New Reweighted ℓ_1-minimization Algorithms for Compressive Sampling Recovery
การหาค่าต่ำสุดของนอร์มหนึ่งแบบถ่วงน้ำหนักรูปแบบใหม่สำหรับการสร้างสัญญาณย้อนกลับของคอมเพรสซีฟแซมปลิ่ง

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ABTRACT

A recent compression method which overlooks the classical Shannon-Nyquist theorem is called *compressive sampling*, also known as *compressed sensing*. The reconstruction of this new compression method is proved to be done with high probability of success by performing ℓ_1-minimization problem. The ℓ_1-minimization reconstruction has been developed to the reweighted algorithm which recovers closely approximate sparse solutions. However, there is no rule that automatically selects the appropriate weighting values. This paper proposes the enhancements of reweighted ℓ_1-minimization by indicating the choice of weighting functions and the suggestion to find the weighting values.

In reconstruction process, the approximate ℓ_1-minimization might recover the fault signal by shifting the zero solutions to the other values. Thus, the *hard selective reweighted* (HSR) algorithm is designed to increase the importance of zero candidates by selecting the near-zero solutions whose numbers are equal to a number of original zero entries scaled by greater weighting value. In general, the locations of zero entries are not known so that the HSR algorithm could not apply to the real-world problems. This problem is coped with by the second proposed *automatic adaptive reweighted* (AAR) algorithm which is used to predict the locations of zero entries without knowing a number of original entries. The idea is to find the smallest frequency bin of solutions which contains empty member then set it to be the threshold and the solutions which are close to zero and the others scaled by larger and smaller weighting values, respectively. The numerical results show comparatively that HSR and AAR algorithms outperform ℓ_1-minimization. Furthermore, both of these algorithms are demonstrated to be applied to manmade and magnetic resonance imaging (MRI) images.

บทคัดย่อ

แนวทางการบีบอัดสัญญาณแบบใหม่ ซึ่งไม่ขึ้นอยู่กับทฤษฎีชานนอน และไนควิสท์ (Shannon-Nyquist theorem) เรียกว่าคอมเพรสซีฟแซมปลิ่ง (compressive sampling) หรือคอมเพรสเซนซิ่ง (compressed sensing) การสร้างสัญญาณย้อนกลับของแนวทางนี้ได้พิสูจน์ และสามารถทำได้โดยการหาค่าต่ำสุดแบบนอร์มหนึ่ง

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(ℓ_1-minimization) ซึ่งต่อมาได้รับการพัฒนามาเป็นระเบียบวิธีการแบบถ่วงน้ำหนัก (reweighted algorithm) และสามารถสร้างสัญญาณกลับได้ใกล้เคียงกับสัญญาณที่มีความเบาบาง (sparse signal) แต่อย่างไรก็ตาม ยังไม่มีแนวทางการเลือกค่าถ่วงน้ำหนัก แบบอัตโนมัติ ด้วยเหตุนี้ บทความฉบับนี้จึงเสนอทางเลือกของฟังก์ชันถ่วงน้ำหนักแบบใหม่ และมุ่งเสนอแนวทางการเลือกค่าถ่วงน้ำหนักที่เหมาะสม

ในกระบวนการสร้างสัญญาณย้อนกลับโดยการหาค่าต่ำสุดแบบนอร์มหนึ่งนั้น มีโอกาสผิดพลาด เนื่องจากมีการย้ายคำตอบที่มีค่าเป็นศูนย์ไปยังตำแหน่งอื่น ๆ ดังนั้นระเบียบวิธีการถ่วงน้ำหนักแบบเลือกค่าอย่างฉับพลัน (hard selective reweighted, HSR) จึงถูกออกแบบเพื่อเพิ่มความสำคัญของตัวแทนคำตอบศูนย์ ซึ่งเลือกจากคำตอบที่มีค่าใกล้ศูนย์มาเป็นจำนวนเท่ากับจำนวนของคำตอบศูนย์ของสัญญาณดั้งเดิม โดยจะทำการเพิ่มขนาดของคำตอบให้ใหญ่ขึ้น แต่โดยทั่วไปตำแหน่งของคำตอบศูนย์นั้นไม่สามารถรู้ได้ จึงส่งผลให้ระเบียบวิธีการถ่วงน้ำหนักแบบเลือกค่าอย่างฉับพลันไม่สามารถประยุกต์ใช้ในปัญหาในโลกความเป็นจริง เพื่อที่จะจัดการกับปัญหานี้จึงได้มีการเสนอเพิ่มเติมระเบียบวิธีการถ่วงน้ำหนักแบบเลือกค่าได้อัตโนมัติ (automatic adaptive reweighted, AAR) ซึ่งใช้สำหรับทำนายหาตำแหน่งของค่าคำตอบศูนย์ โดยไม่จำเป็นต้องทราบจำนวนของคำตอบศูนย์ของสัญญาณดั้งเดิม แนวคิดนี้ใช้วิธีการแบ่งคำตอบออกเป็นช่วงความถี่ต่าง ๆ และตรวจสอบว่าช่วงความถี่ที่มีค่าน้อยที่สุดใดไม่มีสมาชิกอยู่ จะถูกกำหนดให้เป็นตำแหน่งจุดของการเปลี่ยนแปลง และกำหนดให้คำตอบที่มีค่าใกล้ศูนย์ และคำตอบอื่น ๆ ถูกเพิ่มขนาดให้ใหญ่ขึ้น และลดขนาดให้เล็กลงตามลำดับ จากผลการทดสอบเชิงตัวเลข พบว่า ผลการสร้างสัญญาณย้อนกลับที่ถูกต้อง โดยระเบียบวิธีการถ่วงน้ำหนักแบบเลือกค่าอย่างฉับพลันและระเบียบวิธีการถ่วงน้ำหนักแบบเลือกค่าได้อัตโนมัติ ดีกว่าวิธีการหาค่าต่ำสุดแบบนอร์มหนึ่ง นอกจากนั้นระเบียบวิธีทั้งสองยังถูกนำไปประยุกต์กับภาพที่ถูกสังเคราะห์จากมนุษย์ และภาพที่สร้างจากเรโซแนนซ์แม่เหล็ก (magnetic resonance imaging, MRI)

Key Words : Compressive sampling, Compressed sampling, Compressive sensing, Sparsity, Reweighted ℓ_1-minimization, ℓ_1-minimization.

