Research Article

Confidence Intervals for the Coefficient of Variation in a Normal Distribution with a Known Mean and a Bounded Standard Deviation

Wararit Panichkitkosolkul*
Department of Mathematics and Statistics, Faculty of Science and Technology, Thammasat University, Pathum Thani, Thailand

* Corresponding author. E-mail: wararit@mathstat.sci.tu.ac.th
DOI: 10.14416/j.ijast.2017.04.001
Received: 15 April 2016; Accepted: 5 October 2016; Published online: 24 April 2017

Abstract
The natural parameter space is known to be bounded in many real applications such as engineering, science and social science. The standard confidence interval derived from the classical Neyman procedure is unsatisfactory in the case of a bounded parameter space. New confidence intervals for the coefficient of variation in a normal distribution with a known population mean and a bounded standard deviation are proposed in this paper. A simulation study has been conducted to compare the performance of the proposed confidence intervals.

Keywords: Bounded parameter space, Coverage probability, Expected length, Simulation

1 Introduction

The coefficient of variation, introduced by Karl Pearson [1] in 1896, has been one of the most widely used statistical measures of relative dispersion. Important properties of the coefficient of variation are that it is a dimensionless (unit-free) measure of variation and it also can be used to compare several variables obtained by different units. The population coefficient of variation is defined as a ratio of the population standard deviation (σ) to the population mean (μ) given by $\theta = \sigma / \mu$

The coefficient of variation has been widely used in many areas of science, medicine, engineering, economics and others. For example, the uncertainty of fault trees has been analyzed by the coefficient of variation [2]. The coefficient of variation has also been applied to estimate the strength of ceramics [3]. Faber and Korn [4] used the coefficient of variation for measuring the variation of the mean synaptic response of the central nervous system. Hamer *et al*. [5] evaluated the homogeneity of bone tests using the coefficient of variation. The impact of socioeconomic status on hospital use in New York City has also been studied using the coefficient of variation [6]. Miller and Karson [7] used the coefficient of variation as a measure of relative risk and a test of the equality of the coefficients of variation for two stocks. Worthington and Higgs [8] measured the degree of risk in relation to the mean return by the coefficient of variation. Furthermore, the variability of the competitive performance of Olympic swimmers has been studied using the coefficient of variation [9]. Applications of the coefficient of variation in business, climatology and other areas are briefly reviewed in Nairy and Rao [10].

In most applications, the population coefficient of variation is practically unknown. Thus, the sample estimate of the coefficient of variation is required in order to estimate an unknown value. Although the point estimator can be a useful measure for statistical inference, its confidence interval is more useful than the point estimator. A confidence interval provides much more information about the population characteristic of interest than does a point estimate. Namely, the

Please cite this article in press as: W. Panichkitkosolkul, "Confidence intervals for the coefficient of variation in a normal distribution with a known mean and a bounded standard deviation," *KMUTNB Int J Appl Sci Technol*, vol. x, no. x, pp. x–x, (Year).

confidence interval provides an estimated range of values, which is likely to include an unknown population parameter. Several methods available for constructing the confidence interval for θ have been proposed. For instance, McKay [11] presented a confidence interval for θ based on the chi-square distribution, with this confidence interval demonstrating a good performance when θ is less than 0.33. In 1996, Vangel [12] modified McKay's confidence interval based on an analysis of the distribution of a class of approximate pivotal quantities for the normal coefficient of variation. For normally distributed data, Vangel's confidence interval is usually more accurate and nearly exact in comparison to McKay's confidence interval. Panichkitkosolkul [13] proposed a new confidence interval for θ of a normal distribution by modifying McKay's confidence interval. He estimated the population coefficient of variation by the maximum likelihood method. Later, the asymptotic distribution and confidence interval of the reciprocal of the coefficient of variation were proposed by Sharma and Krishna [14]. This confidence interval does not require any assumptions about the population distribution. Miller [15] studied the approximate distribution for the estimate of θ and constructed an approximate confidence interval for θ in a normal distribution. A comparison of confidence intervals for θ obtained by McKay's, Miller's and Sharma-Krishna's methods was undertaken under the same simulation conditions by Ng [16].

An approximately unbiased estimator and two approximate confidence intervals for θ in a normal distribution were introduced by Mahmoudvand and Hassani [17]. Koopmans *et al*. [18] and Verrill [19] presented the confidence intervals for θ in normal and lognormal distributions. Interval estimation for the difference of the coefficient of variation for lognormal and delta-lognormal distributions was constructed by Buntao and Niwitpong [20]. Panichkitkosolkul [21] proposed an asymptotic confidence interval for the coefficient of variation of a Poisson distribution. Curto and Pinto [22] introduced the confidence interval for the coefficient of variation in the case of non-independently and identically distributed random variables. Gulhar *et al*. [23] compared many confidence intervals for the coefficient of variation based on parametric, nonparametric and modified methods. The recent work of Panichkitkosolkul [24] has developed three confidence intervals for the coefficient of variation in a normal distribution with a known population mean. These three proposed confidence intervals consist of normal approximation, shortest-length and equal-tailed confidence intervals.

