Benefit of Elastically Mounted Linear Damping Vibration Isolation for Transportation System

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Abstract—**Aim of the research is to investigate benefit of supporting the damping element elastically. A single degree of freedom base excited vibration isolation system is focused for this study. Such the isolation system can be found in transportation system which its base is exciting. The improved vibration isolation system can help in reducing the damages caused from vibration to the transporting products. The proposed model has a damping element supported elastically by a spring. The damping force characteristic is assumed to be linear function of relative velocity across the damper. Two values of linear damping ratios were employed for investigation. The frequency response obtained from mathematical and numerical examinations are compared to experimental results. The frequency responses for the system with the relaxation spring are also compared to that with rigidly connected damper. Supporting the damping element elastically is found to reduce the vibration level for the high excitation frequencies in around 10 frequency decade from the natural frequency. The stiffness of supporting spring plays the important role in lowering the vibration level. Thus, it can be concluded that the vibration level of the products being transported can be reduced by supporting the damping element elastically.**

Index Terms—**Base excited isolator, elastically supported damper, transportation vibration.**

I. Introduction

In road transportation system, it is known to have vibration transmitting from the vehicle to the products. This vibration can somehow cause to the products being damaged. In the case that the products are kind of delicate farm products, e.g., vegetables or fruits, the products could not be restored but be treated less value or waste. It is unlike the products in kind of machines which can be repaired and fixed. Thus, in the transportation procedure of farm products, the products should be protected in the proper ways.

By preventing vibration causing from unevenness of the road to the vehicle for which transmits to the products as consequential result, it is believed to reduce the amount of the products being damaged. One of equipment used for preventing vibration is known as a vibration isolator. The application of vibration isolator can be found in many places and in many forms as shown in Figure 1, for example, automotive suspension systems or shipping container suspension systems.

(a) (b)

Fig. 1. Various forms of vibration isolators.
(a) Automotive suspension system [1]
(b) Shipping container vibration isolator [2]

The example benefit of vibration isolator is to protect the farm products during transportation. There are numbers of publication mentioning about damage of fruits or farm products during transportation. O’Brien et al. [3] investigated the causes of fruit bruising on transport trucks. The investigation was carried out by using four types of farm products, i.e. peaches, pears, apricots and tomatoes, which were put in a separated-space container. The result showed significantly remarked for the severe level of vibration exceeding 1.0 g that the fruits in the second and third layers from the top were found damaged. Chonhenchob et al. [4] reported level of vibration from truck transportation in Thailand influencing to fresh fruit damage. They found that the physical damage on the transporting fruits were noticeable. The major condition that caused fruit damage was found to be the poor packaging design which lack of vibration protection to the products.

By means of transportation vehicle, Ishikawa et al. [5] found that damage among fruits can be found varying on vibration frequencies. The key factor to the damage was focused on the type of transportation and vehicle used. It was added that between the trucks with leaf-spring suspension and air-ride suspension provide different vibration levels to the fruits. Zhou et al. [6] suggested in their study to apply the shock absorption treatments at the rear position of the truck floor to reduce the damage to the fruits. This is in corresponding to the study of Soleimani et al. [7] which mentioned about the effect of vibration level in truck bed and suspension to fruit damage. The leaf-spring suspension was found to produce greater vibration level compared to that for air-ride suspension. It is seen that by improving suspension characteristic for either vehicle suspension or packaging suspension can help providing fruits or farm products from damage.

Thus in this study, the ordinary linearly damped single degree of freedom base excited vibration isolator as shown in Fig. 2. (a) is applied by means of vibration isolator. It consists of a system mass, *m*, a suspension spring, *k*, and a damper, *c*. It is seen in Fig. 2. (a) that the damper is installed between the system mass and the base excitation. Such the model is called rigidly supported damper in this paper. Whereas the proposed elastically mounted damping system as shown in Fig. 2. (b) and 2 (c) is brought to the study for comparison in terms of vibration isolation performance. These two figures show that the damper is supported by the supporting spring k_r.

Fig. 2. (b) shows that the damper is installed between the system mass and the supporting spring whereas that in Fig. 2. (c) is installed between the base excitation and supporting spring. Theoretically, for the assumption with massless suspension spring and massless damper, these two models give the identical response. This is because spring and damper for either case experience the same force.

