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On sensitivity of control chart for monitoring serially correlated data

Wannaphon Suriyakat*

Faculty of Science and Technology, Pibulsongkram Rajabhat University, Phitsanulok, Thailand

Abstract

The normality and independent observations were designed for double exponentially weighted moving average (DEWMA) and exponentially weighted moving average (EWMA) control charts. DEWMA control chart was modified from the EWMA control chart. The performance of the control chart is measured in terms of the average run length (ARL) that is the average number of samples before getting an out-of-control signal. In a real application, data are dependent and non-normal observations. The exponential distribution has application in any area of the subject such as reliability theory, survival analysis and queuing theory. The purpose of this paper was to study DEWMA and EWMA control charts using the first-order autoregressive (AR(1)), the first-order autoregressive moving average (ARMA(1,1)) and the first-order integrated moving average (IMA(1,1)) models for an exponential distributed process variable to study the efficiency of detecting small process mean shift. A simulation using the R program was applied to study the ARL performances for DEWMA and EWMA control charts for the small process mean shift. Tables of ARLs are presented for the various process mean shift. All ARL at various sets of parameters of the control chart calculation was completed based on 10,000 replications for a scenario. The EWMA control chart is more efficient than the DEWMA control chart in the detection of small process mean shifts as it dependably gives smaller ARL values and quickly detects the process shift at various levels of correlation and shifts in the process mean. The application of serially correlated data in the control chart literature has achieved with wide suitability. The design and application of the DEWMA and EWMA control charts suggest a model in the detection of small process mean shift by process control employees.

Keywords: EWMA, DEWMA, average run length, autocorrelated data, control chart

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1. Introduction

The control chart is the most powerful tool for statistical process control to monitor and control a process. The control chart is a graphical expression of a quality characteristic that has been measured or computed from a sample versus the sample number or time [1]. The centerline (CL), the upper control limit (UCL) and the lower control limit (LCL) are shown on the control chart. The performance of the control chart is usually evaluated using average run lengths (ARLs) and standard deviations of run lengths (SDRLs). The traditional control charts designed for normally independent identically distributed (i.i.d.) observations but measurements from industrial processes are often serially correlated. Recently, many researchers have studied control chart for correlated data, Harris and Ross [2] investigated cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts based on time series model. Weib [3] presented the EWMA control chart to detect mean increases in Poisson the first integer autoregressive processes. Suriyakat [4] studied the performance of the EWMA control chart to detect a small shift in the process mean with an exponential autoregressive integrated moving average process using explicit formula. The geometric moving average (GMV) control chart known as an EWMA control chart was introduced by Roberts [5], and also known to be effective for detecting small to moderate shifts in the parameters [6]. The double EWMA (DEWMA) control chart was extended from the EWMA control chart [7]. The EWMA and DEWMA control charts under Type-I censoring for gamma-distributed lifetimes were used for monitoring the mean level by using average run length, and it is found DEWMA control chart was an efficient chart for the detection of a shift in scale parameter [8]. Recently, Raza et al [9] introduced and compared the DEWMA and EWMA control charts under type?I censoring for Poisson?exponential distribution to monitor of mean level shifts using censored data, the performance of the DEWMA and EWMA control charts was evaluated using the average run length, expected quadratic loss, and performance com-

*Corresponding author; email: suriyakat@psru.ac.th

parison index measures. Raza et al [10] presented conditional expected value (CEV) and conditional median (CM)?based DEWMA control charts for monitoring the type I censored data with Rayleigh distribution.

The DEWMA and EWMA control charts using a non-normal distribution with time series model are studied in the literature (see, e.g., [3, 4]). A time series is a set of observations , each one being recorded at a specific time [11] and may be used simply to provide a compact explanation of the data. The exponential distribution is a continuous random variable and has application in many areas of the subject such as reliability theory, survival analysis, queuing theory. The exponential distribution is also studied in the time series model. Several publications dealing with the subject of studying in the time series model using an exponential white noise have appeared in the literature (see, e.g., [12 – 15]). Therefore, the purpose of this paper was to study DEWMA and EWMA control charts using AR(1), ARMA(1,1) and IMA(1,1) models for an exponential distributed process variable to study the efficiency of detecting small process mean shift. The paper is organized as follows: Section 2 is devoted to the presentation of DEWMA and EWMA control charts. Section 3 describes the proposed AR(1), ARMA(1,1) and IMA(1,1) models for an exponential distribution. Section 4 both average run lengths (ARL's) and standard deviations of run lengths (SDRL's) are presented for the EWMA and DEWMA control charts, and in Section 5 are the concluding remarks.

