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# Parameters tuning optimization of second-order sliding mode control by response surface methodology

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## Abstract

In this paper, tracking performance optimization of a second-order sliding mode control (SMC), namely twisting sliding mode is considered through optimal tuning of its control parameters. Effectiveness of transient response can be achieved by considering as minimization of maximum-overshoot ($M_O$) and settling-time ($t_s$), which are obtained through the Response Surface Methodology (RSM). For the optimal tracking performance with the RSM, the computation process by mean of a central composite design (CCD) is performed through a quadratic equation. Finding results of a simulation confirm that the optimal tracking performance as the minimization of the $M_O$ and $t_s$can be achieved by the optimization tuning control parameters with the RSM.

**Keywords** : Transient response, twistingsliding mode control, Response Surface Methodology, Optimization.

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## 1. Introduction

Sliding mode control (SMC) is a powerful control technique which is robust for parametric uncertainty [1,2]. However, the chattering phenomena of control input during the existence of sliding mode is disadvantage in implementation [3]. Extension idea of chattering reduction by the second-order SMC technique has been widely done [3-6], because the input of the system is derived as a new state variable. The input of the system is dictated by the integration of the switching control input. Additionally, in the vicinity about the second-order SMC sliding surface, the control input is converged to the equivalent control, which is independent of the differential of the control input. Therefore, the second-order SMC is continuously effective in definite vicinity of the 2-sliding set [6, 7].

For the second-order SMC so-call the twisting sliding mode control (SMC), it is an effective control approach for an uncertain nonlinear system [7-9]. In practice, the twisting SMC is simple and has high performance of the control ability [8, 10]; however, the high performance of closed-loop control is a result of the setting value of its control parameters. By mean of the twisting SMC, the aim of find-tuning parameters is an essential problem, because the control parameters are coupled effect with determining of the setting value. The value affects to the characteristic of the tracking performance. Furthermore, even if the tracking responses can be obtained through the trial and error method, it cannot confirm the optimization of the control performance [11]. Thus, in order to obtain acceptable tracking performance, it is necessary to specify the optimal value of control parameters.

Among the various solutions, alternative find-tuning parameters through the optimization approach are widely done [12-15]. The response surface methodology (RSM) is one of development approach which widely used in many control systems [16-19], capable to

present the optimization value by the optimization method that using the statistic theory, and experimental design. Therefore, the motivations of this study are based on the following:

- the RSM application has capability to optimize the transient response of a closed-loop control of the twisting SMC; therefore, the results of the closed-loop control provide the errors of the steady-state nearly to zero;
- the acceptation of specific control parameters of the twisting SMC based on the results of optimizing the transient response of closed-loop control especially no overshoot and fast settling time as in [16].

The remainder of this paper is organized as follows. First, we present the concept of twisting SMC. In section 3, the optimization of the transient response of the twisting SMC with the response surface methodology is presented. In section 4, demonstration the results of the RSM application for tuning control parameters. Finally, the research conclusion is presented

## 2. Twisting SMC

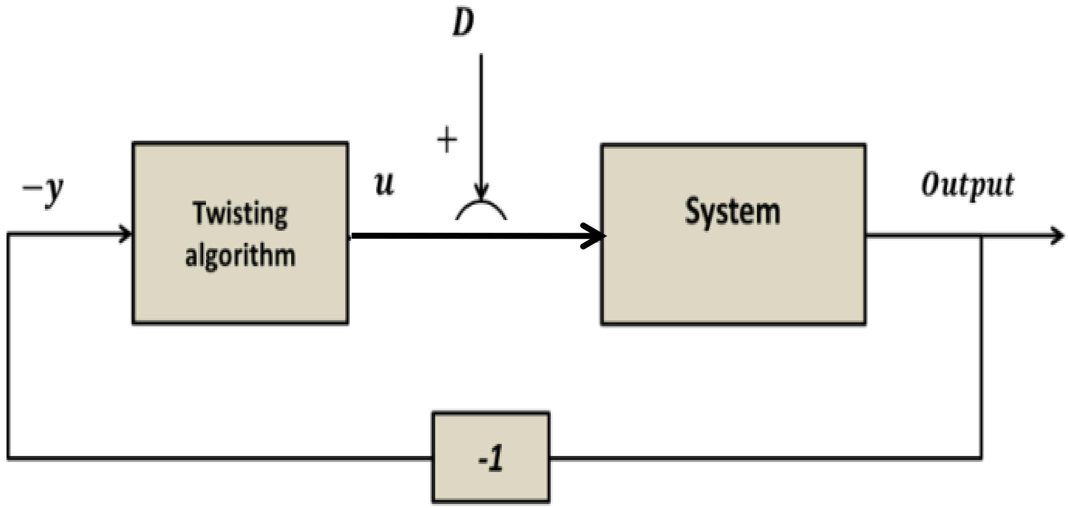


Figure1 Block diagram of the twisting SMC.