คำสำคัญ : คอมเพรสซีฟแซมปลิ้ง คอมเพรสแซมปลิ้ง คอมเพรสซีฟเซนซิ่ง สัญญาณที่มีความเบาบาง การหาค่าต่ำสุดของนอร์มหนึ่งโดยถ่วงน้ำหนัก การหาค่าต่ำสุดของนอร์มหนึ่ง

Introduction

A traditional compression method usually applies a transformation to a sampled signal and then truncates most of coefficients but significant ones. This means that the quality of signal compression depends on type and efficiency of transformation. Up until now there are many transformations which have been currently used such as *Fourier transformation*, *discrete cosine transformation*, *wavelets*, etc. However, the way to choose the transformation is specifically designed for each application so that there are no suitable transformations for all signals. A brief traditional compression process is shown in Figure 1.

Compressive sampling, also known as compressed sensing, is a new founded method to compress signal by exploiting its compressibility. A conventional sampled signal depends on Nyquist-bandlimited sampling rate but this method ignores the Shannon-Nyquist sampling theorem. This new idea was motivated in 2006 (Candès, Romberg and Tao, 2006). They assume that several signals in the world are sparse, i.e. they contain much repetitious information. Thus, it is not necessary to sample the signal following Shannon-Nyquist theorem and random sampling is sufficiently allowed for a considered sparse signal.

Let x be a real-value, finite-length, discrete-time, one-dimensional signal, which can be viewed as an $N \times 1$ column vector in R^N. A signal in R^N can be represented in term of a $N \times N$ basis matrix $\Psi = [\psi_1 | \psi_2 | \ldots | \psi_N]$ by stacking the basis vector ψ_i as columns, i.e.

$$x = \sum_{i=1}^{N} s_i \psi_i \, , \tag{1}$$

$$x = \Psi s \, . \tag{2}$$

where s is the $N \times 1$ column vector of weighting coefficients s_i. This operation is the first stage in Figure 1.

A sparse representation focuses on the elements of non-zero entries in coefficient vector s. If there are K non-zero entries with $K << N$ - only K of the s_i in (1) are non-zero and $(N - K)$ are zero, the signal will be considered as a K-sparse signal. In fact, many natural and manmade signals are sparse with a few large coefficients and many small coefficients (Baraniuk, 2007).

The idea of compressive sampling is to transform the signal x via a measurement matrix $\Phi_{M \times N}$ as defined in (3) and shown in Figure 2.

$$y = \Phi x = \Phi \Psi s = \Theta s \, . \tag{3}$$

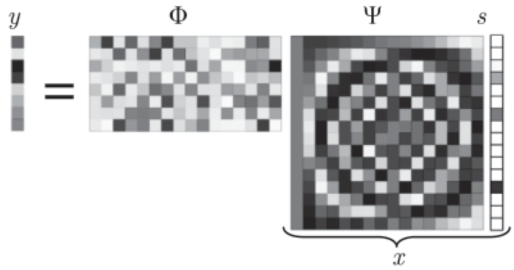


Figure 2 Compressive sampling measurement process (Baraniuk, 2007)

From (3), the matrix product Θ is the representation of measurement matrix Φ and basis matrix Ψ whose columns correspond to non-zero coefficients s_i; i.e. the compressed signal y is a linear combination of K columns of matrix product Θ as shown in Figure 3.

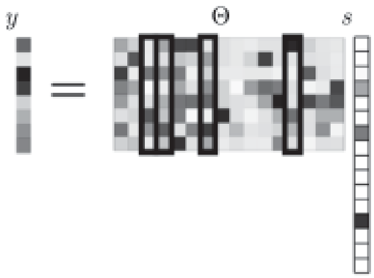


Figure 3 Compressed signal from the linear combination of highlighted columns (Baraniuk, 2007)

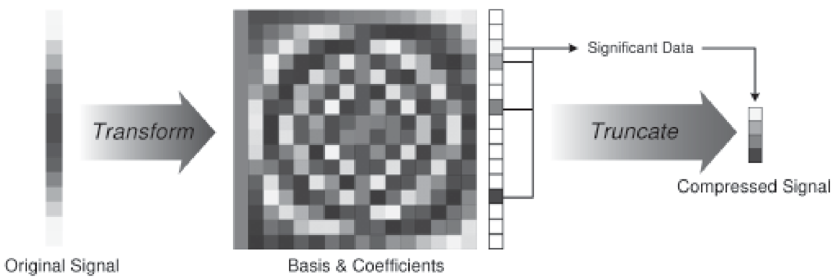


Figure 1 Traditional compression process

It is surprising that with $M < N$ the signal can be reconstruction from the compressed signal y (Candès, Romberg and Tao, 2006). This is to find a solution of system of linear equations with fewer equations than unknowns. As known from linear algebra there are infinitely possible solutions. Thus, this problem cannot be solved unless there are other imposed conditions.

The reconstruction algorithm for K-sparse requires sufficiently $M \approx K$ measurement matrix or slightly more measurements to collect significant coefficients. Because there are K non-zero entries which cause this system likely that there are K unknowns so that the measurement rows $M \approx K$ which approximate to K equations are enough to solve this problem (Baraniuk, 2007).