However, practically, the mass of suspension spring or the mass of damper cannot be neglected. Thus the responses for these two models are different. This is because the inertial force of either spring or damper. In this paper, the system is considered to have the damper installed between the system mass and the supporting spring as shown in Fig. 2. (b). Thus, mass of the damper is combined with the system mass and the damper becomes massless. As such, the supporting spring does not suffer the inertial force resulting from the damper.

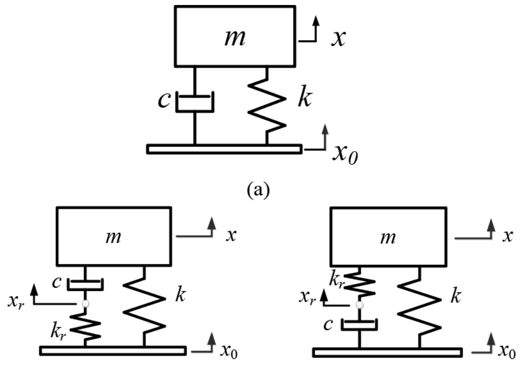


Fig. 2. Base excited vibration isolation models used in this paper.

In addition, number of the studies in vibration isolation performance for the elastically mounted damper can be found, for example in references [8], [9], [10] and [11]. Among these, Snowdon [9] showed the responses for the conventional base excited isolation system with linear damping in the comparison to those for system with relaxation spring in series to the damping. The inputs used were sinusoidal, step-like and pulse-like displacements. The author summarized that the overall response for the system with relaxation spring is better than that for the simple system.

The model of the vibration isolator using the elastically mounted damper was also examined by Carrella et al. [12]. The authors named such the model as the Zener model. They found that the effective damping is inversely varying to the value of damping coefficient. The effective damping tends to zero as the damping coefficient tends to infinity.

The Zener model was also brought to theoretical study by Silva et al. [13]. The original linear system spring was replaced by a nonlinear cubic spring. The response characteristic is also theoretically found to be beneficial on the transmissibility for the excitation frequencies high above the resonance frequency.

To this end, by employing the elastically mounted damper into the base excited vibration isolation system can be beneficial in reducing vibration level at frequencies high above the natural frequency. However, as for the author concerns, the response of the elastically mounted damper for the application of logistic or improving the supporting system for fruit transportation has not been found reported. Therefore, in this paper, the vibration response of the elastically mounted damper regarding the vibration isolation is studied in order to improve the vibration isolation performance of the supporting equipment.

II. Mathematical Modelling and Theoretical Aspect

Firstly, the models with linear damping are considered to demonstrate the effect of relaxation spring. The systems with linear damping are examined analytically for two frequency regions, i.e., when the excitation frequency is equal to the resonance frequency, $\omega = \omega_n$, and for the high excitation frequencies above the natural frequency, $\omega \to \infty$ or $\omega >> \omega_n$. In this study, the responses of the rigidly supported linear damper and the elastically supported linear damper are examined separately in the following sections.

A. Rigidly supported linear damper

Equation of motion for the mass of the SDOF model shown in Fig. 2. (a) can be formed as

$$\ddot{x} + 2\zeta\omega_n\left(\dot{x} - \dot{x}_0\right) + \omega_n^2\left(x - x_0\right) = 0 \tag{1}$$

where the natural frequency $\omega_n = \sqrt{\frac{k}{m}}$ and linear damping ratio $\zeta = \frac{c}{2\sqrt{mk}}$. x, $\dot{x}$ and $\ddot{x}$ represent displacement, velocity and acceleration of the system mass. $x_0$and $\dot{x}_0$ are displacement and velocity of the base excitation. The response for this system is well known as for the fundamental theory of mechanical vibration. Therefore, the description in response for this system is omitted in this paper.

The vibration response or the so called displace transmissibility function (DTF) for the system with rigidly supported damper can be obtained by solving equation of motion shown in (1). By assuming the single harmonic excitation, $x_0 = X_0\cos(\omega t)$, the solution for this model is well known and can be given as a function of frequency, ω, as

$$\left|\frac{X}{X_0}\right| = \sqrt{\frac{\omega_n^4 + \left(2\zeta\omega_n\omega\right)^2}{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}} \tag{2}$$

Equation (2) can be simplified for the case that $\omega = \omega_n$,

$$\left|\frac{X}{X_0}\right| = \sqrt{1 + \frac{1}{\left(2\zeta\right)^2}} \tag{3}$$

Equation (3) shows that, when $\omega = \omega_n$, the DTF level depends wholly on the value of damping ratio. The greater value of damping ratio results in the lower vibration level at this frequency.

