Improving Belief Propagation Decoding of Polar Codes using Bi-Directional Bit-Flipping Strategy

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Abstract. *It is difficult to achieve high throughput and low latency from the cyclic redundancy check (CRC-) aided successive cancellation list (CA-SCL) polar decoder employed in 5G new radio systems. To tackle this problem, the hardware-efficient belief propagation (BP) decoder has been extensively studied. Unfortunately, the performance of the BP decoding for polar codes is quite poor. To improve the performance of this decoder, a novel bi-directional bit-flipping (BF) strategy is proposed. The rule of generating the flip set based on the post log-likelihood ratio (LLR) is combined with the possibility of the two-way flipping. With the help of this idea, the decoding performance of BP decoder is identical to that of the CA-SCL decoder. Moreover, with the same complexity, our proposed BF strategy outperforms both the best known one-directional BF strategy and the previously found bi-directional BF strategy at BLER of* 10^{-4}.

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1. Introduction

Nowadays, channel coding is utilized in digital communication systems to combat random and burst errors. A type of channel coding called polar code has been considered as one of the most favored topics in this field. Erdal Arikan, the inventor of polar codes, and many coding theorists have demonstrated that polar codes can provide the capacity-achieving performance for several classes of channels [1-2]. The key concept for constructing polar codes is known as channel polarization [3]. With this concept, the information bit is sent through the high reliable channel, i.e., the channel with low probability of error, whereas the low reliable channel responds for the frozen bit, generated to perform error correction. Therefore, we call the code constructed from this concept as polar code since the channels are polarized according to the probability.

It is known in the literature that the conventional decoding algorithm for the polar codes is based on the concept of the successive cancellation (SC). Recently, the polar code with cyclic redundancy check (CRC-) aided successive cancellation list (CA-SCL) decoder is selected as the coding scheme for the 5G enhanced mobile broadband (eMBB) control channels due to its excellent decoding performance [4-5]. However, a major drawback of the CA-SCL algorithm is high decoding latency caused by its inherent serial nature [6]. With the serial processing manner of this algorithm, the polar code of medium or long block lengths, e.g., the range of 1,000 to 10,000 bits, suffer from very high decoding latency [7]. This problem motivates the researchers to explore the alternative polar decoders.

Belief propagation (BP) algorithm is recognized in the literature as an iterative message-passing algorithm [8]. Erdal Arikan demonstrated in his groundbreaking work that the BP algorithm can be employed as the polar decoder [9]. It is important to mention that this algorithm can be implemented in a fully parallel manner. Thus, compared with some existing SC-based decoders, the BP decoder can definitely provide the lower latency and the higher throughput [10]. With these particular advantages, the polar code with the BP decoder is received a lot of attention [11-13]. Nevertheless, the performance of polar codes with the BP decoder is rather poor [14]. So, various techniques to improve the performance of BP decoder for polar codes have been proposed [15-17].

The performance of polar codes with the BP decoder can be improved by concatenating it with an outer CRC code [18-19]. With the utilization of this outer code, the BP decoder can determine whether or not the decoding is successful. If the unsuccessful decoding is declared, it is popular to employ the bit-flipping (BF) strategy for adjusting the input of BP algorithm [19]. After that, the additional BP decoding attempt is activated with the ultimate goal of providing successful decoding. Compared with a stand-alone BP decoder, BF aided BP decoder, known as the BPF decoder, can provide a significant performance improvement. However, the decoding performance of BPF decoder is still not identical to that of the CA-SCL decoder, which will be used as the benchmark in this work. Note that BF aided SC decoder is also studied and, like BPF decoder, the performance improvement can also be achieved [20-21]. Also, note that there are a few studies regarding BF aided SCL decoder due to excessive complexity [22-23].

The original concept for the BF strategy is to flip the BP parameters, which are the information involved in the decoding process, in the opposite direction [19]. For example, the hard decision is flipped from bit 0 to bit 1 or the log-likelihood ratio (LLR) value is changed from positive to negative. This BF concept is called in this paper as the one-directional BF strategy. Interestingly, there has been a report that it is also possible to adjust the BP parameters in the same direction [18]. For example, the LLR value can be increased from a positive real value to $+\infty$. The BF strategy that allows the adjustment of BP parameters in the same and opposite directions is named in this work as the bi-directional BF strategy. Nevertheless, it is important to state that the decoding performance of the BPF employing the bi-directional BF strategy is still not satisfactory. Inspired by [18] and [19], the idea of generating a flip set by considering the magnitude of LLR is combined with the bi-directional BF strategy. With this combination, we propose a novel bi-directional BF strategy for a BP-based polar decoder. With the same decoding complexity, the proposed bi-directional BF strategy outperforms the previously found one-directional BF strategies. Furthermore, compared with the benchmark, the proposed BPF decoder can provide the identical decoding performance and higher decoding throughput can be expected from its parallel nature.

