Assessment of Stress-Strain Modeling for Reactive Powder Concrete Deep Beams

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Abstract. *This study focuses on assessing the accuracy of the stress-strain model of concrete, as per the fib model code 2010, in simulating the behavior of deep beams made of reactive powder concrete (RPC). RPC is a modern concrete classified as an ultra-high performance fiber reinforced concrete. The study utilizes finite element analysis (FEA) to obtain numerical results for deep beams and compares them with experimental data gathered from existing literature. The investigation involves three types of deep beams: normal strength concrete (NSC), high strength concrete (HSC), and RPC, allowing for comprehensive comparisons. The findings from the FEA reveal that the fib model code 2010 provides conservative estimations for the loading capacity of RPC deep beams. Consequently, it is recommended that a stress-strain model specially tailored for RPC be implemented to achieve simulation results that closely align with experimental results.*

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1. Introduction

Reinforced concrete deep beams have gained significant importance in structural engineering due to their ability to carry heavy loads over short spans, making them suitable for applications such as transfer girders in tall buildings. However, the large cross-section and the presence of stirrup reinforcement in deep beams contribute to their substantial weight. To address this challenge and enhance their performance, the use of high-performance materials becomes crucial. Reactive powder concrete (RPC), classified as an ultra-high performance fiber reinforced concrete (UHPFRC), offers a practical solution. RPC exhibits superior workability, mechanical properties, and durability when compared to normal strength concrete (NSC) and high strength concrete (HSC) [1]. The utilization of RPC in deep beams improves their ductility, crack control and resistance to seismic forces.

Although some experimental studies have recently investigated the shear behavior of UHPFRC deep beams [2-5], the understanding of their shear behavior remain limited. Experimental testing, while providing fundamental information on structural behavior, can be both expensive and time-consuming. Consequently, finite element analysis (FEA) is often employed to gain a comprehensive understanding of the behavior of structural members. To conduct realistic analyses of deep beams, it is essential to consider reliable stress-strain models that accurately describe the material's behavior under compression and tension. However, stress-strain curves obtained from material tests are not always readily available and may be absent from research reports. Uniaxial tensile and compressive tests are primarily conducted to estimate the material's ultimate strength, resulting in a lack of stress-strain models specially designed for RPC. As a result, approximations based on average compressive strength are commonly employed. The *fib* model code 2010 [6] provides a procedure for formulating stress-strain relationships in both the tension and compression zones for HSC.

The objective of this study is to evaluate the validity of stress-strain models according to the fib model code 2010 to accurately simulate the behavior of RPC deep beams. Numerical results generated using the nonlinear finite element program SOFiSTiK [7] are compared with selected experimental data from literature for NSC, HSC, and RPC deep beams. By assessing the performance of the stress-strain models, this research aims to improve the understanding and prediction of the behavior of RPC deep beams, ultimately contributing to the development of more reliable design guidelines and methodologies.

2. Material Models

2.1 Stress-Strain Model of Concrete

According to the *fib* model code 2010 [6], the stress-strain relationship in compression for concrete can be simulated using Eq. (1).

$$f_c = \left(\frac{k \cdot \eta - \eta^2}{1 + (k-2) \cdot \eta} \right) f_c' \quad (1)$$

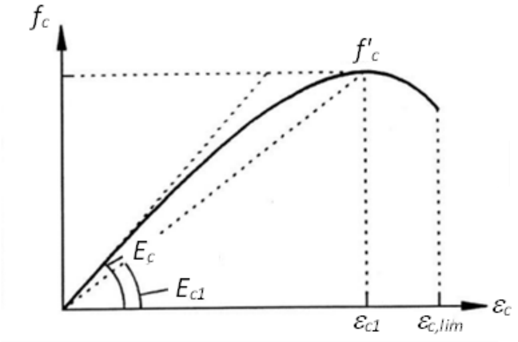


Fig. 1 General stress-strain curve of concrete in compression and the *fib* model parameters [6].

where f_c is compressive stress corresponding to compressive strain ($\varepsilon_c \leq \varepsilon_{c,lim}$), $\varepsilon_{c,lim}$ is maximum compressive strain, f_c' is maximum compressive stress, $\eta = \varepsilon_c / \varepsilon_{c1}$, ε_{c1} is strain at maximum compressive stress, $k = E_c / E_{c1}$ which is plasticity number, E_c is initial elastic modulus, and E_{c1} is secant modulus from origin to peak compressive stress. Fig. 1 shows the general stress-strain curve of concrete in compression with the model parameters. While the stress-strain relationship in tension can be obtained following the concept shown in Fig. 2 along with using Eqs. (2) to (4).