For the case that $\omega >> \omega_n$, (2) can be approximated by

$$\left|\frac{X}{X_0}\right| = \frac{2\zeta\omega_n}{\omega} \tag{4}$$

Equation (4) shows that, the level of DTF reduces by 20 dB/decade as excitation frequency increases. However, it is seen and well known that the greater value of damping ratio results in the higher level of vibration response at high frequency. As also shown in Fig. 3. the greater value of damping ratio helps lowering the level of vibration at the resonance frequency as shown by an arrow (i). In contrast, it causes the higher vibration level at higher excitation frequencies, as indicated by an arrow (ii). This response characteristic can cause the damage to the product at excitation frequencies high above natural frequency. Thus, in order to maintain the lower level of vibration at around the resonance frequency, it is essential to eliminate unwanted characteristic of the linear damping at high excitation frequencies.

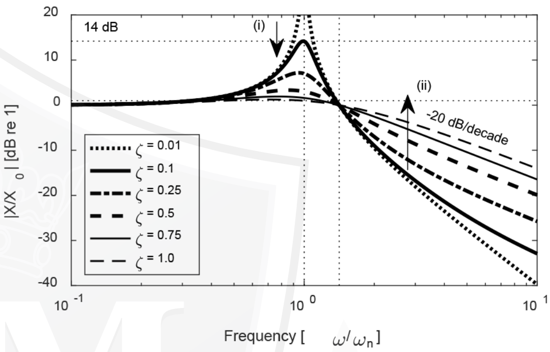


Fig. 3. Displacement transmissibility function for the rigidly mounted damping system.

B. Elastically mounted linear damper

The mathematical model for the elastically supported linear damper is obtained from the model shown in Fig. 2. (b). Theoretically, there is no difference by means of acting force on both the supporting spring and the damper for the models shown in Fig. 2. (b) and (c), as mentioned earlier. However, in order to be consistent to the experimental rig which the active damper is part of the system mass, thus the model in Fig. 2. (b) is employed. Equation of motion for the mass of the model shown in Fig. 2. (b) is given by

$$\ddot{x} + 2\zeta\omega_n\left(\dot{x} - \dot{x}_r\right) + \omega_n^2\left(x - x_0\right) = 0 \tag{5}$$

where $\dot{x}_r$ is the velocity of the connection point.

In addition, since the linear damper and spring experience the same force, therefore there is another first order equation for the force, i.e.

$$2\zeta\omega_n\left(\dot{x} - \dot{x}_r\right) = \kappa\left(x_r - x_0\right) \tag{6}$$

where x_0 is the displacement excitation and κ is the ratio between stiffness of the system spring k and the supporting k_r, i.e. $\kappa = \frac{k}{k_r}$.

The solutions for the elastically mounted damping system can be obtained by solving (5) and (6). For some aspect and convenience, (5) and (6) are written alternatively using relative velocity across the damper, i.e., $y_r = x - x_r$. Thus, (5) and (6) can be written as

$$\ddot{x} + \omega_n^2 x = \omega_n^2 x_0 - 2\zeta\omega_n \dot{y}_r \tag{7}$$

and

$$2\zeta\omega_n \dot{y}_r + \kappa y_r = \kappa x - \kappa x_0 \tag{8}$$

By assuming harmonic base excitation, $x_0 = X_0 \cos(\omega t)$, the solutions can be given by

$$\left|\frac{X}{X_0}\right| = \sqrt{\frac{(\kappa+1)^2 (2\zeta\omega_n\omega)^2 + \omega_n^4\kappa^2}{(\omega_n^2 - \omega^2 + \kappa)^2 (2\zeta\omega_n\omega)^2 + \kappa^2(\omega_n^2 - \omega^2)^2}} \tag{9}$$

For the case that $\omega = \omega_n$, (9) can be simplified to

$$\left|\frac{X}{X_0}\right| = \sqrt{\frac{(\kappa+1)^2}{\kappa^2} + \frac{1}{(2\zeta)^2}} \tag{10}$$

It is seen from (10) that, for the system with relaxation spring, the level of DTF at the natural frequency depends on both the value of damping ratio and stiffness ratio. The effects from these parameters are examined later in the following sections.

