Mathematical models for Environmental-friendly vehicle routing problem

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Abstract

Almost one third of all energy is used for road transportation and one important destructive environmental effect of fuel consumption in road transport is its contribution to atmospheric pollution. The amount fuel consumed and pollution emitted by a vehicle depends on its load, velocity and other factors. There are carbon balance between the total carbon in fuel and the total carbon in all of the combustion that are emitted to gas pollution. This paper presents a Mathematical models of Environmental-Friendly' Vehicle Routing Problem or Fuel & Pollution-Routing Problem (FPRP), an extension of the classical Vehicle Routing Problem (VRP) with a wide and more all-inclusive objective function that accounts not only to minimizing the travel distance, but also minimizing for the amount of greenhouse gas emissions, fuel consumption, total travel times with stochastic velocity and time window, and then solved the problem by LINGO Programming on small-realistic instances. We obtained a reasonable result and also founded that it's significantly more challenging to solve to optimality. Nevertheless, it had the potential of better yielding of reduction emission result than Classical Vehicle Routing Problem (VRP)

Keywords: Fuel Consumption, Road Transportation, Vehicle Routing Problem

1. Introduction

Transportation especially road or truck transport has mainly disastrous impacts on the environment, such as degradation of air quality, resource consumption , toxic effects on ecosystems and humans, and the effect persuaded by Greenhouse Gas (GHG) emissions . Among these, GHG and in particular CO_2 emissions from combustion of fossil fuel to energy are the most concerning as they not only have direct consequences on human health but also have an effect to increasing of global temperature called "Global Warming". There are variety solution can be used to reduce pollution and energy consumption that fleet management or Vehicle Routing Problem (VRP) is one important instrumentality that can get more benefit by route optimization.

Vehicle Routing Problem (VRP) is the significant problem in the section of logistic, transportation and distribution. This problem can be explained as road transportation planning and purposes at designing optimal delivery or collection fleet of vehicles routing on a given network to serve a set of customer under side constraints [5]. By basic objectives of this problem are minimizing traveled distance to minimize transportation cost, number of vehicle used and transportation time along with improving service level that all vehicles start and end at a single depot. Then, the amount of pollutant emission and energy consumption are applied to general objective of VRP that the data and equation of pollutants emission are more based on the methodology for estimating air pollution emissions from transport (MEET) [7], Project of the European Commission (EC). The amounts of pollution emitted by a vehicle and fuel consumption depend on its load and speed, among other factor [12]. This paper presents the an extension of the classical Vehicle Routing Problem model for reduce pollutant emission and fuel consumption by considering uncertainty customer demand and traveling speed for each route and show the results that obtain from our example experimental runs.

2. Literature Review

2.1 Vehicle Routing Problem (VRP)

VRP has been developed from Traveling Salesman Problem (TSP) by adding constraints such as capacity of the vehicle, Time window, etc.[10]. This problems were more used to solve the problem of transportation , distribution and logistics which main objective is to design the optimal vehicle sequences for minimize the transportation cost under various constraints. This problem was first introduced by [6] and ongoing more as more extends the problem framework by adding more different constraint such as additional terms of time (VRP with Time Windows; VRPTW), studied by [7] the several type of vehicles used (Heterogeneous VRP; HVRP), invented by [8], added a condition of good delivery and returned (VRP with Backhauls; VRPB), initiated by [4]. Moreover, "Multi-Period Vehicle Routing Problem", which for the vehicle routing problem with regard to various time periods that was developed [10]

2.2 Emission Characteristics

Bauer et al. [2] were firstly applied pollutant emissions to transportation problem by Intermodal Freight Transportation Network Design Problem Model and also designed the specific equations which can consider in conjunction with energy consumptions and vehicle loaded. Bektas and Laporte [8] then generated new comprehensive model that better than old model and were called “Pollution Routing Problem (PRP)” which GHG emissions rate can consider both vehicle velocity and vehicle load along with traveled route.