$$f_t = 2.12 \cdot \ln(1 + 0.1 f_c') \text{ MPa} \quad (2)$$

$$\varepsilon_{t1} = 0.15 + \frac{w_1}{l_c} \text{ mm/m} \quad (3)$$

$$w_1 = \frac{G_f}{f_t}; \; G_f = 0.073 (f_c')^{0.18} \text{ N/mm} \quad (4)$$

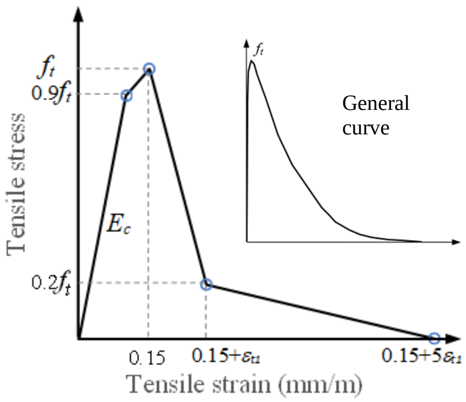


Fig. 2 Tensile stress-strain model of concrete according to the *fib* model code 2010 [6] and general curve.

where f_t is peak tensile stress, ε_{t1} is tensile strain corresponding to crack opening w_1, w_1 is crack opening when tensile stress = $0.2f_t$, l_c is characteristic length taken as total height of section (h), and G_f is fracture energy.

2.2 Stress-Strain Model of Steel Rebar

The stress-strain relationships in compression and tension for steel rebar are assumed with a standard elastic-perfectly plastic relation defined by yield strength (f_y) and elastic modulus (E_s) as depicted in Fig. 3.

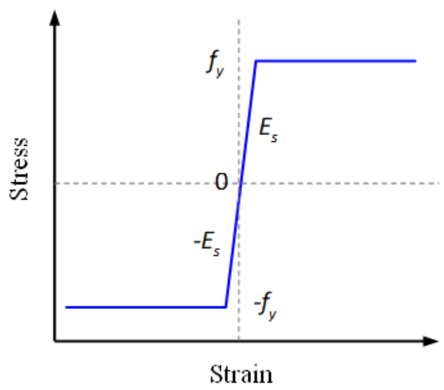


Fig. 3 Stress-strain model of steel rebar

3. Finite Element Analysis

In order to implement the stress-stress models described in Section 2, the nonlinear finite element program SOFiSTiK [7] was adopted. The test results of NSC, HSC, and RPC deep beams carried out by Chen et al. [2] were selected for comparison. The test values of concrete compressive strength, concrete elastic modulus, and yield strength were used in the model.

3.1 Stress-strain curves

The stress-strain data points were calculated using the parameters listed in Table 1 for concrete and Table 2 for steel rebar. For RPC, the parameters corresponding to the highest class of concrete (C120) as recommended in the *fib* model code 2010 [6] were adopted.

Parameter	Material		
	NSC	HSC	RPC
Compression			
1. f_c' (MPa)	46.9	86.5	151.4
2. E_c (GPa)	38.5	41.7	46.2
3. k	1.82	1.41	1.18
4. $\varepsilon_{c,lim}$ (mm/m)	3.5	3.1	3.0
5. ε_{c1} (mm/m)	2.217	2.925	3.867
Tension			
1. f_t (MPa)	3.69	4.81	5.90
2. ε_{t1} (mm/m)	0.057	0.064	0.031

Table 1 Parameters for Stress-Strain Models of Concrete

Parameter	Value
1. f_y (MPa)	435
2. E_s (GPa)	210

Table 2 Parameters for Stress-Strain Models of Steel Rebar

The obtained stress-strain curves in compression and tension for concrete are illustrated in Fig. 4. The stress-strain points were directly inputted into the program. The model incorporated the Poisson's ratio values of 0.2 for concrete and 0.3 for steel rebar.