For the case that $\omega >> \omega_n$, (9) can be simplified to

$$\left|\frac{X}{X_0}\right| = \sqrt{\frac{(\kappa+1)^2 (2\zeta\omega_n)^2}{(\kappa - \omega^2)^2 (2\zeta\omega_n)^2 + \kappa^2\omega^2}} \tag{11}$$

Equation (11) also shows that the vibration response at excitation frequencies high above the natural frequency is dependent on both the value of damping ratio and the stiffness ratio. The effects for these parameters can be analyzed as follows.

1) The influence of linear damping ratio

The vibration behavior for the elastically supported damping system can be influenced by the value of damping ratio. The value of damping ratio can be considered to range from 0 to ∞. When damping ratio is equal to zero, the system becomes undamped system. This is identical to the system with $\kappa \rightarrow 0$. Equation (10) can result to infinity and (11) gives the result of undamped system.

In addition, the system mass is oscillating freely only on the suspension spring as shown in Fig. 2. (a). Thus the resonance frequency is the same as the natural frequency of the system is defined by

$$\omega_n = \sqrt{\frac{k}{m}} \tag{12}$$

When damping ratio approach infinity, $\zeta \rightarrow \infty$, the system mass can be considered oscillating on two springs, i.e., suspension and supporting spring as shown in Fig. 2. (b). This behavior is according to the results reported in [12]. The resonance frequency is then defined by

$$\omega_u = \sqrt{\frac{k + k_r}{m}} \tag{13}$$

The resulting frequency obtained from (13) is higher than that in (12). This means that the resonance frequency for the system with elastically supported damper is affected by the value of damping ratio. The variation of the resonance frequency can be illustrated as shown in Fig. 4. The resonance frequency shifts from ω_n to the higher frequency, ω_u, when value of damping ratio is about 0.5, as shown by an arrow (iii). In addition, when the value of damping ratio becoming much greater approaches infinity, the system becomes undamped vibration system on two parallel springs.

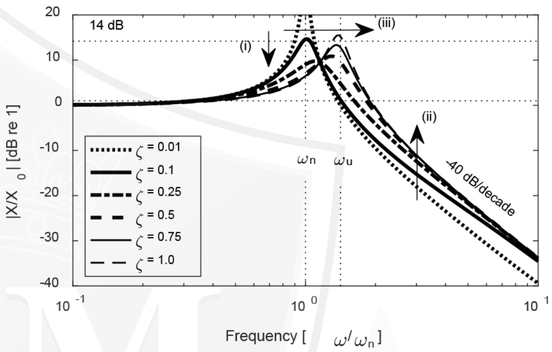


Fig. 4. Displacement Transmissibility Funtion for the elastically mounted damping system with $\kappa = 1$

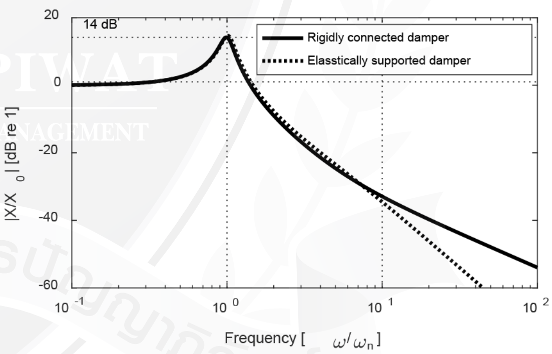


Fig. 5. Comparison of Displacement Transmissibility Funtion for the rigidly connected damper and elastically supported damper systems with $\zeta = 0.1$ and $\kappa = 1$

Furthermore, to show benefit of elastically supported damper, the responses for the system with damping ratio of 0.1 for those with rigidly connected damper and elastically supported damper which presented using the thick solid lines as seen in Fig. 3. and Fig. 4. are compared in Fig. 5. It is noticed that the peak levels are about the same at around 14 dB. But the level of vibration at the excitation frequencies higher than 10 are noticeable different. For that for rigidly connected damper reduces by 20 dB/decade, whereas that for elastically supported reduces by 40 dB/decade. This theoretically results show improvement of vibration isolation by supporting the damping element elastically.