The rest of the paper is organized as follows. The preliminary concepts of polar codes and outline notation are explained in Section 2. The BP decoding algorithms are briefly described in Section 3, and the original bit-flipping strategy is introduced in Section 4. The proposed BPF decoder is presented in Section 5. Simulation results and discussions are given in Section 6. Finally, conclusions are drawn in Section 7.

2. Preliminaries

A brief overview of the polar codes is presented in this section. The information and codeword length of polar codes are denoted by K and N, respectively. Note that $N = 2^n$ bits, where n is a positive integer, and its coding rate is given by $R = K/N$. A number of frozen bits which are redundancy bits for the polar codes, are equal to $N-K$ bits. It is typical to assign all frozen bits with zero. The $N-K$ positions within the codeword must be chosen to allocate frozen bits. These positions are obtained by channel polarization. Thus, channel polarization [1], which is a key to design and construct the polar codes, is first described.

Channel polarization is a technique to create two groups of polarized channels which are high reliable channels and low reliable channels. Each polarized channel will be used to carry one bit. Note that channel polarization is performed only once after the polar code parameters and SNR are specified. Generally, there are many constructions to perform channel polarization [24-27]. One of the most popular methods, named as Gaussian approximation (GA), is utilized to perform channel polarization in this work. Following [27], the error probability for i-th polarized channel is given by

$$P_e(i) = Q\left(\sqrt{E_i/2}\right) \tag{1}$$

where Q denotes a monotone decreasing function, E_i is the expectation of LLR for the i-th polarized channel, and $i = 1, 2, ..., N$. It is important to note that this error probability represents the reliability level for each polarized channel. After finishing the computation of all N error probabilities, these probabilities will be sorted in descending order. Then, we can now define the index set $F = \{F_1, F_2, ..., F_{N-K}\}$ of frozen bits according to the first $N-K$ indices of sorted error probabilities. With these notations, polar codes can be specified by $P(N, K, F)$.

The channel polarization for a polar code $P(4,2,F)$ is briefly described through the following example. First, GA algorithm is used to produce the error probabilities. Suppose that $P_e(1) = 0.9$, $P_e(2) = 0.7$, $P_e(3) = 0.1$ and $P_e(4)$=0.3. Then, the indices of sorted error probabilities are given by 1, 2, 4, and 3, respectively. Finally, the first two indices define the index set of frozen bits $F = \{1,2\}$. This means that the frozen bits will be allocated at the first and second positions of the input for a polar encoder. This selection of polarized channels completely defines a polar code and it can be named as the channel allocation.

After getting F from the channel allocation, the polar encoding can be formulated as follows. The input of a polar encoder, which comprises of $N-K$ frozen bits and K information bits, is denoted by source vector $\mathbf{u} = [\mathrm{u}_1, \mathrm{u}_2, ..., \mathrm{u}_N]$. The frozen bits will be allocated in $\mathbf{u}$ by using $N-K$ indices defined in F. The indices that remain in unallocated positions are responsible for the information bits. These indices will be used to define the index set of the information bits. This set is denoted in this paper by $\mathcal{A}$ and the information vector thus can be written as $\mathbf{u}_{\mathcal{A}} = [\mathrm{u}_{\mathcal{A}_1}, \mathrm{u}_{\mathcal{A}_2}, ..., \mathrm{u}_{\mathcal{A}_K}]$. With source vector $\mathbf{u}$, the polar codeword $\mathbf{c}$ is generated by

$$\mathbf{c} = \mathbf{u} \cdot \mathbf{G}_N = \mathbf{u} \cdot \mathbf{F}^{\otimes n}, \tag{2}$$

where $\mathbf{G}_N = \mathbf{F}^{\otimes n}$ is the generator matrix of the polar codes, $\mathbf{F}^{\otimes n} = \mathbf{F} \otimes \cdots \otimes \mathbf{F}$ (n times) is the n-fold

