Comparison of the Effectiveness of Detecting Variability between Parametric and nonparametric Moving Average Control Charts

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Abstract. *Production process variability is a problem that must be resolved promptly to reduce damage and costs. An important tool in statistical quality control is often the use of control charts as a tool to track process changes because they can show the trend of changes more clearly than other tools. The use of control charts can be both parametric and nonparametric. The use of control charts has both parametric and nonparametric types, each with its own advantages and disadvantages. Therefore, this research aims to study the efficiency in detecting process variation between parametric and nonparametric moving average control charts by using sign test. Using the Monte Carlo simulation technique to gather study results, it was discovered that in every scenario examined, the parametric control chart is able to identify changes more quickly than the nonparametric chart. Moreover, the tensile test results of both carbon fiber bundles and individual fibers, which comprised the experimental dataset, agreed with the simulation outcomes.*

Keywords: Distribution free, moving average, parametric control chart, nonparametric control chart

1. Introduction

The primary driver of industrial performance is high-quality, standard-compliant products. Controlling the production process and identifying anomalies or waste in the process are therefore essential to ensuring that products satisfy consumer demands. There is always the possibility of deviations or changes in the industrial production process. Each product made using the same technique may have variations, such as different weights, thicknesses, flaws, or none at all. Natural variation is the type of variation that is impossible to identify. It is a typical variance that is concealed throughout manufacturing. variance brought on by assignable factors, such as human error, machinery, processes, or raw materials, is another kind of variance. Furthermore, the production process will ultimately still vary even if we make an effort to lessen or regulate the diversity in each component. If the causes of the variation are substantial, the resulting products will fall short of expectations. There must be a statistical technique to verify that the manufacturer is aware that the production process has deviated from the original specifications in order to regulate the quality of the output under control. The control chart is the statistical tool used to manage the production process. Setting standards for the manufacturing process, achieving objectives, and enhancing the process are the three primary purposes of control charts.

Standard control charts, sometimes referred to as control charts or standard control charts, are able to identify changes in the average value of the production process well when the process undergoes significant changes since they are founded on the Shewhart principle of defining control limits. Some people create control charts that highlight historical data because ordinary control charts (memoryless-type control charts) do not. One such example is the Cumulative Sum control chart (CUSUM chart), which was proposed by Page [1]. The Exponentially Weighted Moving Average control chart (EWMA chart) was first suggested by Roberts [2] in 1959. Small process changes can be accurately detected by both of these charts (see details Montgomery [3]). Furthermore, Khoo [4] created the Moving Average control chart (MA Chart), a control chart that effectively detects slight changes by calculating the moving average with the moving average period. This is applicable to both discrete and continuous distribution data. The double moving average control chart (DMA Chart) was created later in 2008 by Khoo and Wong [5] by taking the statistical values from the MA control chart and calculating the moving average again. They used the Monte Carlo Simulation method to show how effective the DMA chart was in comparison to the CUSUM, EWMA, and MA control charts. The DMA control chart was shown to be the most successful when the process had a small change, while the MA control chart was found to be the most effective when the process changes medium. Statistics like mean and variance could not be approximated, and parametric control charts, such as the traditional Shewhart control chart, could not be employed. Nonparametric control charts are required for processes that use data from uncertain distributions. A number of nonparametric or distribution-free control chart formats are proposed like Tukey's control chart as an effective alternative to parametric control charts [6]. Additionally, Yang et al. [7] proposed the nonparametric Cumulative Sum control chart to identify the unknown distribution processes, and Yang et al. [8] also presented the nonparametric Exponentially Weighted Moving Average control chart, also referred to as the EWMA Sign and Arcsine EWMA Sign. In addition to non-normal observations, they can be used in cases where the process distribution is unknown. Furthermore, nonparametric control charts do not react to outliers or other abnormal data. Several literatures have offered comparative investigations of the parametric and nonparametric performance of CUSUM and EWMA control charts, even though the numerical data has been analyzed to determine the process mean [9], [10], and [11]. To track process variation for both qualitative and quantitative data, the nonparametric control chart can be used (see more [12], [13]). The findings showed that, on average, nonparametric control charts outperform parametric control charts. Since the MA and DMA charts are suggested to identify variation through range and standard deviation, comparative investigations of parametric and nonparametric MA and

DMA control charts to identify process variability have not yet been examined. [14, 15]. The purpose of this study is to offer a comparative analysis of the baseline performance in detecting variability between traditional parametric and nonparametric moving average control charts. Specifically, the study evaluates the effectiveness of the nonparametric MA-sign and DMA-sign control charts relative to the parametric MA and DMA control charts in monitoring process variation.

