

## Supplementary data

### Forecasting Municipal Solid Waste Generation in Thailand with Grey Modelling

#### Supplement S1: MSW generation and management in Thailand

The key government offices responsible for analyzing and reporting the waste situation in Thailand are the Pollution Control Department (PCD) and the Department of Local Administration (DLA). The data on MSWG are available publicly for the period 1993-2018 on the state report of pollution and solid waste in the country by the PCD (PCD, 2022a) and online database (PCD, 2022b). Until 2010, most MSWG data were estimated by multiplying the city’s population by an average MSWG rate corresponding to the city scale. From 2011 onward, methods for estimating the amount of MSW evolved to become more precise. The data on MSWG in some cities were then derived from the recorded weights of waste collection trucks and on-site surveys. This inconsistency in data collection methods caused a shift in the MSW time series data with a

significant rise (67%) between 2010 and 2011, as shown in Figure 1.

The PCD also reports on various types of waste disposal and waste utilization. To estimate the energy potential and GHG emissions, this study focused on the collected waste that is impW; this includes open dumping (OD), controlled dumping (CD) of waste of more than 50 ton/day, incineration (IN) without proper pollution control, and open burning (OB). When this study was conducted, the most recent year such information was available was 2017. Phongphiphat et al. (2019) analyzed the PCD data on 2,441 municipalities and reported that the MSW generated in 2017 was disposed of by OD, IN, CD and OB at 16.4%, 0.6%, 0.9%, and 0.4%, respectively. These proportions were used to estimate GHG emissions for the improper disposal of collected waste in all future years.

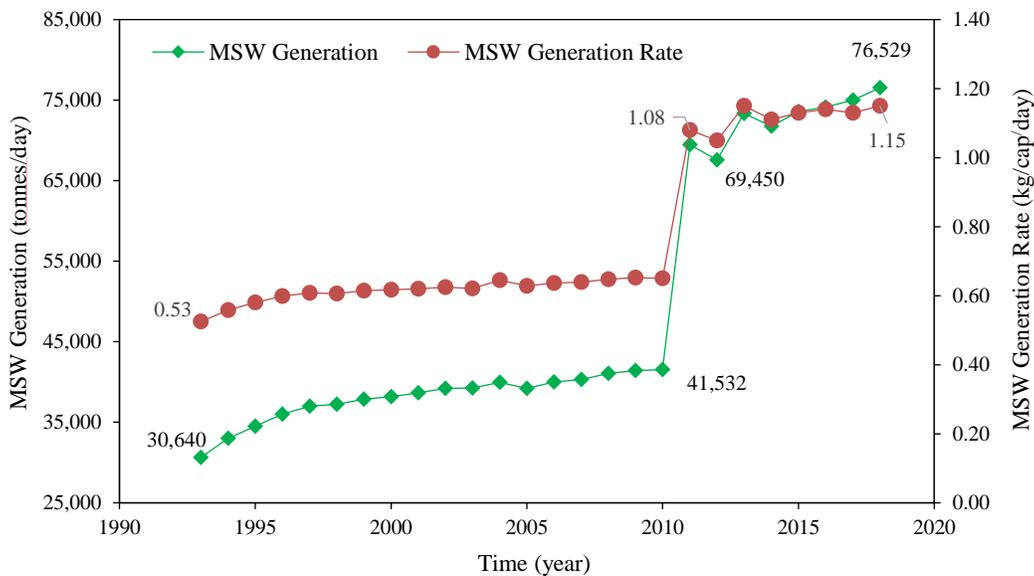


Figure 1. MSW generation and MSW generation rate in Thailand over 26 years

#### Supplement S2: Forecasting using the grey model

##### S2.1 Basic sequences of input data

For running GMs, the input data included a sequence of raw data and its accumulation-generated sequence. The sequence of raw data was denoted by  $X_i^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(k))$  and its accumulation-

generated sequence was  $X_i^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(k))$  where;  $i = 1, 2, \dots, n$  and  $k = 1, 2, \dots, m$ . The  $i$  represents the number of influencing factors, and  $k$  represents the number of years for which data are available (Sifeng and Yi, 2010).

*S2.2 Univariate grey model: GM (1,1) and GM (1,1)-α*

GM (1,1) and GM (1,1)-α are the first-order difference (FOD) equation as shown in Equation (1). The equation was derived with one dependent variable and without influencing factors.

$$\frac{xd^{(1)}(t)}{td} + xa^{(1)}(t) = b \tag{1}$$

and the FOD equation could be solved by using Equation (2)

$$\hat{x}_1^{(0)}(k) = \left[ x_1^{(0)}(1) - \frac{b}{a} \right] (1 - e^a) e^{-a(k-1)}, k = 2, 3, \dots, m \tag{2}$$