Barth et al. [1] has designed a heuristic method by developing a Neighborhood Search for solving PRP to minimize both fuel consumption and workers wage by the suitable driving speed. They got appropriate results and can be used this model to solve problems as more parking point up to 200 points .Then, Xiao et al. [11] have taken oil consumption rate into account of weight function in Vehicle Routing Problem and solved by Simulated Annealing .The results show that can reduce fuel consumption by 5% compared to the original model of the CVRP problem.

Z. Zsigraiova et al.[12] has designed a model that combines VRP and waste collections to minimize pollution emissions and fuel consumption by the theories of MEET the results show that it can reduce emissions by up to 43% and 40 % fuel consumption.

The fuel consumption retrieved from MEET is given by:

$$C_{fuel} = \frac{M_{fuel}(\frac{[CO]}{M_{CO}} + \frac{[CO_2]}{M_{CO_2}} + \frac{[HC]}{M_{HC}} + \frac{[PM]}{M_{PM}})}{\rho_{fuel}}, \quad (1)$$

In the previous equation, C_{feul} represents the fuel consumption (l), *[i]* is the emitted mass of pollutant *i* (g), HC stands for unburned hydrocarbons and PM for the particulate matter, M_i is the molar mass of pollutant *i* (g/mol), and ρ_{fuel} is fuel density (g/l).

$$E_i = \sum_{vehicle\ route} (E_{i,hot} + E_{i,cold}), \quad (2)$$

$$E_{i,hot} = \varepsilon_{i,c} d_{tr}, \quad (3)$$

$$\varepsilon_{i,cold} = (k_1 + av + bv^2 + cv^3 + \frac{d}{v} + \frac{e}{v^2} + \frac{f}{v^3}, \quad (4)$$
$$\times \left[(k_2 + rv + sv^2 + tv^3 + \frac{\mu}{v} - 1)Z + 1 \right]$$

$$E_{i,cold} = \varepsilon_{i,cold} N \quad , \qquad (5)$$

According to MEET, pollutant emissions are calculated by Equation (2)–(5), where , E_i, $E_{i,hot}$ and $E_{i,cold}$ are the total emissions of pollutant *i* (g), the hot emissions of pollutant *i* (g), and the cold emissions of pollutant *i* (g), respectively; ε_i is the hot emission factor for pollutant *i* corrected for load (g/km), and d_{tr} is the travelled distance (km); *v* is the mean velocity (km/h), k_1, *a, b, c, d, e, f* and k_2, *r, s, t, u* are coefficients depending on the vehicle total weight, and *z* is the fraction of transported load; $\varepsilon_{i,cold}$ is the cold emission factor for pollutant *i* (g/cold start), and N is number of cold starts. The values of the coefficients of Eq. (4) are displayed in Table 1and the values of the coefficients of Eq. (5) are displayed in Table 2

Table 1 Values of the coefficients appearing in Eq. (4) for heavy-duty vehicles with gross weight in the range 7.5–16 tones [7]

Pollutant	Parameter
CO	$k_1 = 3.08; a = -0.0135; b = 0; c = 0;$ $d = -37.7; e = 1560; f = -5736$ $k_2 = 1.03; r = 9.77\times10^{-4}; s = 0;$ $t = 0; u = 0$
CO_2	$k_1 = 871; a = -16; b = 0.143; c = 0;$ $d = 0; e = 32031; f = 0 k_2 = 1.26;$ $r = 0; s = 0; t = -2.03\times10^{-7};$ $u = -1.14$
NO_x	$k_1 = 2.59; a = 0; b = 0.143;$ $c = 8.56\times10^{-6}; d = 140; e = 0;$ $f = 0 k_2 = 1.19; r = 0; s = 0;$ $t = 0; u = -0.977$
PM	$k_1 = 0.0541; a = 1.51\times10^{-3}; b = 0;$ $c = 0; d = 17.1; e = 0; f = 0$ $k_2 = 1.02; r = 2.34\times10^{-3}; s = 0;$ $t = 0; u = 0$

Table 2 Values of the coefficients appearing in Eq. (5) for heavy-duty vehicles [7]