The aim of the control approach is to steer the errors to move along the sliding surface and to maintain its first successive derivative null [20, 21], while the removable relative-degree restriction and the chattering phenomenon are the capabilities of second-order SMC[7, 22].

Considering the dynamic system as in the form[7]

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \delta(x,t) + \vartheta(x,t)u \qquad (1)$$

where $x_1 = \sigma$, $x_2 = \dot{\sigma}$, $\delta(x,t)$ and $\vartheta(x,t)$ are the smooth function that the frequency response by the transfer function $G(s)$ can be then used as the SISO model, $u$ is the scalar control input, $t$ is the time. For the selection of second-order SMC, namely twisting SMC(see Figure1), and the relative degree in this task is assumed to be 2, andthe computation of $\ddot{\sigma}$ can be expressed as[7]

$$\ddot{\sigma} = \delta(x,t) + \vartheta(x,t)u \qquad (2)$$

The calculation of the bound is

$$0 < \vartheta_m \le \vartheta \le \vartheta_M, \quad \frac{\dot{\vartheta}}{\vartheta} \le \vartheta_d \qquad (3)$$

where $\vartheta_m$ is the lower bound, $\vartheta_M$ is the upper bound, $\vartheta_d$ is some known positive constants[23]. The control input of twisting SMC for relative degree 2 is given by [7]

$$u = -(R_1 sign\ (\sigma) + R_2 sign\ (\dot{\sigma})) \qquad (4)$$

where $R_1$ and $R_2$ are positive constant and not equal to 0. The finite-time convergence can be described as[7]

For the principle of second-order SMC, the higher order time derivatives of the sliding variable σ is defined as $\sigma(x,t) = \dot{\sigma}(x,t) = \cdots = \frac{d^{r-1}\sigma}{dt^{r-1}} = 0$.

$$(R_1 + R_2)\vartheta_m - C > (R_1 - R_2)\vartheta_M + C, (R_1 - R_2)\vartheta_m > C \quad (5)$$

where $C$ is positive constant, and $R_1$, $R_2$, and $C > 0$.

### 3. Optimization tracking performance with RSM

To make the ultimate tuning of closed-loop control of the twisting SMC, this recommends to consider the optimization of the output response behavior that should be without the overshoot ($M_O$) while has fast settling time ($t_s$) [16]. For the finding tuning control parameter in this work, the response surface methodology was applied in order to find tuning the control parameters $R_1$ and $R_2$ of Eq.(4), which the specific value of $R_1$ and $R_2$ is capable to make the trade-off between fast settling time ($t_s$) and overshoot of the response; in addition, increasing of tracking performance and stability of control system are the result of the specific value of $R_1$ and $R_2$. For the tuning control parameters of the twisting SMC with the RSM, the system model of the fourth-order with relative degree two was used in this study as follow [24]

$$G(s) \cong \frac{K(s^2+k_1 s+k_2)e^{-0.15s}}{(s^3+d_1 s^2+d_2 s+d_3)}$$

$$\cong \frac{K(s^2+k_1 s+k_2)}{(s^3+d_1 s^2+d_2 s+d_3)(1+0.15s)} \quad (6)$$

where $G(s)$ is the transfer function of the system that is defined to be $\delta(x,t)$, $K$ is 29.8, $k_1 = 50$, $k_2 = 833$, $d_1 = 21.2$, $d_2 = 51.3$, and $d_3 = 189.5$. For the optimization of $M_O$ and $t_s$ through the RSM, the formula of the relative between the independent variable and the dependent variable is unknown exactly; therefore, a specific scheme is built in order to fit the second-order polynomial response surface equation that was applied with the quadratic canonical model [16] as follow