2. Time Series Model with Exponential White Noise

Definition 1 A random variable X has an exponential distribution with parameter $\alpha > 0$ if its probability density function (pdf) is

$$f(x;\alpha) = \begin{cases} \alpha \exp(-x\alpha) & , x \geq 0 \\ 0 & , otherwise. \end{cases}$$

The mean, variance and cumulative distribution function (cdf) of exponential distribution are given in the following:

$$\begin{aligned} E(X) &= \frac{1}{\alpha},\ V(X) = \frac{1}{\alpha^2},\ F(x) = P(X \leq x) \\ &= 1 - \exp(-x\alpha);\ x \geq 0. \end{aligned}$$

Definition 2 The first-order autoregressive (AR(1)) process. Let $\{X_t\}$ is a stationary series satisfying the equations

$$X_t = \phi X_{t-1} + Y_t, \quad t = 0, \pm 1, \pm 2, \ldots$$

where $\{Y_t\}$ is an exponential distribution with parameter α, $|\phi| < 1$ and is uncorrelated with X_s for each $s < t$.

Definition 3 The autoregressive moving average processes. The time series $\{X_t\}$ is an ARMA(1,1) process if it is stationary and satisfies (for every t)

$$X_t - \phi X_{t-1} = Y_t + \theta Y_{t-1}, \quad t = 0, \pm 1, \pm 2, \ldots, \phi + \theta \neq 0.$$

Where $\{Y_t\}$ is an exponential distribution with a parameter α.

Definition 4 The integrated moving-average (IMA(1,1)) process. If $\{X_t\}$ is a nonstationary series satisfying the equations

$$X_t = X_{t-1} + Y_t + \theta Y_{t-1}, \quad t = 0, \pm 1, \pm 2, \ldots$$

where $\{Y_t\}$ is an exponential distribution with parameter α and $|\theta| < 1$.

3. The EWMA and DEWMA Control Chart for Monitoring the Process Mean

In this section, we propose to use EWMA and DEWMA to construct a control limit for serially correlated data under exponential white noise. We first consider the EWMA control chart proposed by Roberts [5]. The EWMA statistic is defined as

$$Y_t = (1-\lambda) Y_{t-1} + \lambda X_t,\ t = 1, 2, 3, \ldots$$

where $\lambda \in (0, 1]$ is a constant and the starting value or the process target is $Y_0 = \alpha$. If the observations X_t are dependent random variables with variance σ^2, then the asymptotic of the variance of Y_t statistic for an AR(1) process is [16]

$$V(Y_t) = \sigma_X^2 \frac{\lambda}{2-\lambda} \frac{1+\phi(1-\lambda)}{1-\phi(1-\lambda)}.$$

The control limits are given as follows:

$$\begin{aligned} UCL &= \alpha + E\sigma_Y = E \\ LCL &= \alpha - E\sigma_Y = 0 \end{aligned}$$

The DEWMA statistic is defined via the system of equation

$$Z_t = (1-\lambda) Z_{t-1} + \lambda Y_t,\ t = 1, 2, 3, \ldots$$

where $\lambda \in (0, 1]$ is a constant and the starting value or the process target is $Z_0 = \alpha$. The asymptotic of the variance of Z_t the statistic is

$$V(Z_t) = \sigma_Y^2 \frac{\lambda\left(2 - 2\lambda + \lambda^2\right)}{(2-\lambda)^3}$$

The control limits are given as follows:

$$\begin{aligned} UCL &= \alpha + D\sigma_Z = D \\ LCL &= \alpha - D\sigma_Z = 0 \end{aligned}$$

The average run length (ARL) is the way to define the performances of the EWMA and DEWMA control charts. The ARL is the average number of the

Table 1. Values of the UCL for the DEWMA and EWMA control charts with $ARL_0 = 300$.

λ	AR			ARMA			IMA		
	$\phi = 0.1$	$\phi = 0.5$	$\phi = 0.95$	$\phi = 0.1$ $\theta = 0.9$	$\phi = 0.5$ $\theta = 0.5$	$\phi = 0.95$ $\theta = 0.05$	$\theta = 0.1$	$\theta = 0.5$	$\theta = 0.95$
DEWMA									
0.05	1.252	1.502	21.420	0.133	0.379	20.360	236.103	197.006	14.062
0.1	1.425	2.555	22.802	0.177	1.285	21.662	253.828	141.672	15.047
0.2	1.728	3.045	24.067	0.327	1.560	22.880	263.008	146.633	15.571
0.3	2.019	3.471	24.710	0.561	1.829	23.520	265.914	148.323	15.782
EWMA									
0.05	1.400	2.590	22.516	0.343	1.344	21.400	253.000	141.000	15.000
0.1	1.779	3.014	23.760	0.581	1.624	22.513	262.000	146.000	15.540
0.2	2.271	3.635	24.750	1.058	2.104	23.540	266.500	148.480	15.920
0.3	2.722	4.123	25.258	1.534	2.550	24.000	268.000	249.410	16.220

Table 2. Performance for the EWMA and DEWMA control charts when $\phi = 0.1,\ \theta = 0.9, \lambda = 0.05$.