Pollutant	Vehicle types (tons)	Parameter $\varepsilon_{i,cold}$ (g/cold start)
CO	7.5 – 16	6
CO_2	7.5 – 16	300
NO_X	7.5 – 16	-2
PM	7.5 – 16	0.6

Then, we applied this equation to VRP mathematical model in next section

3. Mathematical Model Formulation

This section describes the mathematical model formulation of our model. First, we define the notations used in this model on a complete graph G = (N, *A*) with *N* = {0, 1, 2…n} as the set of arcs defined between each pair of nodes. Node 0 is the depot. There exist a homogeneous set of vehicles *K* = {1, 2… m}, each with capacity Q and customer has demand q_i and a request to be served within a pre-specified time interval $[a_i, b_i]$ and the distance from *i* to *j* is denoted by d_{ij}.

K : The number of route cycle, which depend on customer demand

N : The number of customer needs to be served

C_0 : Warehouse or Depot

C_i : Customer i

q_i : Demand of customer i (kg)

d_{ij} : Distance between customer i and customer j (km)

v_{ij} : Vehicle Velocity used between customer i and customer j ; in this, we assume the velocity distribution as normal distribution $v_{ij} N(\mu_{ij}, \sigma_{ij}^2)$ (km/h)

t_{ij} : Travel time between customer i and customer j ; depend on distance and velocity used (min)

CW : Vehicle weight; define as 75 tons.

TC : Vehicle Capacity; define as 75 tons.

TM : Total driving duration of each route cycle; here we define as each route must has the total time less than or equal 24 hours or 1440 minutes.

s_j : Service time at customer *j* (min)

y_{ik} : Arrival time of vehicle k at customer *i*

$[a_i, b_i]$: The time window of the vehicle to serve customer *i*. a_i and b_i denote the earliest and the latest time when customer *i* will permit the start of the service (min)

For the path as pollutant emission, E_p, $E_{p,hot}$ and $E_{p,cold}$ are the total emissions of pollutant *p* (g), the hot emissions of pollutant *p* (g), and the cold emissions of pollutant *p* (g), respectively; $\varepsilon_{p,c}$ is the hot emission factor for pollutant *p* corrected for load (g/km), and d_{ij} is the travelled distance (km); v_{ij} is the mean velocity (km/h), k_1, *a, b, c, d, e, f* and k_2, *r, s, t, u* are coefficients depending on the vehicle total weight, and *z* is the fraction of transported load; $\varepsilon_{p,cold}$ is the cold emission factor for pollutant *p* (g/cold start), and N is number of cold starts. C_{feul} represents the fuel consumption (l), *[p]* is the emitted mass of pollutant *p* (g), PM is the particulate matter, M_{fuel} is the molar mass of pollutant *p* (g/mol), and ρ_{fuel} is fuel density (g/l).

$$Minimise \quad P = \sum_{x=1}^{4} E_p \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m} x_{ijk}, \qquad (6)$$

$$\varepsilon_{p,c} = [(k_1 + av_{ij} + bv_{ij}^2 + cv_{ij}^3 + \frac{d}{v_{ij}} + \frac{e}{v_{ij}^2} + \frac{f}{v_{ij}^3})] \quad , \qquad (7)$$
$$\times \left(\left[\left(k_2 + rv_{ij} + sv_{ij}^2 + tv_{ij}^3 + \frac{u}{v_{ij}}\right)\left(\frac{CW + w_{ijk}}{CW + TC}\right)\right] + 1\right)$$

$$E_{p,hot} = \varepsilon_{p,c} d_{ij} \qquad \forall i, \forall j, \forall k, \forall p \qquad (8)$$

$$E_{p,cold} = \varepsilon_{p,cold} N \qquad \forall i, \forall j, \forall k, \forall p \qquad (9)$$

$$E_p = E_{p,hot} + E_{p,cold} \qquad \forall_i, \forall_j, \forall k, \forall p \qquad (10)$$

$$C_{fuel} = \frac{M_{fuel}\left(\frac{[CO]}{M_{CO}} + \frac{[CO_2]}{M_{CO_2}} + \frac{[PM]}{M_{PM}}\right)}{\rho_{fuel}} \qquad (11)$$