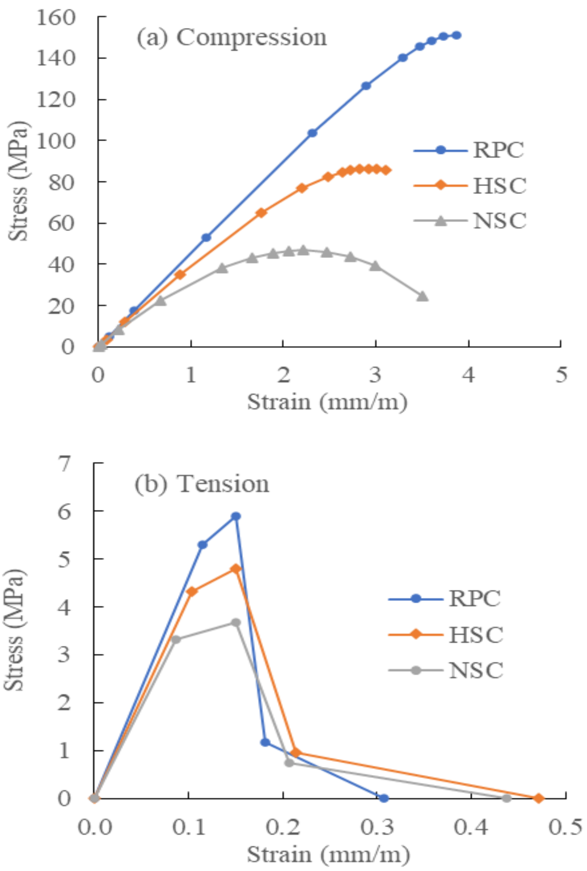


Fig. 4 Stress-strain curves for concrete using in FEA.

3.2 Structural Model

Fig. 5(a) shows the typical cross-section of deep beams tested by Chen et al. [2]. The tested deep beams had a span length of 1000 mm and were simply supported, with a point load applied at the midpoint of the span.

Fig. 5(b) depicts the typical FE model employed for the deep beams. The models utilized four-node shell or QUAD elements, which exhibit plate structural behavior based on the Reissner-Mindlin theory. The QUAD elements incorporate a layer material model to facilitate the analysis of cracked concrete. They also include discrete Kirchhoff conditions and an optional penalty term to account for shear deformation. The nonlinear analysis employed an incremental solution technique based on the modified Newton Raphson method. While the Newton-Raphson method exhibits stability convergence, it does suffer from some disadvantages. These include the computationally intensive inversion of the tangent stiffness matrix in each iteration and potential convergence issues when extreme material nonlinearities are present in a structure. For this case modified Newton-Raphson method is more effective as the current tangent stiffness matrix is replaced with a previous stiffness matrix from the beginning of the increment [8].

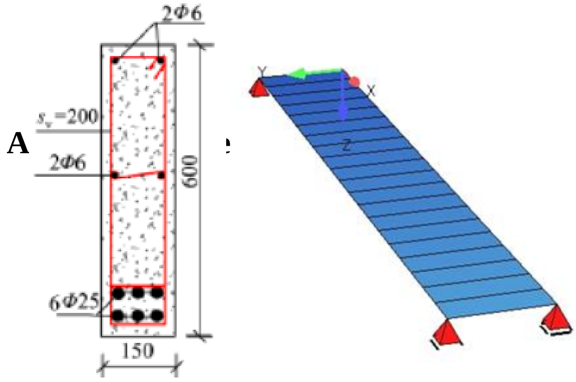


Fig. 5 (a) Typical cross section of the deep beams tested by Chen et al. [2]; (b) Typical FE model of the deep beams

4. Numerical Results

4.1 Load-Deflection Response

Fig. 6 presents the comparison between the numerical curves and the experimental results. It can be observed that the predicted peak loads closely align the test values for NSC and HSC deep beams. However, there is a significant disparity in the predicted peak load for RPC deep beam. The analysis predicts peak loads that are approximately 104% and 107% of the test values for NSC and HSC deep beam, respectively. In contrast, the analysis peak load for RPC deep beam is only about 63% of the corresponding test value. This discrepancy highlights the influence of the stress-strain models for concrete used in the analysis, emphasizing the necessity for a suitable model tailored for RPC. To conduct nonlinear analysis effectively, one must integrate nonlinear material models that accurately capture the behavior of the materials under investigation. When dealing with RPC, it becomes crucial to take into account the impact of steel fibers.

Nevertheless, it should be noted that the current FE model employed in the analysis fails to capture other important characteristics such as elastic stiffness, inelastic stiffness, and post-peak behavior for all types of deep beams. This limitation stems from the inherent characteristics of the FE model itself.

4.2 Cracking and Failure Mode

The findings from the FEA are summarized in Table 3. In the case of NSC and HSC deep beams, the first crack occurs when the load reaches about 46% to 56% of the peak load, and the main reinforcement yields shortly after reaching the peak load. On the other hand, for RPC deep beam, the first crack appears at around 38% of the peak load, and the main reinforcement yields before reaching the peak load. All beams, therefore, fail in flexure, as depicted in Fig. 7.

However, the experimental results indicated that the first crack emerged in NSC and HSC deep beams at approximately 15% to 16% of the peak load, whereas it occurred at about 19% of the peak load for RPC deep beam. Moreover, all deep beams experienced shear failure without any yielding of the main reinforcement. These also implies the effects of the stress-strain models adopted in the FE model.