Kronecker product of $\mathbf{F}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$. This codeword will be mapped into modulated symbols, denoted by $\tilde{\mathbf{c}}$, via the symbol mapper. Then, $\tilde{\mathbf{c}}$ is sent through the noisy channel and the received vector $\mathbf{r}=[\mathrm{r}_1,\mathrm{r}_2,...,\mathrm{r}_N]$ is given by

$$\mathbf{r}=\tilde{\mathbf{c}}+\mathbf{n}, \quad (3)$$

where $\mathbf{n}$ is the noise vector. We assume in this paper that the transmission channel is additive white Gaussian noise (AWGN) channel, i.e., $\mathbf{n}\sim\mathcal{N}(0,N_0/2)$, and the symbol mapper is based on binary phase shift keying (BPSK) modulation, e.g., bit $0\rightarrow+1$ and bit $1\rightarrow-1$.

From the above description, the block diagram of the polar coding scheme used in this work can be graphically depicted in Fig. 1. At the receiver side, soft-output demapper must be employed to provide the input $\boldsymbol{y}=[y_1,y_2,...,y_N]$ for polar decoder. This input is expressed in terms of LLR, e.g.,

$$y(\mathrm{r})=\log\frac{P(\mathrm{r}|0)}{P(\mathrm{r}|1)}=\frac{2\mathrm{r}}{\sigma^2}, \quad (4)$$

for the case of AWGN channel. Finally, the polar decoder provides the estimation, denoted by $\hat{\mathbf{u}}_{\mathcal{A}}$. As mentioned earlier, the SC-based algorithms are widely used as the polar decoder. However, in this work, we will focus on the alternative decoding algorithm, called a BP decoder, and its concept will be subsequently presented in the next section.

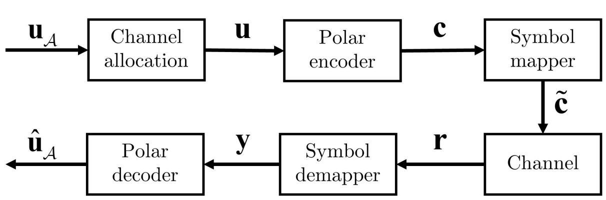

Fig. 1 Block diagram of polar coding scheme.

3. Belief Propagation Decoder

The mechanism of the polar encoder can be viewed as the factor graph defined by the generator matrix $\mathbf{G}$ [13]. The BP algorithm is used to perform polar decoding by iteratively propagating the probabilities over the underlying factor graph. Generally, the factor graph of polar codes $P(N,K,F)$ consists of n stage levels and $N(n+1)$ variable nodes. The example of a factor graph for $P(4,2,\{1,2\})$ is shown in Fig. 2. An index (i,j) is assigned to each variable node to represent i-th row and j-th stage level of factor graph.

A methodology to apply the BP algorithm for polar decoding is shortly presented as follows. For the sake of simplicity, we will refer to this type of decoder shortly as the BP decoder. Following the description given in [28], the BP decoder mainly consists of four steps, which are initialization, right-to-left messages, left-to-right messages, and estimation. These steps are described below.

I. Initialization

Each (i,j) variable node in the factor graph is associated with two types of messages, which are the right-to-left LLR and the left-to-right LLR. These LLRs are usually referred to as the priori LLRs. The first one is denoted by $L_{i,j}$, and can be computed as follows:

$$L_{i,j}=\begin{cases}0, & j\neq n+1\\ y_1, & j=n+1,\end{cases} \quad (5)$$

The expression of the second type of messages, denoted by $R_{i,j}$, is given below

$$R_{i,j}=\begin{cases}0, & j\neq 1\\ 0, & j=1, i\in\mathcal{A}\\ +\infty, & j=1, i\in F\end{cases} \quad (6)$$

After finishing the initialization step, the sub-graph of the factor graph that connects only one box-plus operator can be visualized together with two types of messages in Fig. 3. Note that the BP decoder is processed in an iterative manner. The following two steps are considered as one BP decoding iteration, and the number of decoding iterations is labelled by t.

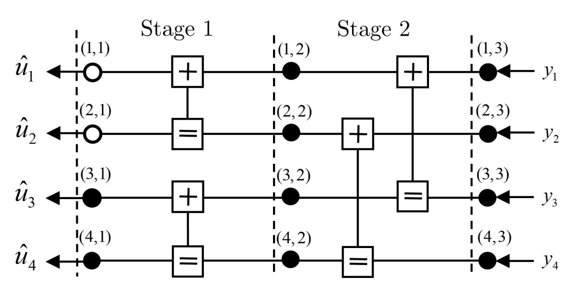

Fig. 2 Factor graph of polar code $P(4,2,\{1,2\})$. At the first stage, black variable nodes represent information bits, while the frozen bits are indicated as white variable nodes.