$$\text{s.t.} \sum_{i=1}^{N} x_{i1k} = 1 \qquad \forall i, i = (2,3,..,n), \forall k \qquad (12)$$

$$\sum_{i=1}^{N} x_{1jk} = 1 \quad \forall j, j = (2,3,..,n), \forall k \qquad (13)$$

$$\sum_{k=1}^{K}\sum_{j=1}^{N} x_{ijk} = 1 \quad \forall i, i = (2,3,..,n) \qquad (14)$$

$$\sum_{k=1}^{K}\sum_{i=1}^{N} x_{ijk} = 1 \quad \forall j, j = (2,3,..,n) \qquad (15)$$

$$\sum_{j=1}^{N}\sum_{i=1}^{N} x_{ijk} - \sum_{j=1}^{N}\sum_{i=1}^{N} x_{jik} = 0 \ \forall k \qquad (16)$$

$$\sum_{j=1}^{N}\sum_{i=1}^{N} q_i x_{ijk} \le TC \ \forall k \qquad (17)$$

$$\sum_{j=1}^{N} w_{ijk} - \sum_{j=1}^{N} w_{jik} = q_i \sum_{j=1}^{N} x_{ijk} \quad \forall i, \forall k \qquad (18)$$

$$w_{1jk} \le TC \qquad \forall j, \forall k \qquad (19)$$

$$w_{i1k} = 0 \qquad \forall i, \forall k \qquad (20)$$

$$q_j x_{ijk} \le w_{ijk} \le TC - q_i \quad \forall i, \forall j, \forall k \qquad (21)$$

$$\Pr\left\{\sum_{j=1}^{N}\sum_{i=1}^{N}\left(\frac{d_{ij}}{v_{ij}}\right) x_{ijk} \le TM\right\} \ge \lambda \ \forall k \qquad (22)$$

$$y_i - y_j + t_i \le M_{ij}(1 - x_{ij}) \ \forall i, \forall j, \forall k \qquad (23)$$

$$a_i \le y_i \le b_i \qquad \forall i, \forall k \qquad (24)$$

$$y_i + t_j + s_j \le L(1 - x_{jo}) \ \forall i, \forall j, \forall k \qquad (25)$$

$$M_{ij} = \max\{o, b_i + s_i + d_{ij}/l_{ij} - a_j\} \ \forall i, \forall j \qquad (26)$$

$$x_{ijk} \in \{0,1\} \ \forall i, \forall j, \forall k \qquad (27)$$

$$w_{ijk} \ge 0 \ \forall i, \forall j, \forall k \qquad (28)$$

Where x_{ijk} is the decision variable. It is equal to 1 if edge $(i, j) \in A$ is used by vehicle *k* and 0 otherwise. Objective Function in Equation (6) state minimizes total emission. Equations (7)-(11) are sub-equations of pollutant emission and fuel consumption. Constraints (12) - (16) ensure each node must be visited only once. Constraint (17) states that total vehicle load do not exceed its capacity. Balance weight load is shown in constraint (18) – (21) and total time for each route constraint (22) is formulated in Chance-constraint programming to control the flexibility of total route time that may exceed *TM*, λ is confidence level of total time which over *TM*. The reason that we formulated in this form because of we need to check the influence of velocity or speed when it is not stable along the route (in this paper we assume velocity in each arc (*i,j*) are normally distributed random variable. Constraints (23) are used to restrict the total load a vehicle carries by its capacity. Time windows are imposed by constraints (24), which are obtained through a linearization of a set of nonlinear inequalities, as in [5] in constraints (25) by constraints (26).

4. Computational Results

In this section, we present experimental results from small – generated problem size and classified them into four categories namely;

Problem A = Modeling with short distance - low velocity (3 node, 5 node and 7 node.)

Problem B = Modeling with short distance – high velocity (3 node, 5 node and 7 node.)

Problem C = Modeling with long distance - low velocity (3 node, 5 node and 7 node.)