In actual fact, the locations of non-zero coefficients s_i in K-sparse signal are not known although a number of equations M equal or exceed a number of unknowns K. Thus, a necessary and sufficient condition that ensures the solution of $M \approx K$ system is that the vector $\bar{s}$ must share the same locations as K non-zero entries. The matrix Θ which applies this ideal preserving the lengths of these particular K-sparse vectors is said to have *restricted isometry property* (RIP),

$$1 - \varepsilon_{\mathrm{RIP}} \le \frac{\|\Theta\bar{s}\|_2}{\|\bar{s}\|_2} \le 1 + \varepsilon_{\mathrm{RIP}}, \qquad (4)$$

for some $\varepsilon_{\mathrm{RIP}} > 0$. So far there are independent and identically distributed (i.i.d.) random Gaussian distribution and Rademacher distribution satisfying the RIP property (Baraniuk, 2007).

Reconstruction algorithms

The $M < K$ system generates infinitely many possible solutions which all lie on the $(N - M)$-dimensional hyperplane $\mathcal{H} = \mathcal{N}(\Theta) + s$ in R^N. The true solution vector is also sparse corresponding to the constraint $\Theta(s + r) = y$ for any vector r in null space $\mathcal{N}(\Theta)$. Thus, our goal is to find the sparsest coefficient vector s in the translated null space.

A suitable method that counts the smallest number of non-zero in coefficient vector s is ℓ_0-minimization,

$$\min \|s\|_0 \text{ sub. to } \Theta s = y, \qquad (5)$$

which ℓ_0 is well-called "zero norm". The other form of ℓ_p-norm for the vector is defined as,

$$\|s\|_p = \sum_{i=1}^{N}\left(|s_i|^p\right)^{\frac{1}{p}}, p > 0. \qquad (6)$$

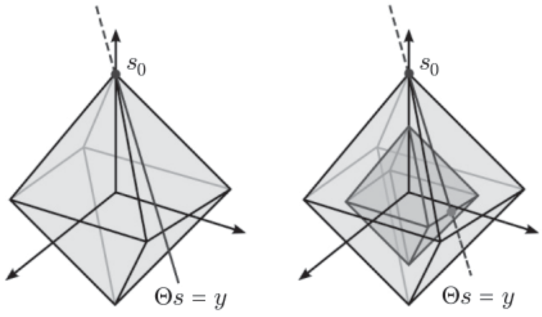
However, the ℓ_0-minimization is hard to solve because an exhaustive enumeration which is NP-complete requires C_K^N possible combinations for all locations of non-zero entries in vector s. Thus, this optimization is adapted to the approximate ℓ_1-minimization,

$$\min \|s\|_1 \text{ sub. to } \Theta s = y. \qquad (7)$$

This is a convex optimization problem that conveniently reduces to a linear programming which requires computational complexity about $O(N^3)$ (Boyd and Vandenberghe, 2004). It is proved that this optimization can exactly reconstruct K-sparse signal vectors with high probability (Candès, Romberg and Tao, 2006).

Reweighted ℓ_1-minimization

The differences of solutions between ℓ_0 and ℓ_1 norms are the locations of K non-zero entries. Although ℓ_1-minimization can search the locations of K non-zero entries, a number of reconstructed zero entries are not equal to the original. Because the objective function of ℓ_1-minimization is designed specifically for a symmetric ℓ_1-ball (the possible capacity cost of objective function $\|s\|_1$) which has probability to touch the hyperplane in a wrong position for which $\|s\|_1 < \|s_0\|_0$, shown as interior ℓ_1-ball (Figure 4(b)) while the ℓ_1-ball of radius $\|s_0\|_1$ touches the true solution at the vertex containing more zero entries and close to K-sparse (Figure 4(a)).



(a) (b)

Figure 4 ℓ_1-ball for reconstructed signal (Candès, Wakin, and Boyd, 2007)

The enhancement of ℓ_1-minimization which reshapes the ℓ_1-ball to counteract the function penalty is possible for a convex optimization. A weighted relaxation ℓ_1-minimization which employs this idea to formulate the objective function $\|Ws\|_1$ with the same constraints is expressed as,

$$\min \|Ws\|_1 \text{ sub. to } \Theta s = y, \qquad (8)$$

where $W = diag([w_1\, w_2, \ldots, w_N]^T)$ is a weighting diagonal matrix of size $N \times N$. The weighted ℓ_1-minimization can be solved via linear programming as same as ℓ_1-minimization. However, the solutions from the convex optimization might present the different solutions. Thus, the suitable weighting matrix can reshape the ℓ_1-ball in order to avoid the fault solution $s \neq s_0$ for which $\|Ws\|_1 \leq \|Ws_0\|_0$ as illustrated in Figure 5.