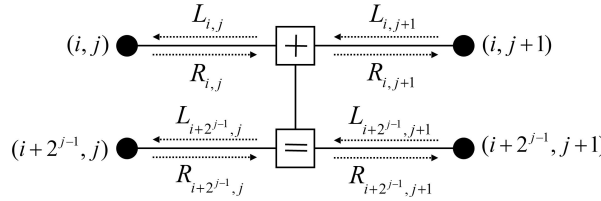

Fig. 3 Sub-graph of BP decoder with one box-plus operator. Box-plus operator is invoked to efficiently represent the calculation of four variable nodes in this factor graph.

II. Right-to-Left Messages

The BP decoding in this step starts from the stage level $j = n$. Then, at the t-th decoding iteration, the BP decoder processes the computation of messages $L_{i,j}^{t}$ and $L_{i+2^{j-1},j}^{t}$ simultaneously as follows:

$$\begin{aligned} L_{i,j}^{t} &= f\left(L_{i,j+1}^{t}, L_{i+2^{j-1},j+1}^{t} + R_{i+2^{j-1},j}^{t-1}\right) \\ L_{i+2^{j-1},j}^{t} &= L_{i+2^{j-1},j+1}^{t} + f\left(L_{i,j+1}^{t} + R_{i,j}^{t-1}\right), \end{aligned} \quad (7)$$

where $f(x,y) = 0.9375\text{sign}(x)\text{sign}(y)\min\left(|x|,|y|\right)$, known in the literature as the scaled min-sum (SMS) algorithm [16]. Note that the box-plus operators in Fig. 2 and Fig. 3 are used to represent $f(x,y)$.

III. Left-to-Right Messages

The computation in this step starts from the stage level $j = 1$. Similar to the previous step, the left-to-right messages in BP decoding can be computed as follows:

$$\begin{aligned} R_{i,j+1}^{t} &= f\left(R_{i,j}^{t}, L_{i+2^{j-1},j+1}^{t} + R_{i+2^{j-1},j}^{t}\right) \\ R_{i+2^{j-1},j+1}^{t} &= R_{i+2^{j-1},j}^{t} + f\left(R_{i,j}^{t} + L_{i,j+1}^{t}\right). \end{aligned} \quad (8)$$

These two steps are repeated until the maximum iteration $I^{\max}$ is reached, i.e., $t = I^{\max}$.

IV. Estimation

Let $\hat{\mathbf{y}} = [\hat{y}_1, \hat{y}_2, ..., \hat{y}_N]$ be the post LLRs from a BP decoder. The i-th post LLR is computed from the right-to-left LLR and the left-to-right LLR at the first stage level as follows:

$$\hat{y}_i = L_{i,1}^{I^{\max}} + R_{i,1}^{I^{\max}}. \quad (9)$$

Finally, based on hard-decision, the estimation of a source vector is computed, as given below

$$\hat{u}_i = \begin{cases} 0, & \text{if} \quad \hat{y}_i \geq 0 \\ 1, & \text{otherwise.} \end{cases} \quad (10)$$

The performance of this BP decoder can surpass the SC decoder. However, its performance is still inferior to the CA-SCL decoder. One of the possible solutions to further improve the performance of the BP decoding will be presented in the next section.

4. Bit-Flipping Aided BP Decoder

Based on the BP decoder discussed in the previous section, it is impossible to know about the validity of the estimation, i.e., output from the BP decoder. It is common in this field of study to use the CRC-polar concatenation code to check this validity. If the invalid estimation is detected by the CRC code, the bit-flipping (BF) strategy can be employed in order to explore for the valid estimation, i.e., $\hat{\mathbf{u}}_{\mathcal{A}} = \mathbf{u}_{\mathcal{A}}$, or the estimation with zero check value. This coding scheme can be referred to as the CRC-polar concatenation code with BF aided BP decoder.