Problem D = Modeling with long distance - high velocity (3 node, 5 node and 7 node.)

Table3shows lower and upper bound of generated data using “Law of large number” which use in our computational runs

Table 3 Lower and upper bound of parameter using in experiment.

Parameter	Lower bound	Upper bound
Demand (q_i)	950 kg.	1800 kg.
Distance (d_{ij})	20 km.	150 km.
Min Velocity	20 km/hr.	79 km/hr.
Max Velocity	80 km/hr.	120 km/hr.

Generally, the driving velocities on road are not constant caused from many factor such as traffic jam, road gradient, weather and environment and etc. Therefore, In each arc (*i,j*), we generated 50 values of velocity under the range of minimum and maximum velocity and use the equation that d_{ij}/v_{ij} to calculate average traveling time by the pre - generated velocity .What's more, we defined the duration of total traveling time on each arc will exceed maximum duration allowed (λ) is probability equal to 90 percentage.

We first converted constraint (22) to deterministic form [9] by following proposition

$$\phi^{-1}(\lambda)\sqrt{\sum_{i=1}^{n} V\left(\frac{d_{ij}}{v_{ij}}\right)} + \sum_{i=1}^{n}[E\left(\frac{d_{ij}}{v_{ij}}\right)] \le TM \qquad (29)$$

Then, all demonstration were run on Lingo Programming (LINGO 5.0 Intel Core i5 2.50 GHz RAM 4.0 GB) by two objective along with Minimizing Emission with time window and Minimizing Distance in order to compare the difference of result in our models with classical VRP model. The results showed that when we added more complicated constraints namely loading weight, stochastic velocity, time window and pollutant emission and fuel consumption equations are significantly changed of optimal route (figure 2) and also found that although distance was minimized but there did not mean that amount of emission and fuel consumption was increased as same. However, it had the potential of better yielding of reduction emission and fuel consumption results. The experimentation result show in the figure 1

Problem	Optimized for EMISSION with time window				Optimized for DISTANCE			
	A	B	C	D	A	B	C	D
TR(min)	129.7	340.7	430.7	1039	129.1	328.2	421.5	1037.6
TT(min)	159.7	370.7	460.7	1069	159.1	358.2	451.5	1067.6
TD(km)	209	201	811	834	206	196	807	807
FC(l)	37.3	21.7	146.2	83.6	40.7	22.0	150.9	89.2
Emission								
CO	314	523	1,219	1,971	318	553	1,169	2,063
CO_2	18,412	106,484	720,378	409,128	200,697	107,834	744,091	436,849
NO_x	1,991	1,125	7,447	4,419	1,957	1,153	7,503	4,772
PM	94	110	376	428	91	112	360	455
Total (g)	186,522	108,242	729,421	415,946	203,064	109,654	744,091	444,140

Figure 1 Comparison of result obtained from different objectives function (especially 7 nodes)

Remark: TR=Travelled time on roads, TT=Total time, TD= Traveled distance, FC = Fuel consumption

Besides, the increasing of time required to calculate for each experiment as increase as problem size due to the adding more of constraints and variables used in the decision .For example, the experiment calculation of the issues in the case of problem 1 in the table 4, which shows that Lingo programming cannot solve the problem greater than 7 cities under an acceptable time.

Table 4 Computation time of each problem.(especially problem A)

Class of problem	Min Emission (hh.mm.ss)	Min Distance (hh.mm.ss)
3 nodes	00.00.35	00.00.27
5 nodes	00.12.47	00.08.03
7 nodes	00.56.32	00.24.04
10 nodes	N/A*	N/A*