Although statistical inference is studied in a natural parameter space, the parameter space is bounded in several real applications, such as engineering, sciences and social sciences. For instance, the blood pressures of patients or the weight of subjects are bounded. However, Mandelkern [25] pointed out the importance of statistical inference where the parameter space is known to be restricted. In addition, he gave the example that the classical Neyman procedure is unsatisfactory in the case of a bounded parameter space. The main reason is that the information regarding the restriction is simply ignored. The other related works are Feldman and Cousins [26] and Roe and Woodroofe [27]. Although a great deal of work has been done on confidence intervals for the coefficient of variation, the confidence intervals for the coefficient of variation with restricted parameter space have not been the subject of much study. Therefore, it would be of significant interest to develop confidence intervals for the coefficient of variation that include additional information on the standard deviation being bounded in order to improve the accuracy of the confidence interval. Motivated by the recent work of Panichkitkosolkul [24], we propose confidence intervals for the coefficient of variation in a normal distribution with a known population mean and a bounded standard deviation in this paper.

2 Confidence Intervals for the Coefficient of Variation with a Known Population Mean

In this section, we review the confidence intervals for the coefficient of variation in a normal distribution with a known population mean proposed recently by Panichkitkosolkul [24]. Three confidence intervals for the coefficient of variation, i.e., normal approximation confidence interval, shortest-length confidence interval and equal-tailed confidence interval, are discussed.

The classical sample estimate of θ is given in Equation (1) as

$$\hat{\theta} = S / \bar{X} \tag{1}$$

where S is the sample standard deviation and $\overline{X}$ is the sample mean. If the population mean is known to be μ_0 then the population coefficient of variation is given by $\theta_0 = \sigma/\mu_0$. The sample estimate of θ_0 is

$$\hat{\theta}_0 = S_0 / \mu_0, \quad (2)$$

where $S_0^2 = n^{-1}\sum_{i=1}^{n}(X_i - \mu_0)^2$. To find the normal approximation confidence interval for θ_0, we have to use the following theorem.

Theorem 1. *Let $X_1, X_2, ..., X_n$ be a random sample from a normal distribution with a known population mean μ_0 and variance σ^2. The unbiased estimator of θ_0 is*

$$\hat{\hat{\theta}}_0 = \hat{\theta}_0 / c_{n+1},$$

where $c_{n+1} = \sqrt{2/n}\left(\Gamma((n+1)/2)/\Gamma(n/2)\right)$, $\Gamma(\cdot)$ is the gamma function and $\hat{\theta}_0$ is shown in Equation (2). The mean and variance of $\hat{\hat{\theta}}_0$ are $E(\hat{\hat{\theta}}_0) = \theta_0$ and $\text{var}(\hat{\hat{\theta}}_0) = \left(\frac{1-c_{n+1}^2}{c_{n+1}^2}\right)\theta_0^2$.

Proof of Theorem 1. See Panichkitkosolkul [24].

2.1 *Normal approximation confidence interval*

Using Theorem 1, we have

$$z = \frac{\hat{\hat{\theta}}_0 - \theta_0}{\sqrt{\text{var}(\hat{\hat{\theta}}_0)}} = \frac{\hat{\theta}_0 / c_{n+1} - \theta_0}{\sqrt{(1-c_{n+1}^2)\theta_0^2 / c_{n+1}^2}}$$

$$= \frac{\hat{\theta}_0 - c_{n+1}\theta_0}{\theta_0\sqrt{1-c_{n+1}^2}} \to N(0,1),$$

where $\to$ denotes the convergence in distribution. Therefore, the 100(1–α)% normal approximate confidence interval for θ_0 is given in Equation (3) as

$$\frac{\hat{\theta}_0}{c_{n+1} + z_{1-\alpha/2}\sqrt{1-c_{n+1}^2}} \le \theta_0 \le \frac{\hat{\theta}_0}{c_{n+1} - z_{1-\alpha/2}\sqrt{1-c_{n+1}^2}}, \quad (3)$$

2.2 *Shortest-length confidence interval*

Panichkitkosolkul [24] introduced the shortest-length confidence interval for θ_0 based on the pivotal quantity $Q = \frac{nS_0^2}{\sigma^2} \sim \chi_n^2$. Thus, the 100(1–$\alpha$)% shortest-length confidence interval for θ_0 is given by

$$\hat{\theta}_0\sqrt{\frac{n}{b}} \le \theta_0 \le \hat{\theta}_0\sqrt{\frac{n}{a}}, \quad (4)$$

where a and b are constant, $a, b > 0$ and $a < b$. The values of a and b in Equation (4) are shown in Table 1 of Panichkitkosolkul [24].