The block diagram of the aforementioned coding scheme is shown in Fig. 4. Compared with the block diagram in Fig. 1, a CRC encoder, a CRC decoder, and bit-flipping are supplemental parts, and their explanations are as follows. Before performing channel allocation, the CRC encoder forms the new information vector by adding k_p parity bits to the original information vector of length K bits. Therefore, the new coding rate for CRC concatenated polar codes is defined by $R = (K + k_p)/N$. The CRC decoder utilizes these parity bits to check the validity of the estimation. If the invalid estimation is declared by the CRC decoder, the BF strategy is used to activate the additional BP decoding with a little modification of the message $R_{i,1}$ in the initialization step. After finishing this additional BP decoding, the estimation can be valid or invalid. The overall decoding process stops when the CRC decoder is satisfied with the valid estimation. Therefore, this means that the BF strategy tells the BP decoder to perform a new decoding attempt. With this mechanism, we define T as the maximum number of additional BP decoding that can be requested by the BF strategy.

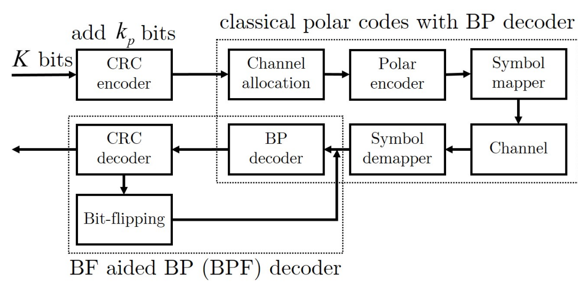

Fig. 4 Block diagram of CRC-polar concatenation code with BF aided BP decoder.

In order to perform the BF strategy described above, the positions to be flipped must be defined. According to some specific rules, these positions, collected only from the indices of unfrozen bits, are used to define the flip set $S = \{s_1, s_2, ..., s_T\}$. If we restrict ourselves to the case of single BF strategy and one-directional BF strategy [19], this flip set is given by

$$S \leftarrow i \in \mathcal{A}, \ |S| = T. \quad (11)$$

After constructing this set, an index from set S will be selected to perform one additional BP decoding. This means that T additional BP decoding attempts activated by the BF strategy utilize T indices defined in set S. Next, using pre-defined rule, hard or soft information associated with the selected index, must be modified. Finally, this modification will be involved in a new BP decoding attempt. Note that multiple BF strategy is also proposed in the literature [29-30] but it is beyond the scope of this work.

The methodologies for generating the flip set and the rules for modifying information have been extensively studied in [18-19]. For example, the most famous one-directional BF strategy proposed in [19], named as the generalized BPF (GBPF) decoder, modifies the soft information, expressed in terms of LLR, as follows. After getting the flip set S, the BF strategy uses the knowledge given in this set to form the new priori LLR $R^*_{i,j}$. At τ-th additional decoding attempt, where $\tau = 1, 2, ..., T$, the new priori LLR for the BP decoding is given by

$$R^*_{i,j} = \begin{cases} -\infty, & i = s_\tau \text{ and } \hat{u}_{s_\tau} = 0 \\ +\infty, & i = s_\tau \text{ and } \hat{u}_{s_\tau} = 1 \\ R_{i,j} & \text{otherwise.} \end{cases} \quad (12)$$

It can be implied from this equation that only one message of the initialization step is modified at each additional BP decoding attempt. Thus, the BF strategy described above is known as the single bit-flip. With the aid of BF strategy, the BP decoding performance can be significantly improved. However, comparing with the benchmark performance, there is room for improvement. We will address our idea regarding generating flip set and the methodology of LLR adjustment for a BPF decoder in the next section.

5. Proposed Bi-Directional Bit-Flipping Strategy

It is important to mention that the magnitude of LLR $(\hat{y}_i)$ represents the level of ambiguity after BP decoding. Therefore, the decoded symbol with a small magnitude of $\hat{y}_i$ implies a very high level of ambiguity. For example, the ratio of the probability of bit 0 and bit 1 approaches to 1 when $|\hat{y}_i|$ is close to zero. Thus, it is intuitive to flip or change the value of the decoded symbols with small magnitudes of $\hat{y}_i$.