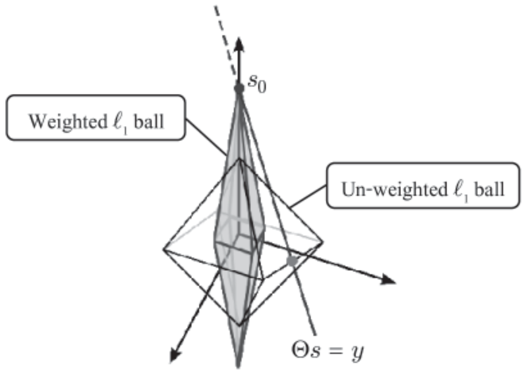


Figure 5 Weighted & un-weighted ℓ_1-ball for coefficient signal vector (Candès, Wakin, and Boyd, 2007)

The conceptual weighting function is designed to control the true solution by counteracting the influence of signal magnitude on the ℓ_1 penalty function. The recommended weights are inversely to the true signal magnitude (Candès, Wakin, and Boyd, 2007),

$$w_i = \begin{cases} \dfrac{1}{|s_{0,i}|} & \text{if } s_{0,i} \neq 0 \\ \infty & \text{if } s_{0,i} = 0 \end{cases}, \qquad (9)$$

where $s_{0,i}$ is the true solution to each entry i. This weighting function guarantees to find the true solution but the locations of K non-zero entries are not already known. Thus, this weighting function is unavoidably applied the solution from ℓ_1-minimization to construct the weights. Otherwise, the numerical computation cannot be defined when $w_i = \infty$ so that the weighting function are revised to the equivalent form,

$$w_i = \frac{1}{|s_i| + \varepsilon}, \quad (10)$$

where $\varepsilon > 0$ for ensuring the division of zero-value component in the reconstructed vector s_i which might estimate the weighting value w_i to infinity.

The equivalent weighting function (10) is not required the location of the true solutions so that the efficiency to find the true solutions might be generally decreased. The achievable process which enhances weighted ℓ_1-minimization is to repeat the algorithm iteratively. A simple reweighted algorithm is shown in Figure 6.

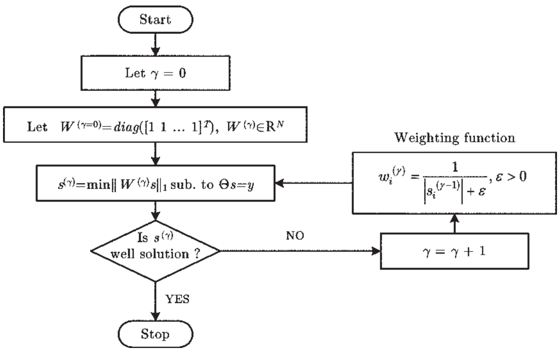


Figure 6 Iterative reweighted ℓ_1-minimization

The first knowing solution vector $s^{(0)}$ obtained from un-weighted ℓ_1-minimization is used to construct a range of weighting matrix $W^{(1)}$. Then, the next weighting matrix will be estimated by the previous solution vector $s^{(\gamma-1)}$. This algorithm repeats until the solutions terminate on convergence or when iteration γ attains a specified maximum number of iterations $\gamma_{\max}$.

In general, the weighting magnitude w_i depends on the choice of parameter ε which controls its distribution changed obviously when varying the value of ε.

However, there are currently no smart and robust rules that would automatically select the parameter ε adapting the dynamic range of weighting values (Candès, Wakin, and Boyd, 2007). Thus, the algorithm that ensures the appropriate weighting magnitude w_i is still an open question.

Methods and Solutions

An innovative proposed idea is to design new weighting function which is more general to select the appropriate weighting value. Since the parameter ε in weighting function is unbounded for some $\varepsilon > 0$ so that it is difficult to vary its value when undergoing the experiment. The way to adapt to the new weighting function is possible for convex optimization. From weighting function (9), the weighting values are mostly to scale the solutions which their values are close to zero by the large factors. Thus, the achievable weighting function which has the similar characteristic designed to reduce the complexity of function is proposed as *hard selective weighing function* (Charunphaisan and Meesombon, 2009A),

$$w_i = \begin{cases} \beta_{\max} & , |s_i| \le \tau \\ \beta_{\min} & , |s_i| > \tau \end{cases}, \quad (11)$$

where τ is the threshold which its value is in the period of possible solutions divided the solutions into 2 groups; there are the solutions which are close to zero multiplied by $\beta_{\max}$ and the other solutions are multiplied by $\beta_{\min}$. Furthermore, the parameters $\beta_{\max}, \beta_{\min} > 0$ are well-defined for the reason that if $\beta_{\max}, \beta_{\min} = 0$, some solutions which are in the summation $\|Ws\|_1$ will be ignored.

According to Figure 5, the vertex of weighted ℓ_1-ball is extended to intersect the hyperplane at the true solution which contains many zero entries. Suppose, the solution s_i which are multiplied by $\beta_{\max}$ are zero candidates so that the weighting value $\beta_{\max}$ should be infinitely larger than $\beta_{\min}$, as a *hard selective ratio*,

$$\frac{\beta_{\max}}{\beta_{\min}} = \infty, \quad (12)$$

However, this ratio cannot be defined as infinity in numerical experiments in order that the infinite ratio is alternatively changed to the suggested formulation,

$$\frac{\beta_{\max}}{\beta_{\min}} = \frac{1}{\mu}, \quad (13)$$

where $\mu \le 1$ is represented as the expanding rate. The ℓ_1-ball can be spanned more elaborately touching the hyperplane when the limit of μ is closer to zero. Additionally, the recommended value of parameter μ is equal to the floating fixed-point accuracy. For example, if the considered signal is of the accuracy at 0.01, the parameter $\mu \le 0.01$ will be the sufficient value. The *hard selective reweighted* (HSR) algorithm, is shown in Figure 7.

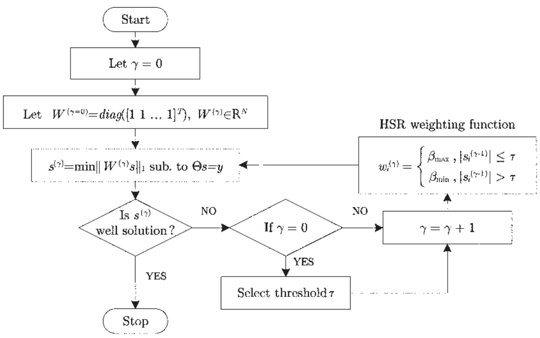


Figure 7 Iterative HSR algorithm including selecting threshold τ process

In reconstruction process, the ℓ_1-minimization is possible to recover the fault signal. For example in Figure 10(b), it shows the scatter plot of reconstructed coefficient vector $s^{(0)}$. As we notice these points spreading widely the diagonal line so that, in this case, the un-weighted ℓ_1 minimization does not offer the exact solution.