2.3 *Equal-tailed confidence interval*

The 100(1–α)% equal-tailed confidence interval for θ_0 based on the pivotal quantity Q is given in Equation (5) as

$$\hat{\theta}_0\sqrt{\frac{n}{\chi^2_{n,1-\alpha/2}}} \le \theta_0 \le \hat{\theta}_0\sqrt{\frac{n}{\chi^2_{n,\alpha/2}}}, \quad (5)$$

where $\chi^2_{n,\alpha/2}$ and $\chi^2_{n,1-\alpha/2}$ are the 100(α/2) and 100(α/2) percentiles of the central chi-squared distribution with n degrees of freedom.

3 Confidence Intervals for the Coefficient of Variation with a Known Population Mean and a Bounded Standard Deviation

In 2008, Wang [28] derived confidence intervals for the mean of a normal distribution when the parameter space is restricted. Following the method proposed by Wang [28], we present confidence intervals for the coefficient of variation of a normal distribution with a known population mean when the population standard deviation is bounded.

The true value of a parameter of interest is usually unknown. However, parameter space is often known to be restricted and the bounds of parameter space are known. We denote m_1 and m_2 as the lower bound and the upper bound of the parameter space. When the parameter space is known to be restricted to the interval (m_1, m_2), it is widely accepted that a confidence interval for a parameter β is the confidence interval of the intersection between the interval (m_1, m_2) and

[L_β, U_β], where L_β and U_β are the lower and upper limits of the confidence interval for β. Therefore, the confidence interval for β when the parameter space is bounded, denoted as CI_B, is defined as

$$CI_B = \left[\max(m_1, L_\beta), \min(m_2, U_\beta)\right]. \quad (6)$$

Four possible confidence intervals in Equation (6) are as follows:

1) if $m_1 > L_\beta$ and $m_2 > U_\beta$ then CI_B is reduced to $CI_B = [m_1, U_\beta]$.

2) if $m_1 > L_\beta$ and $m_2 < U_\beta$ then CI_B is reduced to $CI_B = [m_1, U_\beta]$.

3) if $m_1 < L_\beta$ and $m_2 > U_\beta$ then CI_B is reduced to $CI_B = [L_\beta, U_\beta]$.

4) if $m_1 < L_\beta$ and $m_2 < U_\beta$ then CI_B is reduced to $CI_B = [L_\beta, m_2]$.

When the parameter space of the standard deviation is (m_1, m_2) and the population mean is known, straightforward calculation can show that the population coefficient of variation is also bounded as follows:

$$m_1 < \sigma < m_2$$
$$\Rightarrow \frac{m_1}{\mu_0} < \frac{\sigma}{\mu_0} < \frac{m_2}{\mu_0}$$
$$\Rightarrow \frac{m_1}{\mu_0} < \theta_0 < \frac{m_2}{\mu_0}.$$

According to Wang [28] and Niwitpong [29], the proposed confidence intervals for θ_0 with a bounded standard deviation are given by

$$CI_{\theta_0} = \left[\max\left(\frac{m_1}{\mu_0}, L_{\theta_0}\right), \min\left(\frac{m_2}{\mu_0}, U_{\theta_0}\right)\right], \quad (7)$$

where L_{θ_0} and U_{θ_0} are the lower and upper limits of the confidence intervals for θ_0 respectively. In addition, the existing confidence intervals for θ_0 reviewed in the previous section are used in order to obtain confidence intervals for θ_0 when the standard deviation is bounded.

4 Simulation Studies

The performances of the confidence intervals of the coefficient of variation derived in the previous section are investigated through simulation studies in this section. The estimated coverage probabilities and expected lengths of three confidence intervals for some bounded parameter space are summarized in Tables 1–4. The data are generated from a normal distribution with a known population mean $\mu_0 = 10$ and $\theta_0 = 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45$ and 0.5, sample sizes; $n = 5, 10, 25, 50$ and 100. The parameter space of the standard deviation is set to the interval (1, 5). The 90% and 95% confidence intervals are constructed based on the existing methods with unbounded and bounded standard deviations. Each simulation study is based on 50,000 replicates using the R statistical software [30] version 3.0.2.

In the simulation study, the estimated coverage probabilities of the confidence intervals with a bounded standard deviation are the same as those of the confidence intervals with an unbounded standard deviation. Additionally, all confidence intervals have estimated coverage probabilities close to the nominal confidence level in all situations. The estimated coverage probabilities of all confidence intervals do not increase or decrease according to the values of θ_0. The confidence intervals with a bounded standard deviation have shorter expected lengths than the expected lengths of the confidence intervals with an unbounded standard deviation in all cases. In addition, the expected lengths of all confidence intervals become shorter when the sample sizes increase.