From eq. (12), it is seen that the message is flipped to $-\infty$ when $\hat{u}_{s_\tau} = 0$ (its sign of the original LLR value is positive), and the message $\hat{u}_{s_\tau} = 1$ is flipped to $+\infty$. One can think that the BF strategy changes or flips the message in the opposite direction. From our empirical study, we found that the BF strategy occasionally faces the invalid estimation after finishing the additional BP decoding. Thus, we try to change the priori LLR in the same direction, e.g., flipping the priori LLR to $+\infty$ when $\hat{u}_{s_\tau} = 0$. With this concept, the flip set $S = \{s_1, s_2, ..., s_{T/2}\}$ for our proposed BF strategy is as follows:

$$S \leftarrow i \in \mathcal{A}, \text{ of } \frac{T}{2} \text{ smallest } |\hat{y}_i|. \quad (13)$$

Note that the cardinality of this flip set is a half of $|S|$ given in eq. (11). But the maximum number of additional BP decoding T for both sets is the same since the message at each index defined by S must be flipped two times. With this concept, the flipping operation for this proposed BF strategy can be written by

$$R^*_{i,j} = \begin{cases} -\infty, & i = s_\tau \text{ and } \tau \text{ is an even number} \\ +\infty, & i = s_\tau \text{ and } \tau \text{ is an odd number} \\ R_{i,j} & \text{otherwise} \end{cases} \quad (14)$$

Intuitively, we call this type of the BF strategy as bi-directional BF strategy. Surprisingly, this simple way can provide the additional BP decoding with the valid estimation. The result of the performance improvement and the evidence will be shown in the next section. Before ending this section, the flip set S of the proposed bi-directional BPF algorithm is listed in Algorithm 1, and the proposed bi-directional BF aided BP decoder is summarized in Algorithm 2.

Algorithm 1: Flip Set of Proposed BPF Decoder

Input: $\hat{\mathbf{y}}$, T
Output: $\mathcal{S}$
$\hat{\mathbf{y}}_{\mathcal{A}} \leftarrow \hat{\mathbf{y}}$ `// only the post LLRs of information vector`
$\mathbf{L} = |\hat{\mathbf{y}}_{\mathcal{A}}|$
$idx = \mathbf{Sort}(\mathbf{L}, \text{'ascend'})$
`// obtain in` idx`, the` $|\mathbf{L}|$ `indices of vector` $\mathbf{L}$ `when sorted in ascending order`
$\mathcal{S} = idx[1:(T/2)]$
`// store the first` $T/2$ `indices`
return $\mathcal{S}$;

6. Results and Discussions

We show through the simulations that the utilization of the proposed bi-directional BF strategy improves the decoding performance of BP decoder for polar codes. All the simulation parameters are summarized in Table 1. For the sake of simplicity, the bi-directional BF aided BP decoder is denoted by BPF-2D, whereas the one-directional BF aided BP decoder is denoted by BPF-1D. Note that the GBPF decoder described in Section IV is the best performing BPF-1D [19]. So, in what follows, BPF-1D is used to represent the GBPF decoder. We divide our discussions into three subsections to clearly present the performance advantage of the proposed BPF-2D decoder.

6.1 Optimal Performance under BPF Decoder

To obtain the optimal performance of BPF decoder, two issues must be addressed. The first one is that information bits are assumed to be known at the receiver, i.e., genie-aided manner [18]. This means that the receiver always knows about the erroneous transmission and knows whether bit-flipping must be activated or not. So, the simulations in this section can be conducted without the CRC codes. The second one is about the number of additional decoding T, which is the most important parameter to establish the flip set S of the BF strategy. This parameter is assigned with the maximum value in order to guarantee the optimal performance of BPF decoder.

The optimal BLER performances of BPF decoders with various code rates R are shown in Fig. 5. The number of additional BP decoding T for BPF-1D and proposed BPF-2D are K and $2K$, respectively. Therefore, only for the simulation in this subsection, the computational complexity of proposed decoder is twice that of the BPF-1D. As we expected, it is observed from this figure that the decoding performance of BPF-2D outperforms that of BPF-1D for all cases. At BLER of 10^{-4}, small coding again about 0.2 dB can be achieved from our proposed BF aided BP decoder. Remind that all the results from this figure are optimal performance. So, these results can be considered as the lower bound for both BPF-1D and BPF-2D.