* N/A means problem cannot be solved within 2 days

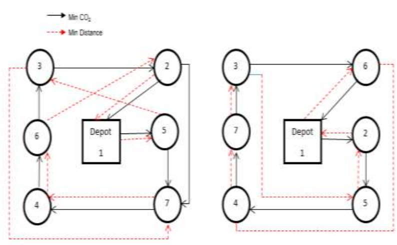

Figure 2 Route solution from difference objective function for 7 nodes problem

(a) Problem A (d) Problem D

5. Summary

The amounts of pollutants were continuously emitted nowadays into the atmosphere by burning of vehicles fuel consumption tends to steadily increasing, especially of the transport sector on the road. There are several factors that have influence on emissions and fuel consumption such as loading, driving velocity, type of engine and etc. This paper therefore presents a Mathematical models of Environmental-Friendly' Vehicle Routing Problem or Fuel & Pollution-Routing Problem (FPRP), an extension of the classical Vehicle Routing Problem (VRP) which considers GHG emissions and fuel consumption associated with the vehicle traveled routes with added the constraints of vehicle loading and time window under their velocity were not constant or velocity data were distributed on normal distribution. By designing for 4 problem as mentioned initially in the size of different ; 3 node , 5 nod and 7 node in order to solve under the two objectives; Minimizing GHG Emissions with time window and Minimizing only distance (Classic VRP). The results showed that although the distance has decreased, there did not mean that the amounts of GHG and fuel consumption during transportation has decreased as same and also found that if we added constraints as time window and its loading in to the account, The results showed that it had the potential of better yielding of reduction emission and fuel consumption result than Classical VRP on average on 4.73% of all four cases of problem. However, our purposed mathematical programming model was implemented by using Lingo programming that it had limitation of solving time when there were more node or customer in the system. Moreover, formulation mathematics model to be stochastic programming also increased its complication. So our future study will apply Meta heuristic such as particle swarm optimization (PSO) to find the solution which has more nodes or customers in the system up to 200 nodes or customers.

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References

[1]Bektas, T. and Laporte, G., 2011. "The Pollution Routing Problem" Transportation Research Part B 45, 1232-1250.

[2]Bauer, J., Bektas_, T., Crainic, T.G., 2010. Minimizing greenhouse gas emissions in intermodal freight transport: an application to rail service design. Journal of the Operational Research Society 61 (3), 530–542.

[3]Barth, M., Boriboonsomsin, K., 2009. Energy and emissions impacts of a freeway-based dynamic eco-driving system. Transportation Research Part D 14 (6),

[4]Bodin, L., B.L. Golden, A. Assad and M.O. Ball, 1983. Routing and scheduling of the vehicle and crews: The state of the art. Comp. Operations Res., 10: 63-212.

[5]Cordeau, J.-F., Laporte, G., Savelsbergh, M.W.P., Vigo, D., 2007. Vehicle routing. In: Barnhart, C., Laporte, G. (Eds.), Transportation, Handbooks in Operations Research and Management Science, vol. 14. Elsevier, Amsterdam, The Netherlands, pp. 367–428 (Chapter 6).

[6]Dantzig, G. B. andRamser, J.H. .1959"The Truck Dispatching Problem." Management Science, Vol. .6,.91- 80

[7]Hickman, J., Hassel, D., Joumard, R., Samaras, Z., Sorenson, S., 1999.Methodology for calculating transport emissions and energy consumption. Report No. SE/491/98. Transport Research Laboratory, Crowthorne, UK.

[8]Laporte, G., 1992. "The Vehicle Routing Problem: An overview of exact and approximate algorithms." European Journal of Operational Research. Volume 59, 345-358.

[9]Li X., Tian P. and Leung S. C.H., .2010"Vehicle routing problems with time windows and stochastic travel and service times: Models and algorithm." Int. J. Production Economics 125, .145 - 137

[10]Tarnchuda P., Kanok-orn R., and Ruengruaedee B., 2011."Cost Reduction of VRP with Mathematic Model Case study: Ubon Aquar is Factory, Ubonratchathani". Operation Research Network Conference 2011, 60-64.

[11]Xiao Y., Zhao Q., Kaku I. and Xu Y. (2012)."Development of a fuel consumption optimization model for the capacitated vehicle routing problem". Computers& Operations Research 39. pp. 1419 - 1431.

[12]Z. Zsigraiova et al. "Operation costs and pollutant emissions reduction by definition of new collection scheduling and optimization of MSW GIS. The case study of Barreiro, Portugal", Waste Management 33 (2013) 793–806.