5 Conclusions

This paper proposes the confidence intervals of the coefficient of variation in a normal distribution with a known population mean when the parameter space of the standard deviation is bounded. The new proposed confidence intervals are based on three existing confidence intervals: normal approximation confidence interval, shortest-length confidence interval and equal-tailed confidence interval. Estimated coverage probabilities and the expected lengths of confidence intervals are considered as the criteria of a good confidence interval. Simulation results indicate that the performances of improved confidence intervals with a bounded standard deviation are the same as the performances of the confidence intervals with an unbounded standard deviation in terms of coverage probability. The confidence intervals with a bounded standard deviation have the advantage of a shorter expected length in all cases.

Table 1: The estimated coverage probabilities of 90% confidence intervals for the coefficient of variation with a known population mean and unbounded and bounded standard deviation

n	θ_0	Coverage Probabilities					
		Unbounded Standard Deviation			Bounded Standard Deviation		
		Approx.	Shortest	Equal-tailed	Approx.	Shortest	Equal-tailed
5	0.10	0.9020	0.8998	0.8992	0.9020	0.8998	0.8992
	0.15	0.9013	0.8984	0.8977	0.9013	0.8984	0.8977
	0.20	0.9045	0.8991	0.8993	0.9045	0.8991	0.8993
	0.25	0.9055	0.9000	0.9021	0.9055	0.9000	0.9021
	0.30	0.9034	0.9009	0.9014	0.9034	0.9009	0.9014
	0.35	0.9035	0.9009	0.8999	0.9035	0.9009	0.8999
	0.40	0.9024	0.8995	0.8989	0.9024	0.8995	0.8989
	0.45	0.9053	0.9019	0.9011	0.9053	0.9019	0.9011
	0.50	0.9030	0.9018	0.8995	0.9030	0.9018	0.8995
10	0.10	0.9003	0.8990	0.8994	0.9003	0.8990	0.8994
	0.15	0.9018	0.9018	0.9003	0.9018	0.9018	0.9003
	0.20	0.8998	0.9005	0.8997	0.8998	0.9005	0.8997
	0.25	0.9012	0.8998	0.8998	0.9012	0.8998	0.8998
	0.30	0.9035	0.9027	0.9021	0.9035	0.9027	0.9021
	0.35	0.8978	0.8970	0.8963	0.8978	0.8970	0.8963
	0.40	0.9016	0.8996	0.8997	0.9016	0.8996	0.8997
	0.45	0.8978	0.8999	0.8975	0.8978	0.8999	0.8975
	0.50	0.9008	0.9012	0.8996	0.9008	0.9012	0.8996
25	0.10	0.8988	0.8999	0.8985	0.8988	0.8999	0.8985
	0.15	0.9004	0.8988	0.8998	0.9004	0.8988	0.8998
	0.20	0.9005	0.8997	0.9000	0.9005	0.8997	0.9000
	0.25	0.9002	0.9010	0.9007	0.9002	0.9010	0.9007
	0.30	0.9026	0.9020	0.9018	0.9026	0.9020	0.9018
	0.35	0.9003	0.9008	0.8990	0.9003	0.9008	0.8990
	0.40	0.9034	0.9016	0.9025	0.9034	0.9016	0.9025
	0.45	0.9012	0.9012	0.9004	0.9012	0.9012	0.9004
	0.50	0.8988	0.8982	0.8984	0.8988	0.8982	0.8984
50	0.10	0.8996	0.8986	0.8996	0.8996	0.8986	0.8996
	0.15	0.8995	0.9006	0.8995	0.8995	0.9006	0.8995
	0.20	0.8994	0.9001	0.8996	0.8994	0.9001	0.8996
	0.25	0.9016	0.9024	0.9012	0.9016	0.9024	0.9012
	0.30	0.8990	0.8980	0.8988	0.8990	0.8980	0.8988
	0.35	0.8980	0.8974	0.8981	0.8980	0.8974	0.8981
	0.40	0.9011	0.8992	0.9005	0.9011	0.8992	0.9005
	0.45	0.8996	0.8993	0.8992	0.8996	0.8993	0.8992
	0.50	0.9005	0.8998	0.9004	0.9005	0.8998	0.9004
100	0.10	0.9013	0.8897	0.9010	0.9013	0.8897	0.9010
	0.15	0.9008	0.8881	0.9002	0.9008	0.8881	0.9002
	0.20	0.8990	0.8878	0.8990	0.8990	0.8878	0.8990
	0.25	0.9008	0.8888	0.9009	0.9008	0.8888	0.9009
	0.30	0.9020	0.8897	0.9019	0.9020	0.8897	0.9019
	0.35	0.8992	0.8874	0.8990	0.8992	0.8874	0.8990
	0.40	0.8988	0.8876	0.8985	0.8988	0.8876	0.8985
	0.45	0.9027	0.8911	0.9026	0.9027	0.8911	0.9026
	0.50	0.9027	0.8903	0.9023	0.9027	0.8903	0.9023