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i \acute{x}_i + \sum_{i<j}\sum \beta_{ij}\, \acute{x}_i \acute{x}_j + \sum_{i=1}^{k} \beta_{ii} \acute{x}_i^2 + e \quad (7)$$

where $Y$ is the scalar output response of interest $\beta_0, \beta_i, \beta_{ij}$ and $\beta_{ii}$ are the regression coefficients $i$ is the linear coefficients, $j$ is the quadratic coefficients, $k \in I^+$, and $e$ is the errors. For the Eq. (7), the prediction output ($Y$) is used for approximation the specification of $M_O$ and $t_s$ that are the measurement output of the closed-loop control with twisting SMC as in Fig. 1, which the disturbance parameter was not into accounted. In the investigation of the $M_O$ and $t_s$ through Eq. (7), the central composite design was applied for determination the factor levels of the parameters $R_1$ and $R_2$ as shown in Table 1. Table 2 shows the data of $M_O$ and $t_s$ which were obtained from closed-loop control with twisting SMC.

For the closed-loop control test, it was simulated (6) through the Matlab programming which based on the system of Eq. (6), and the data of $M_O$ and $t_s$ were obtained by the check-point in the simulation, which the data of $M_O$ and $t_s$ are based on the level of parameters and control value of twisting SMC as shown in Table

2. The quadratic models of the optimization of $M_O$ and $t_s$ are demonstrated in Eq.(8) and Eq. (9), which the prediction value of $M_O$ and $t_s$ can be obtained by the substitution with the value of the level of parameters ($R_1$ and $R_2$) as shown in Table 2. The contour plots of response surfaces of $M_O$ and $t_s$ are shown in Fig. 2 and Fig. 3, respectively.

The errors of $M_O$ and $t_s$ between the prediction by Eq.(8) to Eq. (9) and the data screening in the simulation of closed-loop control by twisting SMC are shown in Table.3, Fig. 4, and Fig. 5, respectively. The errors average of $M_O$ (Eq.(8)) and $t_s$ (Eq.(9)) are shown in Table.3 which are very trivial and can be accepted in the viewpoint of closed-loop control. Table 4 shows the final selection of control value of the twisting SMC which are used for determination the value of $R_1$ and $R_2$ that the optimization of $R_1$ and $R_2$ for closed-loop control with the twisting SMC are 1030 and 999, respectively.

**Table 1.** The value setting of $R_1$ and $R_2$.

| Parameters | Level of parameters | | |
|---|---|---|---|
| | -1 | 0 | 1 |
| $R_1$ | 1000 | 1025 | 1050 |
| $R_2$ | 950 | 974.5 | 999 |

**Table 2.** The specification of $M_O$ and $t_s$ of closed-loop control by twisting SMC.

| Number of Experiment | Level of | | Control value of twisting | | Response of twisting | |
|---|---|---|---|---|---|---|
| | $R_1$ | $R_2$ | $R_1$ | $R_2$ | $M_O$ | $t_s$ |
| 1 | 0 | 1 | 1025 | 999 | 680.317 | 0.339 |
| 2 | 1 | 0 | 1050 | 974.5 | 727.645 | 0.296 |
| 3 | 0 | 0 | 1025 | 974.5 | 702.857 | 0.295 |
| 4 | 1 | 1 | 1050 | 999 | 705.373 | 0.302 |
| 5 | -1 | 0 | 1000 | 974.5 | 680.338 | 0.363 |
| 6 | -1 | -1 | 1000 | 950 | 707.152 | 0.303 |
| 7 | 1 | -1 | 1050 | 950 | 753.451 | 0.3 |
| 8 | -1 | 1 | 1000 | 999 | 651.304 | 0.766 |
| 9 | 0 | 0 | 1025 | 974.5 | 702.857 | 0.295 |
| 10 | 0 | 0 | 1025 | 974.5 | 702.857 | 0.295 |
| 11 | 0 | 0 | 1025 | 974.5 | 702.857 | 0.295 |
| 12 | 0 | 0 | 1025 | 974.5 | 702.857 | 0.295 |
| 13 | 0 | -1 | 1025 | 950 | 729.236 | 0.3 |