2) The influence of stiffness ratio

The value of stiffness ratio influences the vibration response of the system by means of the level of vibration. There are two frequency regions which are affected, i.e., the resonance frequency and the frequencies higher than the natural frequency. The influence of the stiffness ratio to the vibration behavior is as shown in Fig. 6. for which the value of stiffness ratio can be considered to range between $0 < \kappa \ll \infty$.

For $\kappa \rightarrow 0$, this implies that $k_r \ll k$ or equivalent to no supporting spring ($k_r = 0$). The system mass is supported only by the suspension spring. This approach considers the system as the undamped system. The response in (10) becomes infinity and that for (11) becomes $\left|X/X_0\right| = 1/\omega^2$ which are the response for the undamped system.

For $\kappa \rightarrow \infty$, this implies that $k_r \gg k$ or equivalent to the rigidly support damper ($k_r = \infty$). Thus, this approach considers the system the same as the SDOF system.

The value of stiffness ratio is seen to directly influence the system damping. Fig. 6. shows the frequency response function for the system with constant damping ratio, $\zeta = 0.1$.

For the system with $\kappa = 0.1$, $k_r < k$, the system behaves similarly to the undamped SDOF system, i.e., the vibration level around the resonance frequency tends to infinity and the vibration in the high frequency region drops by 40 dB/decade. For the system with $\kappa = 100$, $k_r > k$, the system behaves similarly to the normal SDOF rigidly connected linear damping system.

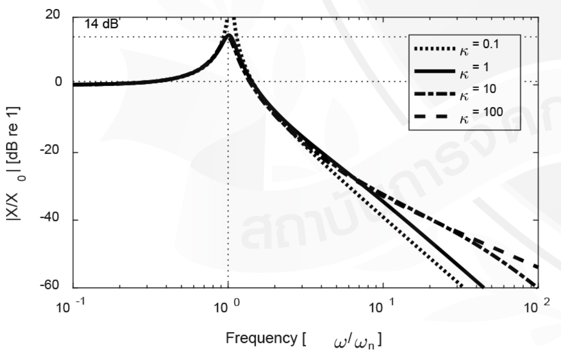


Fig. 6. Displacement transmissibility function for the elastically mounted damping system with $\zeta = 0.1$ for difference values of κ.

However, in this study, the influences of the damping ratio are examined experimentally. Two values of damping ratios were applied respectively whereas the value of stiffness ratio was kept constant.

III. Experimental Setup

The experimental rig is as shown in Fig. 7. The damping element was implemented by using the electromagnetic shaker LMS V101 as the active electrically controllable damper. The rig consisted of a system mass which was the mass of the shaker and additional mass, the suspension spring and the supporting spring. The base excitation was implemented by employing the B&K electromagnetic vibration exciter type 4808. The acceleration signals for both the system mass and the base excitation were captured using B&K 4508B accelerometers. The B&K Pulse LabShop was used to analyze the signals.

The base excitation was defined to be single frequency harmonic excitation. The sinusoidal signal was generated using a Tektronix CFG280 function generator. The amplitude of the signal was controlled to be the same for all applied frequencies which ranged from 6-200 Hz. The amplitude of the base excitation was controlled by means of input displacement. The acceleration signal of the base input was double integrated using the B&K Pulse LabShop. Amplitude of the harmonic base input was controlled to be 30, 40 and 50 μm respectively. Theoretically, the amplitude of the base excitation should not affect the vibration response for the linear vibration system. Thus, varying the excitation input was to ensure the linearity of the system.

In order to simulate the damping force characteristic, the acceleration signals were integrated to be signals of velocity. The integration process was implemented using an STM32 interface board as the schematic illustrated in Fig. 7. Only the signals at fundamental frequency were used. Signals in other frequencies and DC offset were filtered out using passive band pass filter. The integrated signals were then processed to be relative velocity between the system and the base excitation. The signal of relative velocity was then used as feedback signal to the damper. The signal was fed to the shaker amplifier which was acting as the adjustable damping coefficient. Then the amplified signal was fed to the shaker in order to produce the damping force. The process was applied to all excitation frequencies.