Table 2: The expected lengths of 90% confidence intervals for the coefficient of variation with a known population mean and unbounded and bounded standard deviation

n	θ_0	Expected Lengths					
		Unbounded Standard Deviation			Bounded Standard Deviation		
		Approx.	Shortest	Equal-tailed	Approx.	Shortest	Equal-tailed
5	0.10	0.1487	0.1178	0.1352	0.1185	0.0813	0.1050
	0.15	0.2221	0.1759	0.2020	0.2069	0.1561	0.1878
	0.20	0.2960	0.2345	0.2693	0.2639	0.2203	0.2472
	0.25	0.3703	0.2933	0.3369	0.2822	0.2609	0.2721
	0.30	0.4441	0.3518	0.4041	0.2754	0.2760	0.2710
	0.35	0.5192	0.4113	0.4724	0.2557	0.2736	0.2553
	0.40	0.5928	0.4696	0.5393	0.2309	0.2606	0.2328
	0.45	0.6664	0.5279	0.6063	0.2065	0.2428	0.2095
	0.50	0.7437	0.5891	0.6766	0.1836	0.2213	0.1869
10	0.10	0.0864	0.0777	0.0833	0.0619	0.0490	0.0585
	0.15	0.1296	0.1166	0.1249	0.1241	0.1078	0.1192
	0.20	0.1729	0.1556	0.1667	0.1718	0.1538	0.1656
	0.25	0.2161	0.1944	0.2084	0.2092	0.1922	0.2031
	0.30	0.2597	0.2336	0.2504	0.2260	0.2185	0.2222
	0.35	0.3027	0.2723	0.2919	0.2184	0.2241	0.2175
	0.40	0.3450	0.3104	0.3327	0.1975	0.2134	0.1986
	0.45	0.3887	0.3497	0.3747	0.1696	0.1918	0.1718
	0.50	0.4326	0.3892	0.4171	0.1431	0.1663	0.1454
25	0.10	0.0495	0.0475	0.0489	0.0321	0.0280	0.0312
	0.15	0.0741	0.0712	0.0732	0.0733	0.0699	0.0723
	0.20	0.0989	0.0950	0.0977	0.0989	0.0950	0.0977
	0.25	0.1238	0.1189	0.1222	0.1238	0.1189	0.1222
	0.30	0.1484	0.1425	0.1465	0.1477	0.1423	0.1460
	0.35	0.1731	0.1662	0.1709	0.1630	0.1605	0.1618
	0.40	0.1978	0.1900	0.1953	0.1562	0.1607	0.1563
	0.45	0.2226	0.2138	0.2198	0.1302	0.1403	0.1313
	0.50	0.2472	0.2375	0.2441	0.1001	0.1113	0.1014
50	0.10	0.0339	0.0332	0.0337	0.0207	0.0189	0.0204
	0.15	0.0508	0.0498	0.0505	0.0508	0.0498	0.0505
	0.20	0.0678	0.0665	0.0674	0.0678	0.0665	0.0674
	0.25	0.0848	0.0831	0.0843	0.0848	0.0831	0.0843
	0.30	0.1017	0.0997	0.1011	0.1017	0.0997	0.1011
	0.35	0.1186	0.1162	0.1179	0.1180	0.1159	0.1174
	0.40	0.1355	0.1329	0.1347	0.1251	0.1254	0.1249
	0.45	0.1526	0.1496	0.1517	0.1065	0.1114	0.1071
	0.50	0.1695	0.1661	0.1684	0.0747	0.0808	0.0754
100	0.10	0.0236	0.0227	0.0236	0.0139	0.0130	0.0137
	0.15	0.0354	0.0340	0.0353	0.0354	0.0340	0.0353
	0.20	0.0472	0.0453	0.0471	0.0472	0.0453	0.0471
	0.25	0.0590	0.0567	0.0589	0.0590	0.0567	0.0589
	0.30	0.0708	0.0680	0.0706	0.0708	0.0680	0.0706
	0.35	0.0826	0.0793	0.0823	0.0826	0.0793	0.0823
	0.40	0.0945	0.0907	0.0942	0.0934	0.0899	0.0931
	0.45	0.1062	0.1020	0.1059	0.0869	0.0856	0.0871
	0.50	0.1180	0.1133	0.1176	0.0551	0.0554	0.0555