$$M_O = 703.18 + 24.61R_1 - 25.47R_2 + 1.94R_1R_2 + 1.29R_2^2 \quad (8)$$

$$t_s = 0.30 - 0.089R_1 + 0.084R_2 - 0.12R_1R_2 + 0.086R_1^2 \quad (9)$$

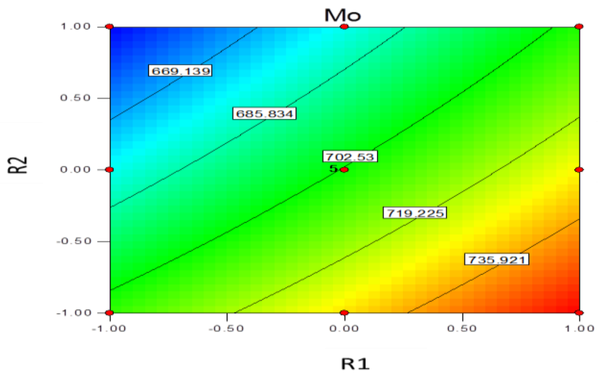


Figure 2 Optimization value of $M_O$

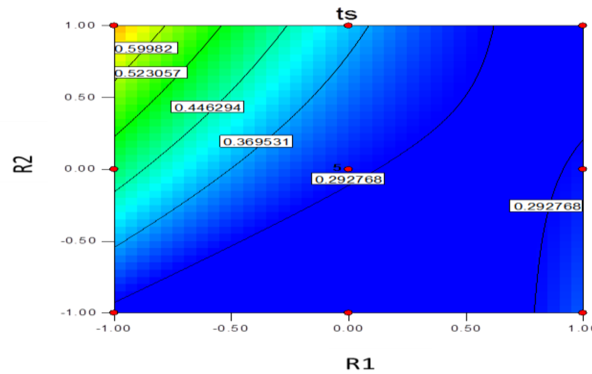


Figure 3 Optimization value of $t_s$.

Table 3 The prediction value by quadratic equation and closed-loop control by twisting SMC.

| Number of experiment | Level of parameters | | Predict Response | | Twisting SMC | | Error | |
|---|---|---|---|---|---|---|---|---|
| | $R_1$ | $R_2$ | $M_O$ | $t_s$ | $M_O$ | $t_s$ | $M_O$ | $t_s$ |
| 1 | 0 | 1 | 679 | 0.384 | 680.317 | 0.339 | 1.317 | 0.045 |
| 2 | 1 | 0 | 727.79 | 0.297 | 727.645 | 0.296 | 0.145 | 0.001 |
| 3 | 0 | 0 | 703.18 | 0.3 | 702.857 | 0.295 | 0.323 | 0.005 |
| 4 | 1 | 1 | 705.55 | 0.261 | 705.373 | 0.302 | 0.177 | 0.041 |
| 5 | -1 | 0 | 678.57 | 0.475 | 680.338 | 0.363 | 1.768 | 0.112 |
| 6 | -1 | -1 | 707.27 | 0.271 | 707.152 | 0.303 | 0.118 | 0.032 |
| 7 | 1 | -1 | 752.61 | 0.333 | 753.451 | 0.3 | 0.841 | 0.033 |
| 8 | -1 | 1 | 652.45 | 0.679 | 651.304 | 0.766 | 1.146 | 0.087 |
| 9 | 0 | 0 | 703.18 | 0.3 | 702.857 | 0.295 | 0.323 | 0.005 |
| 10 | 0 | 0 | 703.18 | 0.3 | 702.857 | 0.295 | 0.323 | 0.005 |
| 11 | 0 | 0 | 703.18 | 0.3 | 702.857 | 0.295 | 0.323 | 0.005 |
| 12 | 0 | 0 | 703.18 | 0.3 | 702.857 | 0.295 | 0.323 | 0.005 |
| 13 | 0 | -1 | 729.94 | 0.216 | 729.236 | 0.3 | 0.704 | 0.084 |
| | | | | | Average | | 0.60 | 0.035 |

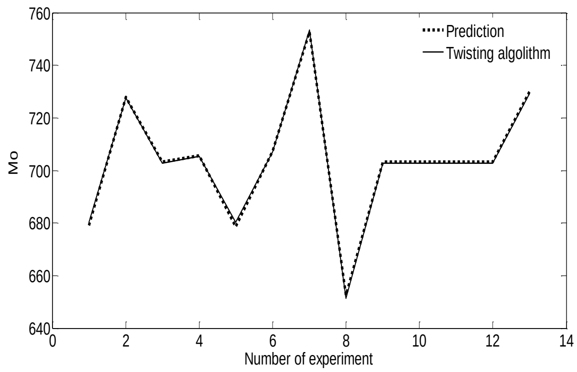


Figure 4 Comparison data of $M_O$ between the and the twisting SMC.