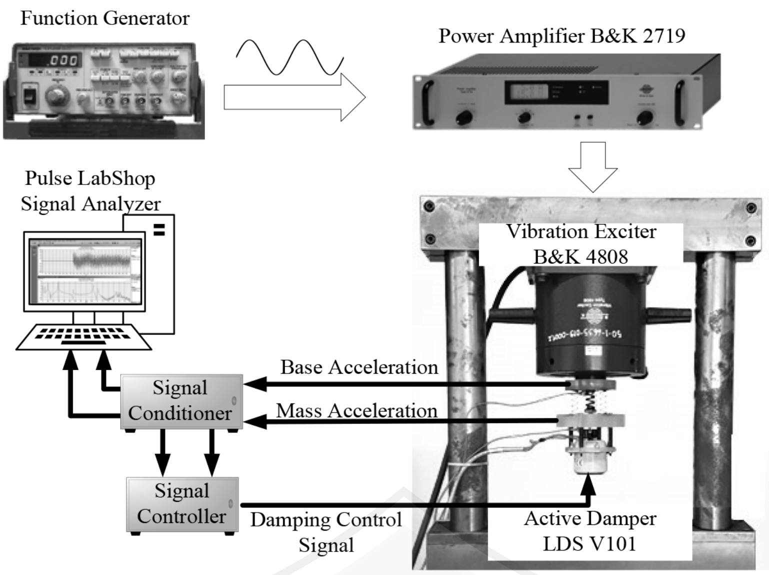


Fig. 7. Experimental rig and measuring components.

The damping was supported elastically by a spring as shown in Fig. 8. There were three suspension spring with the total stiffness of 3 kN/m. The spring stiffness of the electromagnetic shaker is 3.15 kN/m. Therefore the total stiffness for the system suspension was around 6.15 kN/m. Stiffness of the relaxation spring was around 6 kN/m. Thus the stiffness ratio, κ, was around 1.

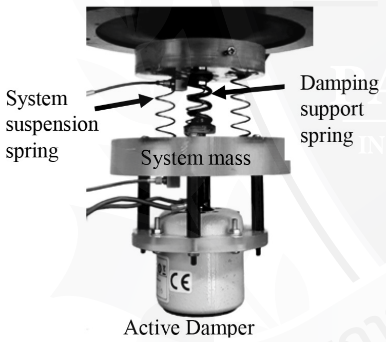


Fig. 8. Assembly of elastically mounted damping vibration isolation system.

Linearity of the experimental rig was carried out by testing on a rigidly mounted damper. Three levels of harmonic base excitation were applied respectively. The vibration response of the system was displayed in the form of displacement transmissibility function (DTF) which is the amplitude ratio between the base excitation and the absolute displacement. The level of vibration at around the resonance frequency was controlled to be around 14 dB as the DTF shown in Fig. 9. This vibration level refers to the value of damping ratio around 0.1.

Fig. 9. also shows that the difference in excitation amplitude did not affect the response of the system. Furthermore, by comparing the experimental results to the theoretical result, it is also seen that the experimental results follow the theoretical result smoothly. Therefore, the experimental rig was working linearly. Such the results assure that the results obtained from this experimental rig will not be affected by any non-linearity of the rig but the effects of supporting spring.

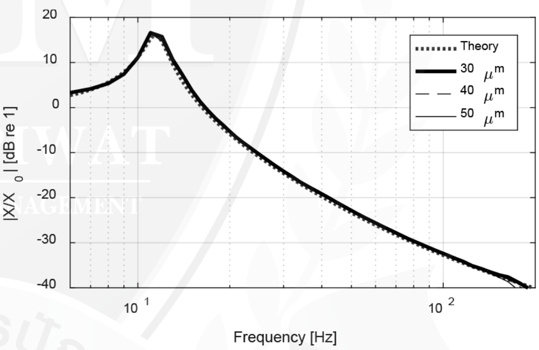


Fig. 9. Displacement Transmissibility Function for the system with rigidly mounted damping under three excitation amplitudes in comparison to the theoretical result.