$\alpha + \delta$	AR(1)				ARMA(1,1)				IMA(1,1)			
	DEWMA D = 1.252		**EWMA E = 1.400**		**DEWMA D = 0.133**		**EWMA E = 0.343**		**DEWMA D = 236.103**		**EWMA E = 253.000**	
	ARL	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
1.00	300.5830	227.8410	299.2177	255.8091	300.2397	231.248	299.6991	282.1490	300.0160	15.6887	299.9737	16.4437
1.05	197.4920	130.663	**193.9646**	150.6475	**197.4080**	140.324	220.8649	205.2657	288.0712	15.4666	**286.7980**	16.2456
1.10	146.6900	84.8862	**137.7283**	99.2872	**142.9830**	87.1489	168.0187	153.5321	276.1806	14.7511	**274.5802**	15.8050
1.15	116.237	59.0084	**105.3290**	70.0818	**111.6680**	60.9768	133.1056	118.4791	266.2992	14.6848	**263.2337**	15.2829
1.20	95.2838	40.6877	**84.8635**	51.5819	**91.8699**	44.6205	107.1719	93.1459	256.4204	13.9437	**253.0287**	14.8260
1.40	62.4160	18.2616	**47.7030**	22.7658	56.3807	19.9168	**55.0258**	45.8441	225.3072	13.0659	**219.7015**	13.7684
1.60	49.1726	11.1799	**33.4061**	13.8455	42.6875	12.8323	**34.5289**	27.8799	201.6364	12.1787	**194.7259**	12.8720
1.80	41.8678	8.3621	**26.6598**	10.5440	34.6175	9.7276	**24.5456**	19.5876	183.6678	11.4827	**175.4715**	12.0420
2.00	37.1276	6.8787	**21.8290**	8.3762	29.5170	7.9448	**18.0758**	14.3802	169.2786	10.8164	**159.4881**	11.3877
2.50	29.6418	5.0958	**15.3927**	5.8292	21.9584	5.9232	**11.1077**	9.2792	142.6826	9.6664	**131.2124**	10.0904
3.00	25.1912	4.5102	**11.8922**	4.6197	17.6031	4.9212	**7.7510**	6.6089	125.1712	8.6722	**112.3858**	9.1932

Bold font represents the lowest ARL_1

sample points that are plotted before a point is beyond the control limits.

Definition 5 The stopping time

Let $\{Y_t, t = 1, 2, 3, \ldots\}$ be a sequence of random variables and $\tau = \min\{t : Y_t < LCL \text{ or } Y_t > UCL\}$ be the stopping time. Then, the ARL is defined to be the expected value of τ, $E(\tau)$.

4. Simulation Results

The performances of the EWMA and DEWMA control charts with serially correlated data are compared in this section. The upper limits are considered for these charts so that the simulation studies conducted here using R. In Table 1, the value of the upper limits for the DEWMA and EWMA control charts with the in-control (ARL_0) is 300. The upper limit of DEWMA control chart is smaller than the EWMA control chart for almost all models.

Algorithms are written in the R program [17] to compute the out-of-control $ARL(ARL_1)$, based on 10,000 repetitions for a shift of $\alpha(\alpha + \delta)$ and δ is shift size. The simulation procedure includes the following:

1. Pseudorandom numbers are generated by the R function.

2. The control statistic (DEWMA and EWMA) is calculated for all models.

3. The control statistic is compared with an experimental LCL and UCL (an experimental D or E is used) and thus a run length is obtained and recorded.

4. After 10, 000 simulation runs, the 10,000 derived run-length values consist of a sample of the random variable run length, and the sample means and squared root of sample variance give the ARL and SDRL, respectively.