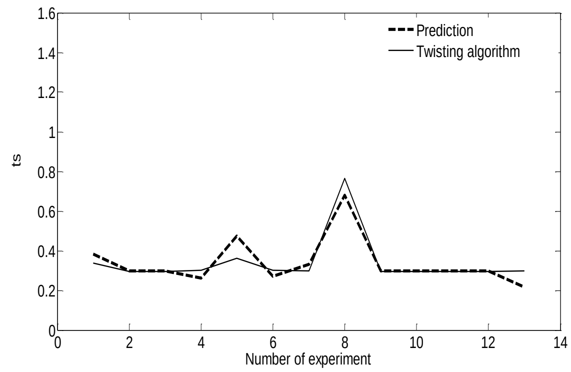


Figure 5 Comparison data of $t_s$ between the prediction prediction and the twisting SMC.

Table 4. Selection data for tuning parameters of closed-loop control of twisting SMC.

| Method | Level of control parameters | | Control value of twisting SMC | | Output | |
|---|---|---|---|---|---|---|
| | $R_1$ | $R_2$ | $R_1$ | $R_2$ | $M_O$ | $t_s$ |
| Optimization | 0.20 | 1 | 1030 | 999 | 684.88 | 0.33 |
| Minimum $t_s$ | 0 | 0 | 1025 | 974.5 | 702.86 | 0.29 |
| Minimum $M_O$ | -1 | 1 | 1000 | 999 | 652.44 | 0.67 |

To make clear the proposed of the optimization $M_O$ and $t_s$ through the RSM tuning, the next section shows the result of applied the RSM in order to optimization the control parameters of the twisting SMC, which were tested in the simulation with Matlab programming.

## 4. Results and discussion

In this section, the response surface calculation of $M_O$ and $t_s$ were checked with the substitution the level of control parameters ($R_1$ and $R_2$) into Eq. (8) and Eq. (9) as the results of $M_O = 684.31$ and$t_s = 0.34$, respectively. To check the characteristic of the set-point tracking performance (see Fig. 6) by the reference value is equal to 650, the finding shows that the optimization for the trade-off between fast convergence and small overshoot is the result of optimizing the values of $R_1 = 1030$ and $R_2 = 999$, and the phase plane plot of $s$ and $\dot{s}$ of the optimization $M_O$ and $t_s$ are shown in Fig. 7 which $s$ and $\dot{s}$ are convoluted to zero.

For the errors of tuning control parameters with the RSM (see Fig.8), the $M_O$ and $t_s$ by the calcution with Eq. (8) and Eq. (9) are over than the $M_O$ and $t_s$ by the closed-loop control with twisting SMC. These effected from the determination of the sampling interval of the level control parameters ($R_1$ and $R_2$), which can increased the errors of tuning control parameter if the range of the level of parameters ($R_1$ and $R_2$) are extended and high standard deviation. Furthermore, since the measurement of the $M_O$ and $t_s$ are obtained through the check-point; therefore, this can cause to arise the errors if the data of measurement is not accurate. However, the optimization value of $R_1$ and $R_2$ with the RSM tuning can be accepted for setting the control parameters of twisting SMC as the errors between the check-point ($M_O$ and $t_s$) through the closed-loop control test in the simulation and the prediction value ($M_O$ and $t_s$) with Eq. (8) and Eq. (9) are very trivial that can be accepted in the viewpoint of control systems.

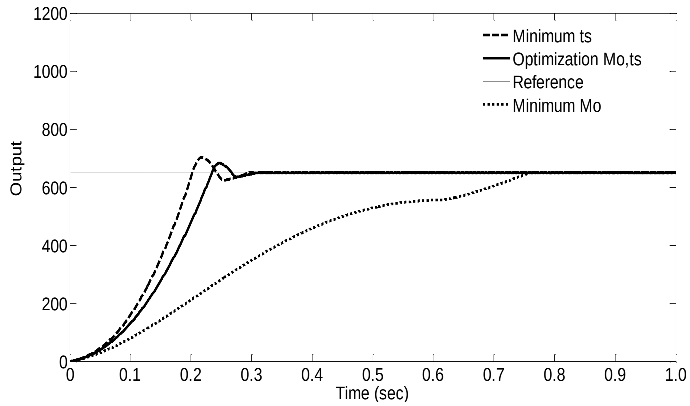


Figure 6 Set-point tracking of difference tuning.