Table 3: The estimated coverage probabilities of 95% confidence intervals for the coefficient of variation with a known population mean and unbounded and bounded standard deviation

n	θ_0	Coverage Probabilities					
		Unbounded Standard Deviation			Bounded Standard Deviation		
		Approx.	Shortest	Equal-tailed	Approx.	Shortest	Equal-tailed
5	0.10	0.9524	0.9504	0.9488	0.9524	0.9504	0.9488
	0.15	0.9539	0.9518	0.9504	0.9539	0.9518	0.9504
	0.20	0.9550	0.9510	0.9517	0.9550	0.9510	0.9517
	0.25	0.9543	0.9493	0.9510	0.9543	0.9493	0.9510
	0.30	0.9543	0.9493	0.9503	0.9543	0.9493	0.9503
	0.35	0.9532	0.9501	0.9499	0.9532	0.9501	0.9499
	0.40	0.9554	0.9493	0.9500	0.9554	0.9493	0.9500
	0.45	0.9534	0.9500	0.9493	0.9534	0.9500	0.9493
	0.50	0.9539	0.9506	0.9494	0.9539	0.9506	0.9494
10	0.10	0.9502	0.9487	0.9486	0.9502	0.9487	0.9486
	0.15	0.9483	0.9493	0.9480	0.9483	0.9493	0.9480
	0.20	0.9524	0.9477	0.9503	0.9524	0.9477	0.9503
	0.25	0.9511	0.9511	0.9491	0.9511	0.9511	0.9491
	0.30	0.9517	0.9496	0.9500	0.9517	0.9496	0.9500
	0.35	0.9521	0.9501	0.9503	0.9521	0.9501	0.9503
	0.40	0.9527	0.9502	0.9505	0.9527	0.9502	0.9505
	0.45	0.9524	0.9506	0.9507	0.9524	0.9506	0.9507
	0.50	0.9519	0.9515	0.9500	0.9519	0.9515	0.9500
25	0.10	0.9500	0.9487	0.9496	0.9500	0.9487	0.9496
	0.15	0.9509	0.9513	0.9500	0.9509	0.9513	0.9500
	0.20	0.9506	0.9498	0.9491	0.9506	0.9498	0.9491
	0.25	0.9521	0.9498	0.9503	0.9521	0.9498	0.9503
	0.30	0.9518	0.9511	0.9510	0.9518	0.9511	0.9510
	0.35	0.9524	0.9502	0.9509	0.9524	0.9502	0.9509
	0.40	0.9506	0.9500	0.9505	0.9506	0.9500	0.9505
	0.45	0.9496	0.9496	0.9485	0.9496	0.9496	0.9485
	0.50	0.9501	0.9486	0.9489	0.9501	0.9486	0.9489
50	0.10	0.9488	0.9493	0.9481	0.9488	0.9493	0.9481
	0.15	0.9501	0.9500	0.9499	0.9501	0.9500	0.9499
	0.20	0.9517	0.9500	0.9505	0.9517	0.9500	0.9505
	0.25	0.9498	0.9496	0.9499	0.9498	0.9496	0.9499
	0.30	0.9511	0.9515	0.9512	0.9511	0.9515	0.9512
	0.35	0.9497	0.9504	0.9495	0.9497	0.9504	0.9495
	0.40	0.9513	0.9498	0.9504	0.9513	0.9498	0.9504
	0.45	0.9492	0.9482	0.9489	0.9492	0.9482	0.9489
	0.50	0.9488	0.9485	0.9485	0.9488	0.9485	0.9485
100	0.10	0.9500	0.9498	0.9498	0.9500	0.9498	0.9498
	0.15	0.9503	0.9495	0.9501	0.9503	0.9495	0.9501
	0.20	0.9497	0.9500	0.9498	0.9497	0.9500	0.9498
	0.25	0.9511	0.9508	0.9507	0.9511	0.9508	0.9507
	0.30	0.9502	0.9497	0.9497	0.9502	0.9497	0.9497
	0.35	0.9510	0.9511	0.9511	0.9510	0.9511	0.9511
	0.40	0.9519	0.9513	0.9518	0.9519	0.9513	0.9518
	0.45	0.9518	0.9522	0.9515	0.9518	0.9522	0.9515
	0.50	0.9526	0.9524	0.9523	0.9526	0.9524	0.9523

Table 4: The expected lengths of 95% confidence intervals for the coefficient of variation with a known population mean and unbounded and bounded standard deviation