Tables 2-4 show the figures of simulation ARL given $\text{ARL}_0 = 300$, $\lambda = 0.05$ with different shift sizes: 0.00, 0.05, 0.1, 0.15, 0.20, 0.40, 0.60, 0.80, 1.00, 1.50, and 2.00. For all tables, the shift sizes increase, while the figures for ARL_1 decrease significantly, starting approximately at $\text{ARL}_0 = 300$. It is observed that the EWMA chart has the best performance for all models considered. When the mean increase simultaneously, the EWMA chart has a minimum ARL_1. However, when the mean increases from 1.05 – 1.20 based on $\phi = 0.1,\ \theta = 0.9, \lambda = 0.05$, the DEWMA chart has the best performance for the ARMA(1,1) model found in Table 2.

5. Conclusions and Discussion

The traditional control chart has been used extensively for process control, which is valid under the normal independence assumption of observations. In the real world applications, there are many types of dependent observations in which the traditional control chart cannot be used. The purpose of this paper was to study DEWMA and EWMA control charts using the first-order autoregressive (AR(1)), the first-order autore-

Table 3. Performance for the EWMA and DEWMA control charts when $\phi = 0.5$, $\theta = 0.5$, $\lambda = 0.05$.

$\alpha + \delta$	AR(1)				ARMA(1,1)				IMA(1,1)			
	DEWMA D = 2.250		EWMA E = 2.590		DEWMA D = 0.127		EWMA E = 1.344		DEWMA D = 131.313		EWMA E = 141.000	
	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.00	300.9760	227.0050	300.7160	252.8826	302.1567	232.7950	300.6725	255.2814	299.5039	15.7775	299.9304	16.2332
1.05	201.1068	131.6905	**194.1897**	148.3545	198.0802	129.3294	**194.2527**	157.2317	287.3571	15.1747	**286.7640**	16.0429
1.10	145.3392	81.6349	**137.6778**	97.6671	145.1894	83.6679	**138.1604**	98.9604	275.7411	14.8281	**274.4729**	15.6488
1.15	113.4026	54.9036	**105.0592**	67.4729	115.7281	58.8883	**106.0439**	70.6266	265.3977	14.6978	**263.2705**	15.4378
1.20	97.4464	42.5902	**84.4638**	51.0492	95.8502	41.8511	**84.4722**	51.6620	255.6588	14.1841	**253.1369**	14.7989
1.40	63.3718	18.2104	**48.0968**	22.5989	62.3689	18.2036	**47.9859**	22.7614	224.4891	13.1989	**219.5317**	13.8439
1.60	49.8326	10.7384	**34.1942**	14.1889	49.2161	11.3431	**33.8064**	14.1942	201.2044	12.3361	**194.0966**	12.7060
1.80	42.8038	8.4065	**26.8601**	10.0956	41.9737	8.4132	**26.5521**	10.5691	183.0617	11.3783	**174.5660**	12.0698
2.00	37.5182	6.8680	**22.3503**	8.1648	36.8899	6.9881	**21.8563**	8.3398	168.3457	10.8082	**159.2056**	11.4755
2.50	30.4138	5.2220	**15.9029**	5.6857	29.5159	5.2471	**15.3637**	5.9088	141.7723	9.5291	**130.7778**	10.1606
3.00	26.0872	4.3452	**12.5014**	4.4939	25.1534	4.4353	**11.9758**	4.7199	124.1507	8.7031	**111.9462**	9.2050

Bold font represents the lowest ARL_1

Table 4. Performance for the EWMA and DEWMA control charts when $\phi = 0.95$, $\theta = 0.05$, $\lambda = 0.05$.

$\alpha + \delta$	AR(1)				ARMA(1,1)				IMA(1,1)			
	DEWMA D = 21.420		EWMA E = 22.516		DEWMA D = 20.360		EWMA E = 21.400		DEWMA D = 14.062		EWMA E = 15.000	
	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL	ARL	SDRL
1.00	299.2810	207.9840	300.0697	227.7582	300.1024	207.8172	299.9306	227.4250	299.9992	16.4231	299.9274	17.2035
1.05	204.4376	118.2978	**196.0685**	130.5010	203.9524	116.6352	**196.5608**	128.8560	286.7340	16.0775	**285.6875**	16.9336
1.10	152.4874	71.5897	**143.8889**	82.0547	153.7275	72.3268	**145.1179**	82.9082	274.9691	15.7376	**272.8503**	16.6181
1.15	127.1698	50.3570	**114.3833**	57.3040	127.1637	49.8661	**114.5719**	57.9010	263.6656	15.3146	**260.7110**	15.8651
1.20	110.5070	35.1162	**96.2238**	42.1280	110.5389	35.2437	**95.5628**	41.1581	253.5760	15.0567	**249.9386**	15.8966
1.40	79.6826	15.1395	**61.9618**	17.7302	79.8274	14.8807	**62.0415**	17.7200	219.9641	13.9586	**214.2049**	14.6212
1.60	67.1816	9.9831	**48.9100**	11.2403	67.1949	10.0219	**49.0990**	11.1388	194.5663	12.8784	**187.5289**	13.6771
1.80	59.6928	7.8569	**41.8662**	8.5405	59.5166	7.9940	**41.8034**	8.3957	175.3479	12.1265	**166.6444**	12.9175
2.00	54.3138	6.8629	**36.8632**	6.8487	54.3507	6.9270	**36.9040**	7.0392	159.4014	11.4997	**149.9435**	12.3626
2.50	45.7408	5.3966	**29.5845**	5.2452	45.7956	5.4085	**29.4296**	5.3010	131.3488	10.1038	**120.2107**	11.0022
3.00	40.4782	4.6030	**25.1080**	4.4459	40.4065	4.6602	**25.0133**	4.4210	112.8009	9.2146	**100.0195**	9.8897