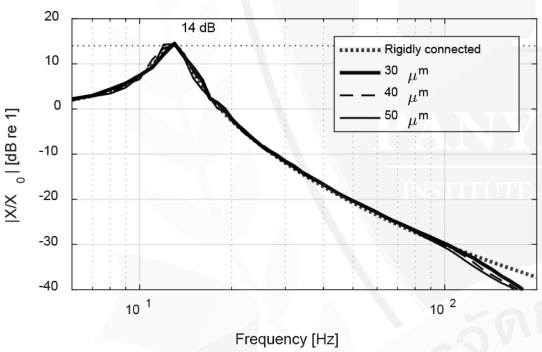
IV. Results and Discussions

As the damping element was electronically controllable, the value of damping ratio was adjusted by referring to the level of DTF at the resonance frequency. The level of DTF at the resonance frequency was controlled to be the same for every level of base excitation and stiffness ratio. By doing so, the levels of DTF at the resonance frequency are equal as seen in Fig. 10. (a) and (b), the level of DTF was controlled to be around 14 dB and 9 dB, respectively. The corresponding value of damping ratios to the vibration level were 0.1 and 0.18, respectively. Therefore, the effects of elastically mounted damper can be seen for the high excitation frequencies.

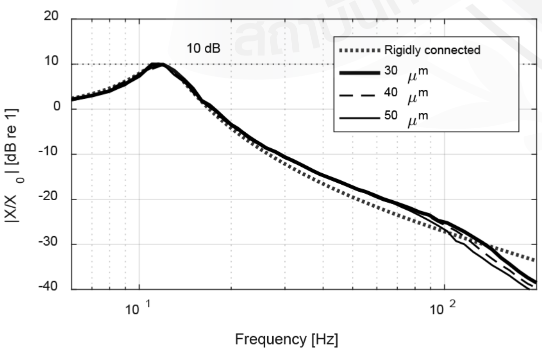
The dotted lines in Fig. 10 (a) and (b) show the DTF for theoretical results of the rigidly mounted damper. These lines are used as the reference to examine the responses for the system with elastically supported damper. Thus the difference in level of DTF for the elastically mounted damper can be noticed.

The responses for the system with elastically supported damper are presented for three excitation amplitude, i.e., 30 μm, 40 μm and 50 μm. Similar to those for rigidly connected damper, difference levels of excitation amplitude were applied to check linearity of the system. It is seen that there is no significant variation in the responses caused from different excitation amplitudes. The responses from these three excitation amplitudes appear similarly for both values of applied damping ratio.

In Fig. 10. at the excitation frequencies higher than 100 Hz, the responses for the system with elastically supported damper are seen to reduce. Such the response characteristic is according to the theoretical examination shown in Fig. 5. The responses for both systems are in the similar level for around 10 frequency decade from the resonance frequency. After that, for the higher excitation frequencies, the responses for the elastically supported damper drop by 40 dB/decade.



(a)



(b)

Fig. 10. Displacement Transmissibility Function for the elastically mounted damping system with $\kappa = 1$, (a) $\zeta = 0.1$ and (b) $\zeta = 0.16$.

To this end, the reduction in the vibration level at high frequencies are said to be the beneficial effects of having the elastically supported damper. Therefore, the application of elastically supported damper definitely helps to protect or isolate the system mass or the product from vibration caused by the exciting base. Nevertheless, the further study needs to be carried out in order to examine the important physical function of the supporting spring.

V. Conclusion

This study investigates the performance of the vibration isolator having elastically mounted linear damping. The theoretical results between the systems with rigidly mounted damping and elastically mounted damping were discussed and compared. The experimental investigation was performed to verify the benefit of the elastically mounted damping theoretically found. The experimental study was done by employing electromagnetic shaker to be as the active damper. The experimental results reveal the benefit of mounting the damping element elastically. The levels of vibration, for excitation frequencies 10 decade higher the natural frequency, are noticeable much lower than that for the rigidly connected damper. The experimental results show accordingly to those obtained theoretically which the level of vibration is found to be reduce by 40 dB per frequency decade. Thus, the level of vibration for the system with elastically supported damper can be much lower than that for the system with rigidly connected damper for the excitation frequencies much higher above the natural frequency. It can be said that the detrimental effects of the linear damper can be eliminated by supporting it elastically. Therefore, by having the damping element support elastically can help improve the vibration isolation performance of the isolation system and reduce damage caused from vibration at high excitation frequencies.

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