Another point of view, this means that some zero entries in the reconstructed coefficient vector $s^{(0)}$ might spread around to the other solutions. One possibility to recall the original zero entries is to find the locations of all zero candidate entries. In this paper, we assume the locations of solutions which their values are close to zero will be set as zero entries. Moreover, if a number of zero entries in the original coefficient vector $s_0 = \Psi^{-1} x_0$ are correctly known and the ℓ_1-minimization presents the fault reconstructed vector $s^{(0)}$, the threshold τ in reweighted ℓ_1-minimization will be computed as follows. Firstly, calculate a number of absolute elements of fault reconstructed vector $s^{(0)}$ in each frequency bin (that is histogram of absolute reconstructed vector $|s^{(0)}|$). After that, count the cumulative sum along difference frequency index until the sum is greater than or equals to a number of original zero entries. Finally, the last cumulative sum is the value of the threshold τ.

For example, Figure 8 shows the cumulative sum of histogram of absolute coefficient vector $|s^{(0)}|$ via ℓ_1-minimization. We know that the original coefficient vector s_0 contains 455 zero entries while the zero entries of absolute coefficient vector $|s^{(0)}|$ are merely about 370 so that the threshold is defined as 0.05 for containing equally 455 zero candidate solutions.

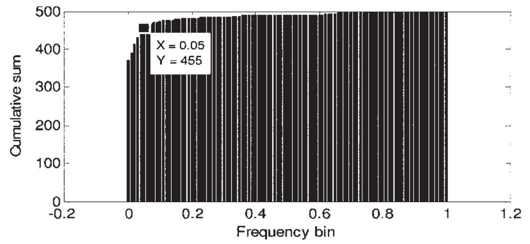


Figure 8 Cumulative sum of histogram values of absolute reconstructed vector $\left|s^{(0)}\right|$ using unweighted ℓ_1-minimization

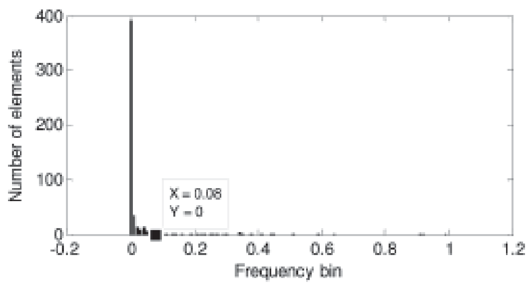


Figure 9 Histogram values of absolute reconstructed vector $\left|s^{(0)}\right|$ using unweighted ℓ_1-minimization

However, a number of original zero entries are not known in practical so that an alternative simple algorithm without knowing a number of original zero entries is also proposed by finding only the smallest frequency bin of histogram of absolute coefficient vector $\left|s^{(0)}\right|$ which is empty, called *automatic adaptive reweighted* (AAR) algorithm (Charunphaisan and Meesombon, 2009B). A brief concept of algorithm is to assume the near-zero solutions being the zero candidates and their entries are punctuated by the nearest empty frequency bin of histogram of absolute coefficient vector $\left|s^{(0)}\right|$. For example, Figure 9 shows the histogram of fault absolute coefficient vector $\left|s^{(0)}\right|$. There are not only zero entries in zero index, but also the other zero entries might be shifted to the other values around zero index. The nearest empty frequency bin of this example is about 0.08 so that the algorithm has decided to select the threshold τ =0.08 for containing the near-zero solutions to be zero candidate entries.

Numerical results

This section demonstrates the numerical experiments that the concept of ℓ_1-ball analysis can do manifestly in practical. However, reweighted ℓ_1-minimization is currently not known the method to define appropriate parameter in weighting function (10). Thus, the results can present sufficiently the percentage of successful reconstruction via ℓ_1-minimization, HSR and AAR algorithms.

The first experiment considers a normalized uniformly distribution K-sparse signal vector of size 250-dimensional and defines the measurement matrix Φ as a zero-mean normal (Gaussian) matrix of size 50×250, generated once for 200 trails of each K-sparse. This experiment categorizes the reconstructed signal which has $\text{PSNR} \geq 80$ as the exact reconstructed signal. Let the expanding rate $\mu = 0.01$ and set 5 iterations for all computational experiments.

In Figure 11, the comparison graph shows that the percentages of HSR and AAR algorithms outperform ℓ_1-minimization about 12.63% and 13.38%, respectively.