Algorithm 2: Proposed Bi-Directional BPF Decoding algorithm

Input: $\mathbf{y}, I^{\max}, \mathcal{A}, \mathcal{F}, T, \mathcal{S}$
Output: $\hat{\mathbf{u}}_{\mathcal{A}}$
$\hat{\mathbf{y}} \leftarrow$ BP($\mathbf{y}, I^{\max}, \mathcal{A}$) ; `// original BP decoding`
$\hat{\mathbf{u}} \leftarrow$ the hard-decision ($\hat{\mathbf{y}}$) in (1)
$\hat{\mathbf{u}}_{\mathcal{A}} \leftarrow \hat{\mathbf{u}}$; `// only information vector`
if CRC_detect ($\hat{\mathbf{u}}_{\mathcal{A}}$) does satisfy **then**
  return $\hat{\mathbf{u}}_{\mathcal{A}}$;
else
  for $t = 1$ **to** $T/2$ **do**
    for $i = 0$ **to** 1 **do**
      $R \leftarrow 0, R_{\mathcal{F},1} \leftarrow +\infty$
      $R^*_{\mathcal{S}(t),1} \leftarrow (2i-1) \times \infty$
      $\hat{\mathbf{y}} \leftarrow$BP($\mathbf{y}, I^{\max}, \mathcal{A}, R$)
      $\hat{\mathbf{u}} \leftarrow$ the hard-decision ($\hat{\mathbf{y}}$) in (1)
      $\hat{\mathbf{u}}_{\mathcal{A}} \leftarrow \hat{\mathbf{u}}$
      if CRC_detect ($\hat{\mathbf{u}}_{\mathcal{A}}$) does satisfy **then**
        return $\hat{\mathbf{u}}_{\mathcal{A}}$;
  return $\hat{\mathbf{u}}_{\mathcal{A}}$;

Channel	Additive white Gaussian noise
Modulation	Binary phase shift keying
Code construction	Gaussian approximation
Constructed by GA	At E_b/N_0 = 2.5 dB
Maximum iteration	$I_{\max}$ = 50
CRC code	$g(x) = x^{12} + x^{11} + x^3 + x^2 + x + 1$

Table 1 Simulation configurations

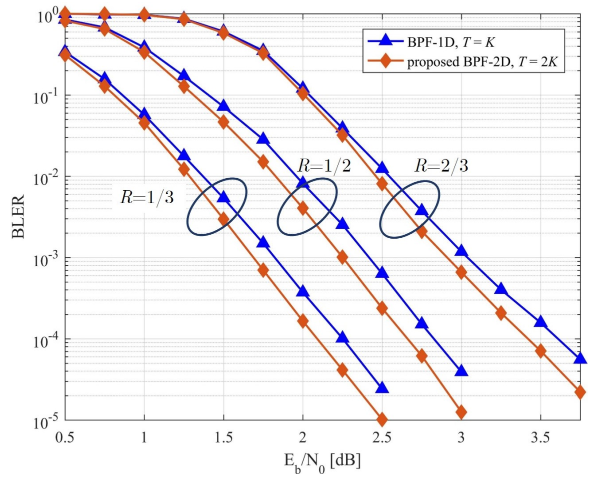

Fig. 5 Optimal BLER performances for polar codes with BF aided BP decoders. Codeword length of all polar codes is $N = 1024$. The performance is demonstrated at R = 1/3, 1/2, and 2/3.

6.2 A Fair Comparison between BPF-1D and BPF-2D

In order to provide a fair comparison, BPF-1D and BPF-2D must use the same computational complexity. This can be done by using the same number of additional BP decoding T for both types of decoders. Fig. 6 shows the decoding performances of three types of the polar decoder, which are BPF-1D, BPF-2D invented in [18] and our proposed BPF-2D. Similar to the optimal case discussed in the previous subsection, our proposed BPF-2D decoder still outperforms BPF-1D for all values of T. Moreover, this simple idea can also provide the performance gain, by approximately 0.25 dB with T = 240 at BLER of 10^{-4}, over the previously found BPF-2D.

The performance comparison at two short codeword lengths and small values of T are conducted in order to show the effectiveness of our proposed BF strategy. It is obviously seen in Fig. 7 that the proposed BPF-2D outperforms BPF-1D for all values of additional BP decoding attempts. Note that the observation points of SNR are selected to clearly demonstrate two decoding performances at the same graph. We remind the reader again that the performance improvement provided by the proposed BPF-2D, presented in this subsection, can be achieved without any increase in decoding complexity.

6.3 Comparison with CA-SCL Decoding

We show in this section the performance comparison between the proposed BPF-2D and the practical CA-SCL decoder. Considering the requirements of uplink transmission in 5G standard [5,19], we focus on the target BLER of 10^{-3} and the list size L for CA-SCL decoder is 4. It is seen from Fig. 8 that the performance of our proposed decoder with T = 120 is identical to that of the CA-SCL decoder. It is worth to mention that this excellent performance of BPF-2D decoder is achieved with a reasonable number of additional BP decoding. It can also be observed that the performance of BPF-2D decoder can surpass this benchmark performance by increasing the parameter T. With this result, it is not exaggerated to state that the proposed bi-directional BPF decoder is a promising candidate to be used as the polar decoder for the 5G uplink network.