n	θ_0	Expected Lengths					
		Unbounded Standard Deviation			Bounded Standard Deviation		
		Approx.	Shortest	Equal-tailed	Approx.	Shortest	Equal-tailed
5	0.10	0.2120	0.1518	0.1743	0.1752	0.1083	0.1376
	0.15	0.3177	0.2275	0.2612	0.2804	0.2010	0.2378
	0.20	0.4236	0.3032	0.3482	0.3224	0.2723	0.2954
	0.25	0.5289	0.3786	0.4348	0.3219	0.3051	0.3091
	0.30	0.6344	0.4542	0.5216	0.3046	0.3114	0.3004
	0.35	0.7413	0.5307	0.6094	0.2795	0.3026	0.2806
	0.40	0.8449	0.6049	0.6946	0.2526	0.2861	0.2565
	0.45	0.9527	0.6821	0.7833	0.2254	0.2654	0.2308
	0.50	1.0583	0.7577	0.8701	0.2009	0.2436	0.2069
10	0.10	0.1103	0.0960	0.1029	0.0810	0.0618	0.0730
	0.15	0.1657	0.1443	0.1546	0.1583	0.1326	0.1466
	0.20	0.2203	0.1919	0.2056	0.2171	0.1890	0.2032
	0.25	0.2765	0.2408	0.2581	0.2550	0.2336	0.2437
	0.30	0.3306	0.2879	0.3085	0.2610	0.2555	0.2557
	0.35	0.3868	0.3369	0.3610	0.2447	0.2543	0.2442
	0.40	0.4415	0.3845	0.4120	0.2179	0.2375	0.2203
	0.45	0.4966	0.4324	0.4634	0.1874	0.2128	0.1912
	0.50	0.5526	0.4812	0.5157	0.1582	0.1853	0.1624
25	0.10	0.0829	0.0760	0.0795	0.0575	0.0469	0.0536
	0.15	0.1244	0.1139	0.1193	0.1206	0.1079	0.1152
	0.20	0.1657	0.1518	0.1590	0.1654	0.1512	0.1586
	0.25	0.2073	0.1899	0.1989	0.2049	0.1892	0.1973
	0.30	0.2486	0.2277	0.2385	0.2271	0.2181	0.2219
	0.35	0.2904	0.2660	0.2786	0.2226	0.2258	0.2214
	0.40	0.3319	0.3040	0.3184	0.1989	0.2118	0.2004
	0.45	0.3735	0.3422	0.3583	0.1674	0.1859	0.1703
	0.50	0.4145	0.3797	0.3977	0.1369	0.1565	0.1400
50	0.10	0.0409	0.0399	0.0404	0.0250	0.0226	0.0243
	0.15	0.0613	0.0598	0.0606	0.0612	0.0597	0.0606
	0.20	0.0818	0.0798	0.0809	0.0818	0.0798	0.0809
	0.25	0.1022	0.0997	0.1011	0.1022	0.0997	0.1011
	0.30	0.1228	0.1198	0.1214	0.1227	0.1198	0.1214
	0.35	0.1431	0.1396	0.1416	0.1416	0.1388	0.1403
	0.40	0.1634	0.1594	0.1617	0.1447	0.1459	0.1445
	0.45	0.1839	0.1794	0.1819	0.1201	0.1261	0.1211
	0.50	0.2044	0.1994	0.2022	0.0846	0.0918	0.0859
100	0.10	0.0283	0.0280	0.0282	0.0165	0.0154	0.0162
	0.15	0.0424	0.0419	0.0422	0.0424	0.0419	0.0422
	0.20	0.0566	0.0559	0.0563	0.0566	0.0559	0.0563
	0.25	0.0707	0.0699	0.0704	0.0707	0.0699	0.0704
	0.30	0.0849	0.0838	0.0844	0.0849	0.0838	0.0844
	0.35	0.0991	0.0979	0.0985	0.0991	0.0979	0.0985
	0.40	0.1132	0.1118	0.1126	0.1107	0.1101	0.1103
	0.45	0.1273	0.1258	0.1267	0.0989	0.1014	0.0993
	0.50	0.1416	0.1398	0.1408	0.0624	0.0662	0.0631

Acknowledgments

The author is grateful to the anonymous referees for their constructive comments and suggestions, which have significantly enhanced the quality and presentation of this paper.