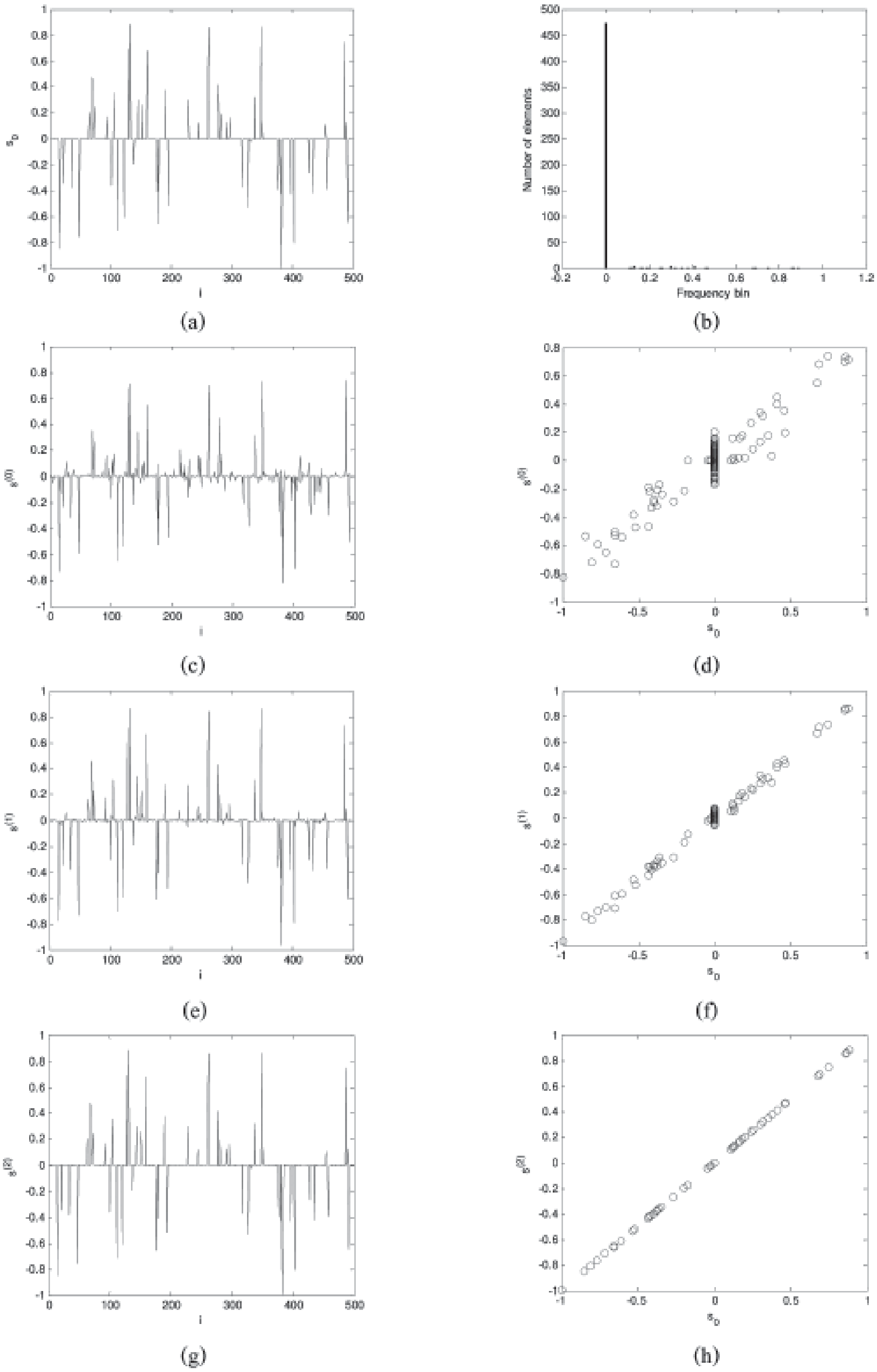


Figure 10 Sparse signal recovery using HSR algorithm. (a) Original cofficient vector s_0 on the interval [-1,1], length $N = 500$, with 45 spikes and (b) its histogram. (c) Reconstructed cofficient vector $s^{(0)}$ and (d) scatter plot (coefficient-by-coefficient of s_0 versus its reconstruction) using unweighted ℓ_1-minimization. (e) Reconstructed cofficient vector $s^{(1)}$ after the first reweighted iteration and (d) its scatter plot. (g) Reconstructed cofficient vector $s^{(2)}$ after the second reweighted iteration and (h) its scatter plot.

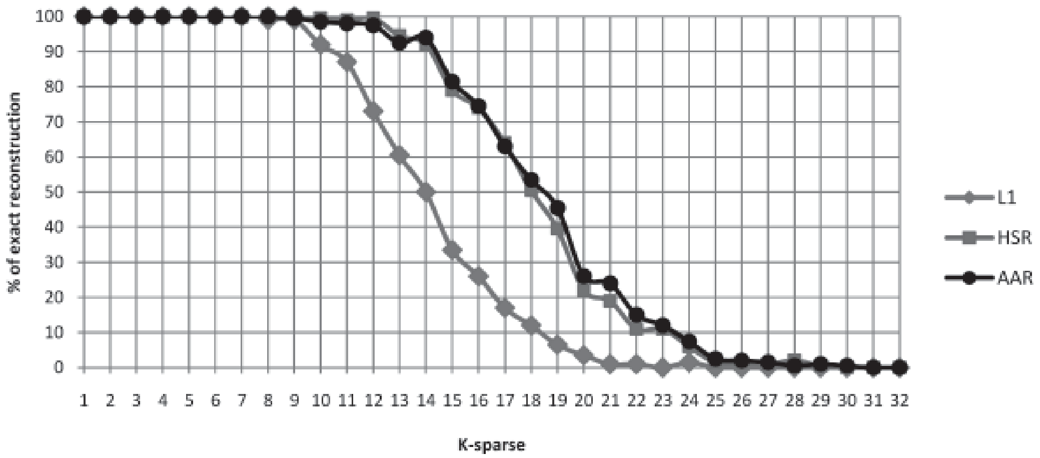


Figure 11 Comparison of ℓ_1-minimization, HSR and AAR algorithms in compressive sampling reconstruction