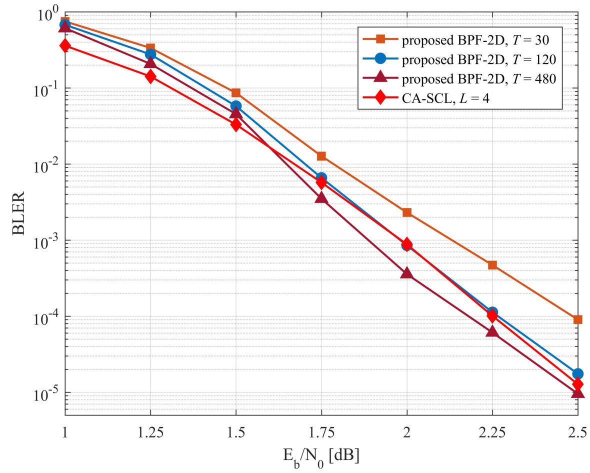

Fig. 8 BLER performance comparison between the proposed BPF-2D and CA-SCL decoding under $P(2048,1024)$ polar code.

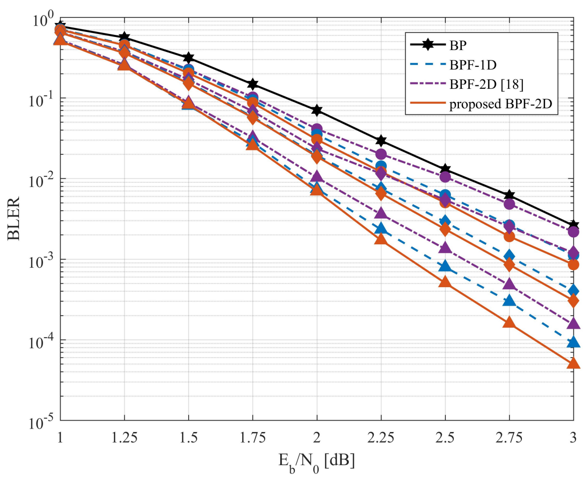

Fig. 6 BLER performance comparison between the proposed BPF decoder and some existing BPF decoders under the same decoding complexity. $P(1024,512)$ CRC-polar concatenated code is used for this simulation. The circle, diamond, and triangle marks are used to label T = 4, 16, and 240, respectively.

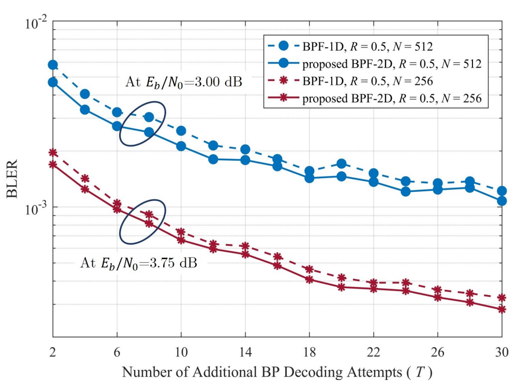

Fig. 7 BLER performance comparison between BPF-1D and proposed BPF-2D at small values of additional BP decoding attempt. $P(512,256)$ and $P(256,128)$ CRC-polar concatenate codes are used to demonstrate the performance advantage of our decoder at $T < 30$.

7. Conclusions

Due to the particular advantage of high parallelism, the belief propagation algorithm is very suitable for the implementation of low latency polar decoder. We propose in this paper a novel bi-directional bit-flipping strategy that suits well with the belief propagation based polar decoder. The proposed strategy can flip the LLRs defined by the flip set in the same and opposite directions, and this yields the improvement of decoding performance. With the same computational complexity, our results reveal that the proposed bit flipping strategy exhibits a better BLER performance compared to the original bit flipping strategy. Moreover, our proposed bi-directional BF strategy aided belief propagation decoder yields the same performance as the practical CA-SCL decoder. Our future research will be focused on the invention of bit flipping aided belief propagation with the performance beyond the practical CA-SCL decoder.

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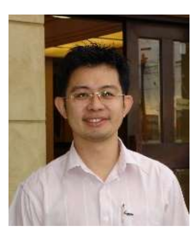

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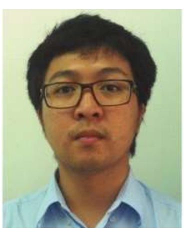

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