References

[1] K. Pearson, “Mathematical contributions to the theory of evolution? III. Regression, heredity and panmixia,” *Philosophical Transactions of the Royal Society A*, vol. 187, pp. 253–318, 1896.
[2] K. Ahn, “Use of coefficient of variation for uncertainty analysis in fault tree analysis,” *Reliability Engineering & System Safety*, vol. 47, pp. 229–230, 1995.
[3] J. Gong and Y. Li, “Relationship between the estimated Weibull modulus and the coefficient of variation of the measured strength for ceramics,” *Journal of the American Ceramic Society*, vol. 82, pp. 449–452, 1999.
[4] D. S. Faber and H. Korn, “Applicability of the coefficient of variation method for analyzing synaptic plasticity,” *Biophysical Journal*, vol. 60, pp. 1288–1294, 1991.
[5] A. J. Hamer, J. R. Strachan, M. M. Black, C. Ibbotson, and R. A. Elson, “A new method of comparative bone strength measurement”, *Journal of Medical Engineering & Technology*, vol. 19, pp. 1–5, 1995.
[6] J. Billings, L. Zeitel, J. Lukomink, T. S. Carey, A. E. Blank, and L. Newman, “Impact of socioeconomic status of hospital use in New York City,” *Health Affairs*, vol. 12, pp. 162–173, 1993.
[7] E. G. Miller and M. J. Karson, “Testing the equality of two coefficients of variation,” *American Statistical Association: Proceedings of the Business and Economics Section*, pp. 278–283, 1977.
[8] A. C. Worthington and H. Higgs, “Risk, return and portfolio diversification in major painting markets: the application of conventional financial analysis to unconventional investments,” Discussion Papers in Economics, Finance and International Competitiveness, No.148, 2003.
[9] D. B. Pyne, C. B. Trewin, and W. G. Hopkins, “Progression and variability of competitive performance of Olympic swimmers,” *Journal of Sports Sciences*, vol. 22, pp. 613–620, 2004.
[10] K. S. Nairy and K. A. Rao, “Tests of coefficients of variation of normal population,” *Communications in Statistics - Simulation and Computation*, vol. 32, pp. 641–661, 2003.
[11] A. T. McKay, “Distribution of the coefficient of variation and the extended t distribution,” *Journal of the Royal Statistical Society: Series B*, vol. 95, pp. 695–698, 1932.
[12] M. G. Vangel, “Confidence intervals for a normal coefficient of variation,” *The American Statistician*, vol. 15, pp. 21–26, 1996.
[13] W. Panichkitkosolkul, “Improved confidence intervals for a coefficient of variation of a normal distribution,” *Thailand Statistician*, vol. 7, no. 2, pp. 193–199, 2009.
[14] K. K. Sharma and H. Krishna, “Asymptotic sampling distribution of inverse coefficient-of-variation and its applications,” *IEEE Transactions on Reliability*, vol. 43, no. 2, pp. 630–633, 1994.
[15] E. G. Miller, “Asymptotic test statistics for coefficient of variation,” *Communications in Statistics - Theory and Methods*, vol. 20, pp. 3351–3363, 1991.
[16] K. C. Ng, “Performance of three methods of interval estimation of the coefficient of variation,” *InterStat: Statistics on the Internet*, vol. 2005, pp.1–8, 2005.
[17] R. Mahmoudvand and H. Hassani, “Two new confidence intervals for the coefficient of variation in a normal distribution,” *Journal of Applied Statistics*, vol. 36, pp. 429–442, 2009.
[18] L. H. Koopmans, D. B. Owen, and J. I. Rosenblatt, “Confidence intervals for the coefficient of variation for the normal and lognormal distributions,” *Biometrika*, vol. 51, pp. 25–32, 1964.
[19] S. Verrill, “Confidence bounds for normal and log-normal distribution coefficient of variation,” *Madison*, no.EPL-RP-609, pp. 1–15, 2003.
[20] N. Buntao and S. Niwitpong, “Confidence intervals for the difference of coefficients of variation for lognormal distributions and delta-lognormal distributions,” *Applied Mathematical Sciences*, vol. 6, pp. 6691–6704, 2012.
[21] W. Panichkitkosolkul, “Asymptotic confidence interval for the coefficient of variation of Poisson distribution: a simulation study,” *Maejo International Journal of Science and Technology*, vol. 4, pp. 1–7, 2010.

[22] J. D. Curto and J. C. Pinto, "The coefficient of variation asymptotic distribution in the case of non-iid random variables," *Journal of Applied Statistics*, vol. 36, pp. 21–32, 2009.

[23] M. Gulhar, B. M. G. Kibria, A. N. Albatineh, and N. U. Ahmed, "A comparison of some confidence intervals for estimating the population coefficient of variation: a simulation study," *SORT*, vol. 36, pp. 45–68, 2012.

[24] W. Panichkitkosolkul, "Confidence intervals for the coefficient of variation in a normal distribution with a known population mean," *Journal of Probability and Statistics*, vol. 2013, pp. 1–11, 2013.

[25] M. Mandelkern, "Setting confidence intervals for bounded parameters," *Statistical Science*, vol. 17, pp. 149–172, 2002.

[26] G. J. Feldman and R. D. Cousins, "Unified approach to the classical statistical analysis of small signals", *Physical Review D*, vol. 57, pp. 3873–3889, 1998.

[27] B. P. Roe and M. B. Woodroofe, "Setting confidence belts," *Physical Review D*, vol. 63, pp. 3009–3015, 2001.

[28] H. Wang, "Confidence intervals for the mean of a normal distribution with restricted parameter space," *Journal of Statistical Computation and Simulation*, vol. 78, no. 9, pp. 829–841, 2008.

[29] S. Niwitpong, "Confidence intervals for the mean of lognormal distribution with restricted parameter space," *Applied Mathematical Sciences*, vol. 7, no. 4, pp. 161–166, 2013.

[30] R. Ihaka and R. Gentleman, "R: a language for data analysis and graphics," *Journal of Computational and Graphical Statistics*, vol. 5, pp. 299–314, 1996.