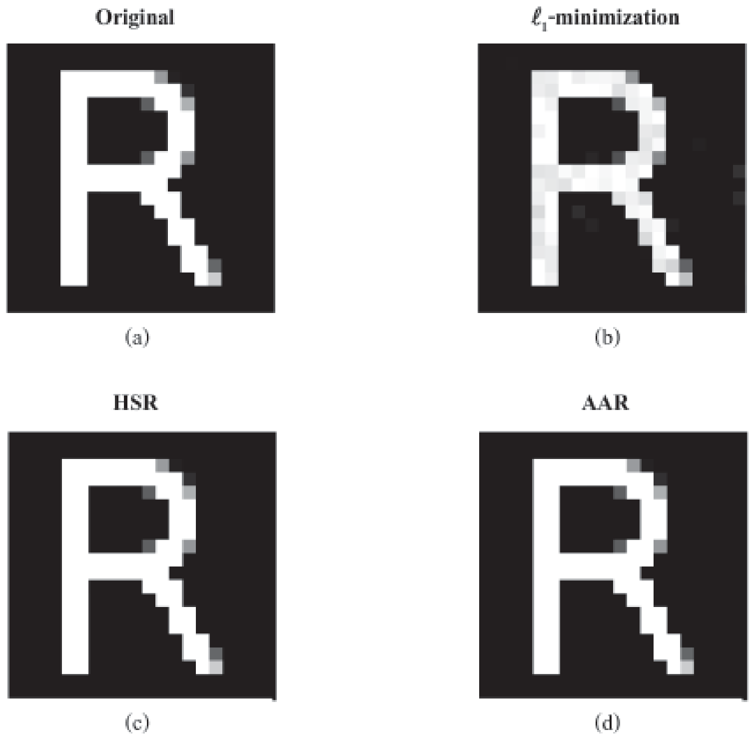


Figure 12 Example of manmade images in compressive sampling: (a) original image (b) ℓ_1-minimization, PSNR=61.83 dB (c) reconstructed image when using HSR algorithm, PSNR=539.21 dB and (d) reconstructed image when using AAR algorithm, PSNR=545.02 dB

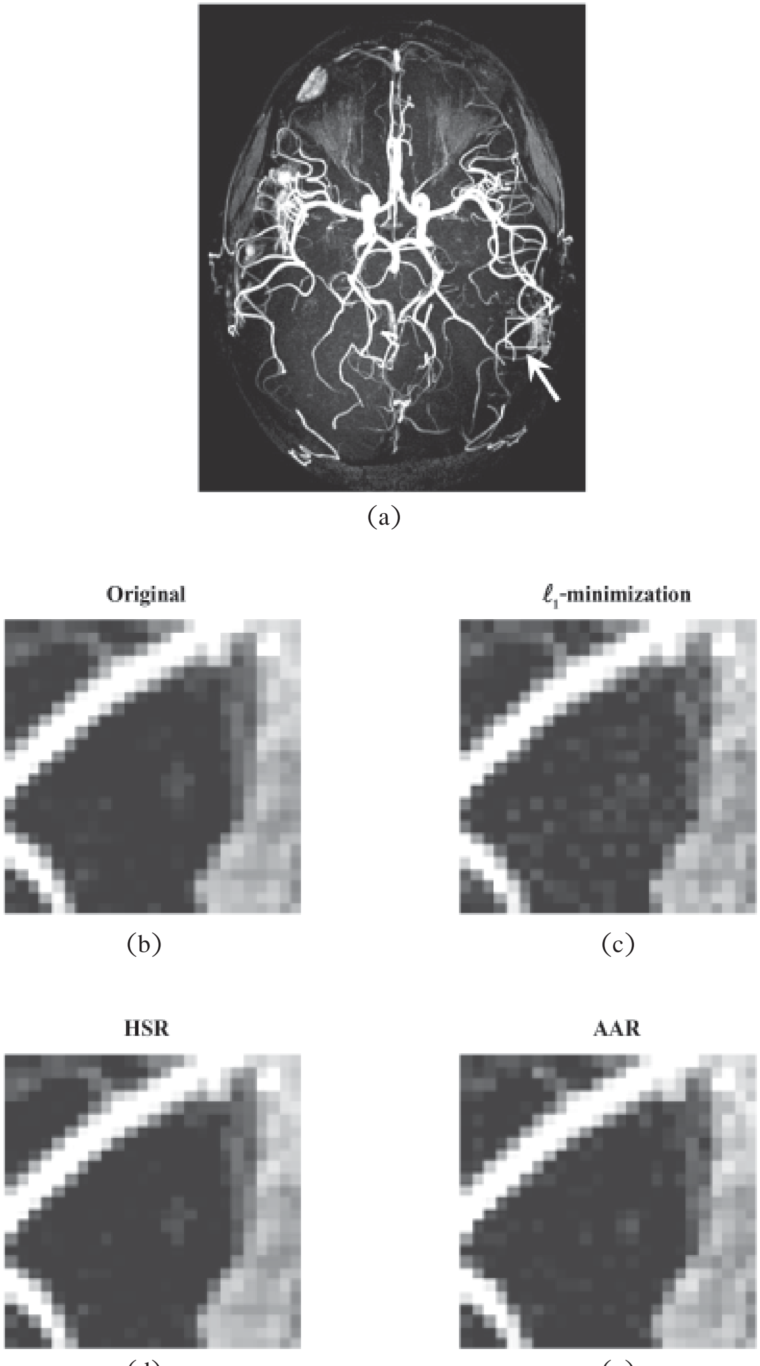


Figure 13 Angiogram MRI images in compressive sampling reconstruction: (a) original MRI image of size 432×338 (Dyck and Wilson, 2006) (b) original cropped MRI image of size 25×25 (framed in Figure 16 (a)) (c) reconstructed cropped MRI image when using ℓ_1-minimization, PSNR=71.30 dB (d) HSR algorithm, PSNR=597.56 dB and (e) AAR algorithm, PSNR=110.88 dB