The inconsistency between the test and analysis results suggests that the stress-strain relationships recommended by the *fib* model code 2010 model are not appropriate for RPC.

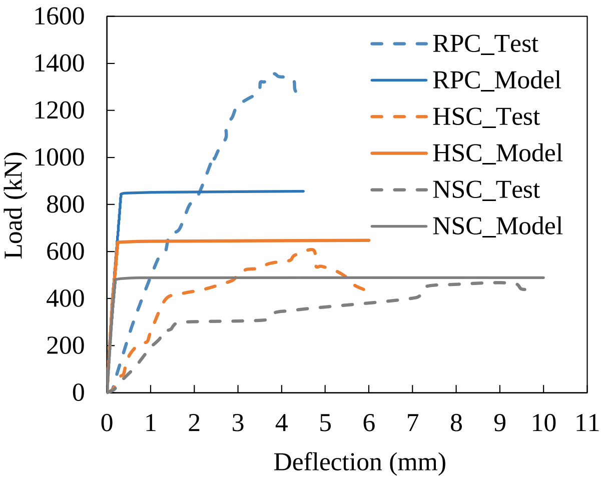
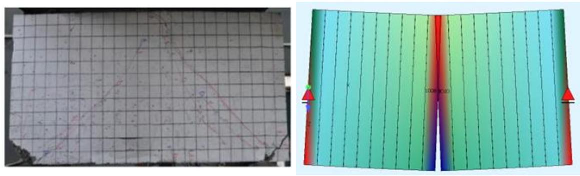


Fig. 6 Comparison of load-deflection curves

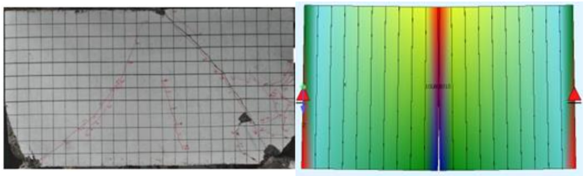
Item	Material		
	NSC	HSC	RPC
P_{cr} (kN)	278.2$^{(1)}$ (56%)	298.2$^{(1)}$ (46%)	326.2$^{(1)}$ (38%)
P_p (kN)	494.2$^{(2)}$ (100%)	654.2$^{(2)}$ (100%)	862.2$^{(3)}$ (100%)
w_p (mm)	0.02	0.14	0.07
P_y (kN)	491.5$^{(3)}$	652.8$^{(3)}$	858.2$^{(2)}$
w_y (mm)	0.24	0.18	0.06
P_{csh} (kN)	491.2$^{(4)}$	-	-
w_{csh} (mm)	0 .24	-	-
FM	F	F	F

Table 3 Parameters for Stress-Strain Models of Concrete

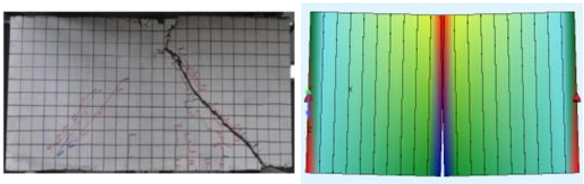
P_{cr} = load at first cracking; P_p = peak load; P_y = load at yielding of main reinforcement; P_{csh} = load at crushing of concrete; w_p, w_y, w_{csh} = crack width at peak load, yielding, crushing; FM = failure mode; F = flexural mode; $^{(1), (2), (3)}$ = sequence of events; the values in the parentheses are percentages of peak value.



(a) NSC Deep Beam



(c) HSC Deep Beam



(b) RPC Deep Beam

Fig. 7 Comparison of crack patterns

5. Conclusions

In conclusion, the validity of the stress-strain model of concrete, as per the *fib* model code 2010, was evaluated in simulating the behavior of RPC deep beam in comparison to NSC and HSC deep beams. Based on the study results, the following conclusions can be drawn:

1) The *fib* model code 2010 demonstrates the capability to accurately predict the loading capacity of NSC and HSC deep beams with an error margin of less than 7%. However, it significantly underestimates the loading capacity of RPC deep beam, providing conservative predictions.

2) The findings highlight the critical need for a suitable stress-strain model specific for RPC in order to accurately simulate the behavior RPC deep beams. The existing stress-strain model recommended by the *fib* model code 2010 is not adequate for capturing the unique characteristics and performance of RPC.

Other standard codes such as ACI 318, NF P18-710, and JSCE should also be investigated their capacity to predict the loading capacity of RPC. Moreover, to enhance the accuracy of predictions and improve the understanding of the behavior of RPC deep beams, it is essential to develop a stress-strain model that is specially tailored to RPC. This will enable more reliable simulations and facilitate the design of RPC deep beams with optimized performance and load-bearing capacity.

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Biographies

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