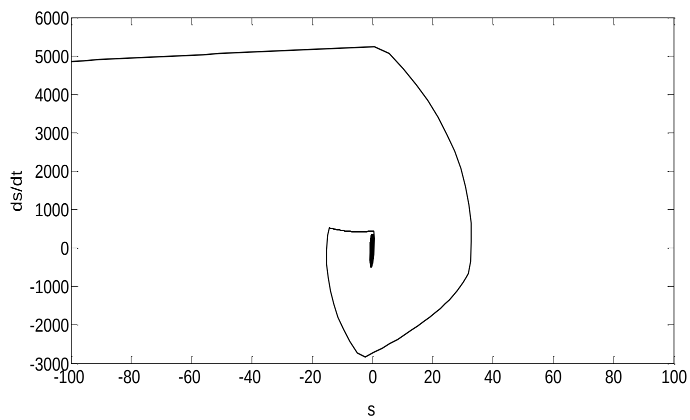


Figure 7 Phase plane plot of $S$ and $\dot{S}$

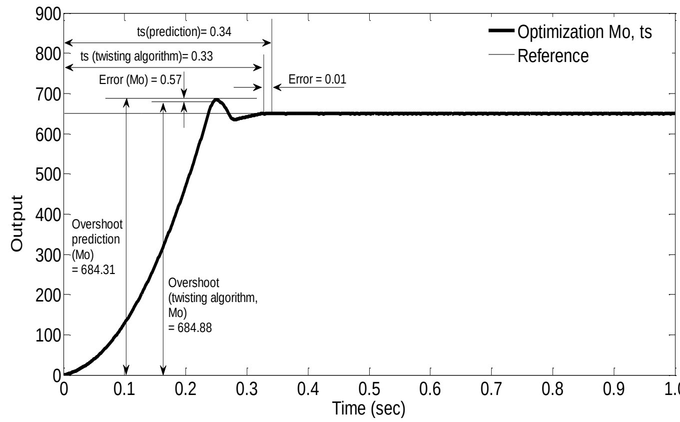


**Figure 8** Errors of tuning with RSM.

## 5. Conclusions

The response surface methodology (RSM) is developed in the optimization of the transient response by considered as the optimal control parameters of the twisting SMC. The proposed solution can be considered as the optimization of the maximum overshoot ($M_O$) and the settling time ($t_s$), which were obtained through the computation with the Response Surface Methodology (RSM). For the computation process of the RSM, the central composite design (CCD) is applied to design the experiment while the calculation process is obtained through the quadratic equation.

As the consequences, it can be achieved through the simulation testsof closed-loop control by twisting SMC, and the finding shows the minimization of $M_O$ and $t_s$ can be achieved by the optimization tuning control parameters with the RSM. In addition, the errors of tuning control parameters through the optimization with RSM are caused by the determination of the sampling interval of the level control parameters ($R_1$and $R_2$) that may increase the errors of tuning control parameter if the range of the level of parameters $R_1$ and $R_2$ are extended and high standard deviation. However, these can be accepted in the viewpoint of closed-loop control system which the errors of the prediction are very trivial.

## 6. Acknowledgment

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## 7. References

[1] Utkin, V., Variable structure systems with sliding modes. Automatic Control, IEEE Transactions on, 1977. **22**(2): p. 212-222.

[2] Utkin, V.I., Sliding mode control design principles and applications to electric drives. Industrial Electronics, IEEE Transactions on, 1993. **40**(1): p. 23-36.

[3] Bartolini, G., A. Ferrara, and E. Usani, Chattering avoidance by second-order sliding mode control. Automatic Control, IEEE Transactions on, 1998. **43**(2): p. 241-246.

[4] Bartolini, G., et al., A survey of applications of second-order sliding mode control to mechanical systems. International Journal of Control, 2003. **76**(9-10): p. 875-892.

[5] Bartolini, G., E. Punta, and T. Zolezzi, Approximability Properties for Second-Order Sliding Mode Control Systems. Automatic Control, IEEE Transactions on, 2007. **52**(10): p. 1813-1825.