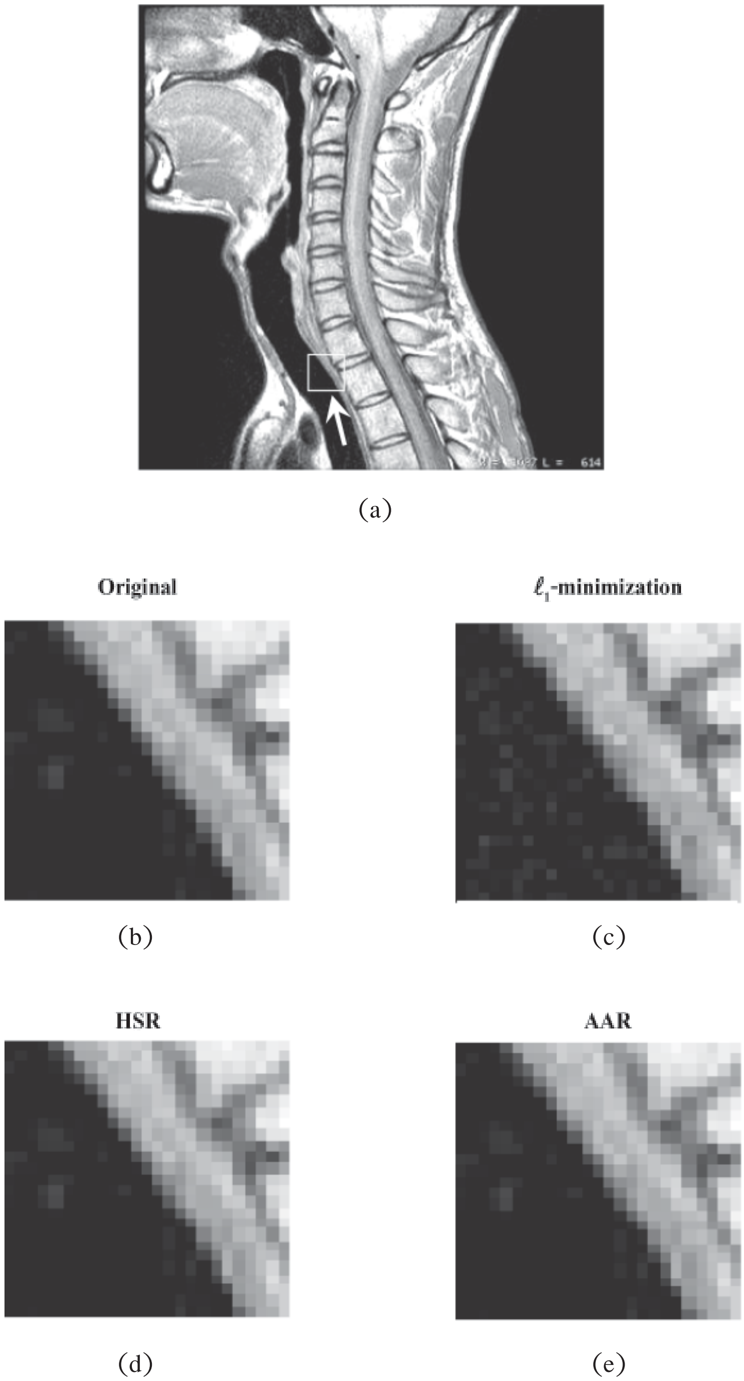


Figure 14 Throat MRI images in compressive sampling reconstruction: (a) original MRI image of size 336×337 (Slocum, 2009) (b) original cropped MRI image of size 25×25 (framed in Figure 16 (a)) (c) reconstructed cropped MRI image when using ℓ_1-minimization, PSNR=69.26 dB (d) HSR algorithm, PSNR=574.92 dB and (e) AAR algorithm, PSNR=585.90 dB

The second experiment applies these algorithms to recover an example of manmade image of size 25×25 with 34.50% K-sparse. The numerical result shows that PSNRs of its reconstructions via ℓ_1-minimization, HSR and AAR algorithms as shown in Figure 12.

In the last experiment, an angiogram MRI image of size 432×338 (Figure 13(a)) with 58.14% K-sparse and a throat MRI image of size 336×337 (Figure 14(a)) with 60.29% K-sparse are cropped to the undersized image of size 25×25 which are the representation of original image with 64.80% K-sparse and 59.04% respectively.

Figure 13 and Figure 14 show the reconstructions of angiogram and neck MRI images respectively, via ℓ_1-minimization, HSR and AAR algorithms.

Results and Discussions

HSR algorithm can recover the exact signal with high probability but this algorithm needs to know a number of original zero entries. However, AAR algorithm is designed to cope with this problem although the percentage of exact reconstruction is little lower than HSR algorithm. This means that AAR algorithm can supersede HSR algorithm in case that a number of original zero entries are not known.

Even though, from the results, the HSR sometimes yields higher PSNR than that of AAR (sometimes it is the other way round), the main purpose of this paper is to provide methods that give better performance than ℓ_1-minimization reconstruction.

It is also an open question in compressive sampling that when and what kind of signals for the compressive sampling reconstruction process which should work since the main result (Candès, Romberg and Tao, 2006) is only proved with high probability of success when the signal is sparse enough.

In this paper, the enhancements of reweighted algorithms are proposed to find the appropriate weighting matrix. As yet there are many undetermined questions for some properties as below,

- What condition does the algorithm converge?
- How many iterations does the solution converge?
- Can these weighting functions and algorithms apply to the other applications?

Furthermore, since our experiments are only applied by the linear programming so that there are many interesting tools to solve this optimization problem such as Dantzig selector, basis pursuit and total variance minimization in addition to ℓ_1-Magic (Candès and Romberg, 2005). For further works, we will search out the theoretical supports for above questions and apply to the other applications such as signal recognition, remote sensing image, image processing, etc.

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