in the Farm Forestry Toolbox software package and named FIO-teak2 for recommended application to optimize the log product value of standing teak trees in the plantations of Western Thailand based on log grade specifications, commonly used during the inventory process. Furthermore, using the under bark taper equations, regional differences in bark thickness can be an important factor in the teak inventory and will be investigated further.

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REFERENCES


Fonweban J, Gardiner B, Macdonald E, Auty D. Taper functions for Scots pine (Pinus sylvestris L.) and sitka spruce (Picea sitchensis (Bong.) Carr.) in Northern Britain. Forest 2011;84:49-60.