$\hat{x}_1^{(0)}$  and  $x_1^{(0)}$  is a sequence of predictions,  $a$  is a coefficient, and  $b$  is a control parameter (Huang, 2012). Both  $a$  and  $b$  can be calculated by the ordinary least squares method (OLS) as

$$[a \ b]^T = (B^T B)^{-1} B^T Y_n \tag{3}$$

Where;

$$Y_n = \begin{pmatrix} -[\alpha x_1^{(1)}(1) + (1 - \alpha) x_1^{(1)}(2)] & 1 \\ -[\alpha x_1^{(1)}(2) + (1 - \alpha) x_1^{(1)}(3)] & 1 \\ \dots & \dots \\ -[\alpha x_1^{(1)}(m - 1) + (1 - \alpha) x_1^{(1)}(m)] & 1 \end{pmatrix} \tag{4}$$

$$Y_n = \begin{pmatrix} x_1^{(0)}(2) \\ x_1^{(0)}(3) \\ \dots \\ x_1^{(0)}(m) \end{pmatrix} \tag{5}$$

Where;  $\alpha$  is between 0 to 1 (Huang, 2012). The  $\alpha < 0.5$  means that the old data in the sequence are more significant to the model,  $\alpha > 0.5$  means that the latest or new data are more significant, and  $\alpha = 0.5$  means that the sequences of data are generated without preference. For  $\alpha = 0.5$ , GM (1,1)-α is equivalent to GM (1,1) (Sifeng and Yi, 2010).

*S2.3 Multivariate grey model: GM (1, n)*

GM (1,n) denotes grey model for multivariate forecasting, where “n-1” is the number of independent variables. The GM (1,n) was defined as

$$x_1^{(0)}(k) + az_1^{(1)}(k) = \sum_{i=2}^n b_i x_i^{(1)}(k) \tag{6}$$

GM (1,n) consists of the basic sequences and the coefficient for the solution, similar to the form of GM (1,1). However, GM (1,n) needs different equations to solve its parameters as shown in equation (7)-(13) (Zhang, 2013):

$$\hat{x}_1^{(0)}(k) = \sum_{i=2}^n \beta_i x_i^{(1)}(k) - \alpha z_1^{(1)}(k) \tag{7}$$

Where;

$$\alpha = \frac{a}{1+0.5a} \tag{8}$$

$$\beta_i = \frac{b_i}{1+0.5a}, \text{ where } i = 2, 3, \dots, n \tag{9}$$

The variables  $a$  and  $b_i$  can be determined by OLS as

$$[a \ b_2 \ \dots \ b_n]^T = (D^T D)^{-1} D^T Y_n \tag{10}$$

Where;  $D$  and  $Y$  can be calculated using equations (11)-(13)

$$D = \begin{bmatrix} -z_1^{(1)}(2) & -x_2^{(1)}(2) & \dots & -x_n^{(1)}(2) \\ -z_1^{(1)}(3) & -x_2^{(1)}(3) & \dots & -x_n^{(1)}(3) \\ \dots & \dots & \dots & \dots \\ -z_1^{(1)}(m) & -x_2^{(1)}(m) & \dots & -x_n^{(1)}(m) \end{bmatrix} \tag{11}$$

$$Y_n = \begin{bmatrix} x_1^{(0)}(2) \\ x_1^{(0)}(3) \\ \dots \\ x_1^{(0)}(m) \end{bmatrix} \tag{12}$$

Where;

$$z_1^{(1)}(k) = \frac{1}{2} x^{(1)}(k) + x^{(1)}(k - 1), k = 2, 3, \dots, m \tag{13}$$

*S2.4 Grey relational analysis*

Grey relational analysis (GRA) is a mathematical method for identifying the level of relationship between dependent and independent variables. It was used, therefore, an indicator in selecting the appropriate independent variables for a multivariate model. According to Wu and Chen (2005), the formula for GRA is given as

$$\gamma(Y_0 Y_i) = \frac{1}{m} \sum_{k=1}^m \xi(Y_0(k), Y_i(k)), \tag{14}$$

$i = 1, 2, \dots, n$  and  $k = 1, 2, \dots, m$

Where;  $\gamma(Y_0, Y_i)$  is the grey relational grade distributed between 0 and 1, and  $\xi(Y_0(k), Y_i(k))$  is the grey relational coefficient.  $\xi(Y_0(k), Y_i(k))$  was defined as

$$\xi(Y_0(k), Y_i(k)) = \frac{\min_k |\min_k |Y_0(k) - Y_i(k)| + \rho \max_k |Y_0(k) - Y_i(k)|}{|Y_0(k) - Y_i(k)| + \rho \max_k |Y_0(k) - Y_i(k)|} \quad (15)$$

Where  $Y_0(k)$  and  $Y_i$  are the normalized sequences of dependent and independent variables, respectively;  $\rho$  is called the distinguished coefficient, varied between 0 and 1 and frequently determined as 0.5.

## REFERENCES

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