Bold font represents the lowest ARL_1

gressive moving average (ARMA(1,1)) and the first-order integrated moving average (IMA(1,1)) models for an exponential distributed process variable to study the efficiency of detecting small process mean shift. The EWMA control chart is more efficient than the DEWMA control chart in the detection of small process mean shifts as it dependably gives smaller ARL values and quickly detects the process shift at various levels of correlation and shifts in the process mean. Future research should be focused on other charts and the real-life situation data with exponential white noise as an alternative and should be applied to evaluate the ARL of all charts.

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References

[1] D. C. Montgomery, Introduction to statistical quality control, John Wiley & Sons, 2009.

[2] T. J. Harris, W. H. Ross, Statistical process control procedures for correlated observations, The Canadian journal of chemical engineering 69(1) (1991) 48 – 57.

[3] C. H. Weiβ, Detecting mean increases in Poisson INAR (1) processes with EWMA control charts, Journal of Applied Statistics 38(2) (2011) 383 – 398.

[4] W. Suriyakat, Performance of exponentially weighted moving average control chart for exponential ARIMA processes, Far East Journal of Mathematical Sciences 100(3) (2016) 371.

[5] S. W. Roberts, Control chart tests based on geometric moving averages, Technometrics 1(3) (1959) 239 – 250.

[6] P. E. Maravelakis, J. Panaretos, S. Psarakis, An examination of the robustness to non normality of the EWMA control charts for the dispersion, Communications in Statistics-Simulation and Computation 34(4) (2005) 1069 – 1079.

[7] S. E. Shamma, A. K. Shamma, Development and evaluation of control charts using double exponentially weighted moving averages, International Journal of Quality & Reliability Management 9(6) (1992).

[8] S. M. M. Raza, M. Riaz, S. Ali, On the performance of EWMA and DEWMA control charts for censored data, Journal of the Chinese Institute of Engineers 38(6) (2015) 714 – 722.

[9] S. M. M. Raza, A. F. Siddiqi, EWMA and DEWMA Control Charts for Poisson?Exponential Distribution: Conditional Median Approach for Censored Data, Quality and Reliability Engineering International 33(2) (2017) 387 – 399.

[10] S. M. M. Raza, S. Ali, M. M. Butt, DEWMA control charts for censored data using Rayleigh lifetimes, Quality and Reliability Engineering International 34(8) (2018) 1675 – 1684.

[11] P. J. Brockwell, R. A. Davis, M. V. Calder, Introduction to time series and forecasting (Vol. 2), New York: springer 2002.

[12] J. Andel, On AR (1) processes with exponential white noise, Communications in Statistics-Theory and Methods 17(5) (1988) 1481 – 1495.

[13] M. A. Amaral Turkman, Bayesian analysis of an autoregressive process with exponential white noise, Statistics 21(4) (1990) 601 – 608.

[14] H. Fellag, M. Ibazizen, Estimation of the first-order autoregressive model with contaminated exponential white noise, Journal of Mathematical Sciences, 106(1) (2001) 2652 – 2656.

[15] M. Ibazizen, H. Fellag, Bayesian estimation of an AR (1) process with exponential white noise, Statistics 37(5) (2003) 365 – 372.

[16] M. B. Vermaat, F. H. Van der Meulen, R. J. M. M. Does, Asymptotic behaviour of the variance of the EWMA statistic for autoregressive processes. Statistics & Probability Letters 78(12) (2008) 1673 – 682.

[17] R Core Team R: A language and environment for statistical computing, R Foundation for Statistical Computing, Vienna, Austria, Available from: https://www.R-project.org/. (accessed 2017).