[6] Boiko, I., et al., Analysis of Chattering in Systems With Second-Order Sliding Modes. Automatic Control, IEEE Transactions on, 2007. **52**(11): p. 2085-2102.

[7] Levant, A., Principles of 2-sliding mode design. Automatica, 2007. **43**(4): p. 576-586.

[8] Taleb, M., A. Levant, and F. Plestan, Pneumatic actuator control: Solution based on adaptive twisting and experimentation. Control Engineering Practice, 2013. **21**(5): p. 727-736.

[9] Polyakov, A. and A. Poznyak, Lyapunov function design for finite-time convergence analysis: “Twisting” controller for second-order sliding mode realization. Automatica, 2009. **45**(2): p. 444-448.

[10] Reichhartinger, M. and M. Horn, Application of Higher Order Sliding-Mode Concepts to a Throttle Actuator for Gasoline Engines. Industrial Electronics, IEEE Transactions on, 2009. **56**(9): p. 3322-3329.

[11] Dongcheol, K. and R. Sehun, Design of an optimal fuzzy logic controller using response surface methodology. Fuzzy Systems, IEEE Transactions on, 2001. **9**(3): p. 404-412.

[12] Grefenstette, J.J., Optimization of Control Parameters for Genetic Algorithms. Systems, Man and Cybernetics, IEEE Transactions on, 1986. **16**(1): p. 122-128.

[13] Dai, C., et al., Seeker optimization algorithm for tuning the structure and parameters of neural networks. Neurocomputing, 2011. **74**(6): p. 876-883.

[14] Chu, C.-W., B. Erik Ydstie, and N.V. Sahinidis, Optimization of IMC-PID Tuning Parameters for Adaptive Control: Part 1, in Computer Aided Chemical Engineering, M.C.G. E.N. Pistikopoulos and A.C. Kokossis, Editors. 2011, Elsevier. p. 758-762.

[15] Kao, C.-C., C.-W. Chuang, and R.-F. Fung, The self-tuning PID control in a slider–crank mechanism system by applying particle swarm optimization approach. Mechatronics, 2006. **16**(8): p. 513-522.

[16] Demirtas, M. and A.D. Karaoglan, Optimization of PI parameters for DSP-based permanent magnet brushless motor drive using response surface methodology. Energy Conversion and Management, 2012. **56**(0): p. 104-111.

[17] Sahraian, M. and S. Kodiyalam, TUNING PID CONTROLLERS USING ERROR-INTEGRAL CRITERIA AND RESPONSE SURFACES BASED OPTIMIZATION. Engineering Optimization, 2000. **33**(2): p. 135-152.

[18] Iskender, B., N. Özdemir, and A. Karaoglan, Parameter Optimization of Fractional Order PI λ D μ Controller Using Response Surface Methodology, in Discontinuity and Complexity in Nonlinear Physical Systems, J.A.T. Machado, D. Baleanu, and A.C.J. Luo, Editors. 2014, Springer International Publishing. p. 91-105.

[19] Romero, J.A., R. Sanchis, and P. Balaguer, PI and PID auto-tuning procedure based on simplified single parameter optimization. Journal of Process Control, 2011. **21**(6): p. 840-851.

[20] Eker, I., Second-order sliding mode control with experimental application. ISA Transactions, 2010. **49**(3): p. 394-405.

[21] Eker,i., Second-Order Sliding Mode Control with PI Sliding Surface and Experimental Application to an Electromechanical Plant. Arabian Journal for Science and Engineering, 2012. **37**(7): p. 1969-1986.

[22] Carbajal-Espinosa, O., et al., Robust Dynamic Control of an Arm of a Humanoid Using Super Twisting Algorithm and Conformal Geometric Algebra, in Computational Kinematics, F. Thomas and A. Perez Gracia, Editors. 2014, Springer Netherlands. p. 261-269.

[23] M. Taleb, A. Levant, and a.F. Plestan, Twisting algorithm adaptation for control of electropneumatic actuators, in 12 th IEEE Workshop on Variable Structure SystemsJanuary 2012, IEEE: Mumbai.

[24] Yildiz, Y., et al., Spark-Ignition-Engine Idle Speed Control: An Adaptive Control Approach. Control Systems Technology, IEEE Transactions on, 2011. **19**(5): p. 990-1002.