2. Research Design

The theories related to this research are divided into four sections: Section 1 discusses data characteristics, section 2 focuses on parametric control charts, section 3 addresses nonparametric control charts, and section 4 examines the performance measurement of the charts, as follows:

2.1. Distributions

This section presents the characteristics of data with continuous distributions, including normal distribution, lognormal distribution, and Laplace distribution, with details outlined as follows:

2.1.1 Normal Distribution

When determining process capability indices such as Cp and Cpk, the process distribution is frequently modeled using the normal distribution. These indices offer a means of determining whether the process is functioning within acceptable bounds (specifications) and whether there is variation within the process in relation to the specification limits by presuming that the process data has a normal distribution. The probability density function (PDF) of a normal distribution, which describes the likelihood of a continuous random variable taking on a given value, is expressed as follows (for more details [16]):

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}\ ;\ x=0,1,2,\dots \tag{1}$$

The expectation or mean μ of a normal distribution is the average value that the variable tends to take. It is the point of symmetry for the distribution and represents the "center" of the data. In mathematical terms:

$$E(X)=\mu \tag{2}$$

where x represents a random variable that follows a normal distribution.

The variance, denoted as σ^2, quantifies the degree to which the values of a dataset are dispersed around the mean. It tells you how much the values deviate from the expected value on average as

$$V(X)=\sigma^2 . \tag{3}$$

The standard deviation is σ, and it provides a measure of how widely the values in the distribution are spread around the mean.

2.1.2 Lognormal Distribution

A lognormal distributed refer to probabilistic distribution of an unknown variable that's logarithmic value corresponds to the distribution that is normal. When X has a lognormal distribution, *Y=ln(X)* has a normal distribution, according to this. It is frequently employed to represent positively skewed variables that are required to be non-negative, such income, stock prices, or physical measures like particle sizes. The probability density function (PDF) of a lognormal random variable such as X can be expressed as follows (see in detail [17]):

$$f(x)=\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}}\ ,\ x>0 \tag{4}$$

where μ is the logarithm's midpoint and σ is its normal deviation.

The expected outcome regarding X equals

$$E(X)=\mu \tag{5}$$

The variance of X denotes

$$V(X)=2\sigma^2 . \tag{6}$$

If $Y = \ln(X)$ and $Y \sim N(\mu, \sigma^2)$, then X has lognormally dispersion given component μ with σ.

2.1.3 Laplace Distribution

A continuous probability distribution with a strong peak at the mean and exhibits larger tails than the typical distribution consists of the Laplace dispersion, sometimes referred to as the double exponential distribution. Pierre-Simon Laplace is the reason behind its name. The following formula provides the probability density function (PDF) of a Laplace-distributed randomized component X (more detailed information is provided in [18]):

$$f(x)=\frac{1}{2\sigma}e^{-\frac{|x-\mu|}{\sigma}}\ ,\ x>0 \tag{7}$$

where μ represents the location parameter, equivalent to the mean., $b > 0$ is the scale parameter (related to spread). The desired outcome regarding X can be

$$E(X)=e^{\mu+\frac{\sigma^2}{2}}. \tag{8}$$

A variation associated with X represents

$$V(X)=(e^{\sigma^2}-1)e^{2\mu+\sigma^2}. \tag{9}$$

2.2. Parametric Control Chart

The moving average control chart and the double moving average control chart are the two parametric control chart kinds that are the subject of this study. These are explained below:

2.2.1 Parametric Moving Average for Range Control Chart (PMA$_R$ chart)

A moving average control chart [1] is a time-varying control chart with unequal weights that was created to count variables like the overall amount of inconsistencies in the material's assessment unit. Furthermore, the moving average control chart was created by [2] to detect variation through range. Let us assume that the findings from studies originate from a typical distribution. When considering a time moving average, the breadth is defined as

$$PMA_{R_i}=\begin{cases}\frac{1}{i}\sum_{j=1}^{t}R_j & ,i<k\\ \frac{1}{k}\sum_{j=t-k+1}^{t}R_j & ,i\geq k\end{cases}. \tag{10}$$

The variable *I* denotes the current time step or index in the time series at which the moving average is calculated, while *k* denotes the fixed window size that determines the number of preceding observations included in the moving average when sufficient data points are available. The expectation of the PMA statistics under known parameter case when *I* is less than or greater than and equal to k depends on

$$E\left(PMA_R\right)=d_2\sigma. \tag{11}$$

Variability among the variables PMA statistics under known parameter case for both $i<k$ and $i\geq k$ are

$$Var\left(PMA_{R_i}\right)=\begin{cases}\frac{d_3^2\sigma^2}{i} & ,i\leq k\\ \frac{d_3^2\sigma^2}{k} & ,i>k\end{cases}. \tag{12}$$

The control limits of the *PMA$_R$* statistics are given as follows:

- For the case, $i<w$,

$$LCL=d_2\sigma-3\sqrt{\frac{d_3^2\sigma^2}{i}}=d_2\sigma-3d_3\sigma\sqrt{\frac{1}{i}}=D_9^*\sigma \tag{13}$$

$$UCL=d_2\sigma+3\sqrt{\frac{d_3^2\sigma^2}{i}}=d_2\sigma+3d_3\sigma\sqrt{\frac{1}{i}}=D_{10}^*\sigma \tag{14}$$

- For the case, $i\geq w$,

$$LCL=d_2\sigma-3\sqrt{\frac{d_3^2\sigma^2}{w}}=d_2\sigma-3d_3\sigma\sqrt{\frac{1}{w}}=D_{11}^*\sigma \tag{15}$$

$$UCL=d_2\sigma+3\sqrt{\frac{d_3^2\sigma^2}{w}}=d_2\sigma+3d_3\sigma\sqrt{\frac{1}{w}}=D_{12}^*\sigma \tag{16}$$

where D_9^*, D_{10}^*, D_{11}^*, D_{12}^* refers to the factor of the *PMA$_R$* chart (for more detail see [2]).

2.2.2 Parametric Double Moving Average Control Chart (PDMA$_R$ chart)

Khoo and Wong [5] suggested a parametric double-moving average control chart (PDMA chart). In order to track process variation and identify slight to moderate shifts, Phantu et al. [13] recently presented a double moving average control chart based on range. The gathered twofold moving average of the PMAR statistic is what makes up the observations of PDMAR statistics. The span's PDMAR at that moment is described as

$$PDMA_{R_i} = \begin{cases} \dfrac{PMA_{R_i} + PMA_{R_{i-1}} + PMA_{R_{i-2}} + ...}{i} &, i \le k \\ \dfrac{PMA_{R_i} + PMA_{R_{i-1}} + ... + PMA_{R_{i-w+1}}}{k} &, k < i < 2k-1 \\ \dfrac{PMA_{R_i} + PMA_{R_{i-1}} + ... + PMAR_{i-w+1}}{k} &, i \ge 2k-1 \end{cases} \quad (17)$$

where PMA_R stands for the PMA chart's statistic. A straightforward, unweighted moving average serves as the foundation for this time-weighted moving control chart. $X_1, X_2, ...$ are assumed to be drawn from a normal distribution. The PMA statistic for a certain time i period

$$PMA_{R_i} = \frac{R_i + R_{i-1} + ... + R_{i-k+1}}{k}, \ i > k.$$

There are no measurements available for the timeframe to calculate a moving average of span k. Throughout these time frames, $i \le k$, the PMA_R is defined as corresponding to the mean of all the measurements obtained before the respective periods (i). According to a $PDMA_R$ chart's in-control process, the means are

$$E(PDMA_R) = d_2\sigma \quad (18)$$

and the variance derived from a controllable procedure of the PDMAR chart are

$$Var(PDMA_R) = \begin{cases} \sum_{j=1}^{i} \dfrac{1}{j} \dfrac{d_3^2\sigma^2}{i^2} &, i \le w \\ \dfrac{d_3^2\sigma^2}{w^2} \left[\sum_{j=i-w+1}^{w-1} \dfrac{1}{j} + (i-w+1) \right] &, w < i < 2w-1. \\ \dfrac{d_3^2\sigma^2}{w^2} &, i \ge 2w-1 \end{cases} \quad (19)$$

From (5) and (6), the management of constraints of the $PDMA_R$ chart can be established as follows:

- For the case, $i \le w$,

$$LCL = d_2\sigma - 3\sqrt{\frac{d_3^2\sigma^2}{i^2}} = d_2\sigma - 3\frac{d_3\sigma}{\sqrt{i^2}} = D_{13}^*\sigma \quad (20)$$

$$UCL = d_2\sigma + 3\sqrt{\frac{d_3^2\sigma^2}{i^2}} = d_2\sigma + 3\frac{d_3\sigma}{\sqrt{i^2}} = D_{14}^*\sigma \quad (21)$$

where $D_{13}^* = \left(d_2 - 3\dfrac{d_3}{\sqrt{i^2}}\right)$ and $D_{14}^* = \left(d_2 + 3\dfrac{d_3}{\sqrt{i^2}}\right)$.

- For the case, $w < i < 2w-1$,

$$LCL = d_2\sigma - 3\frac{d_3\sigma}{\sqrt{w^2}}\sqrt{\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(i-w+1)}{i}} = D_{15}^*\sigma \quad (22)$$

$$UCL = d_2\sigma + 3\frac{d_3\sigma}{\sqrt{w^2}}\sqrt{\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(i-w+1)}{i}} = D_{16}^*\sigma \quad (23)$$

where $D_{15}^* = d_2 - 3d_3\sqrt{\dfrac{\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(i-w+1)}{i}}{w^2}}$

and $D_{16}^* = d_2 + 3d_3\sqrt{\dfrac{\sum_{j=i-w+1}^{w-1}\frac{1}{j} + \frac{(i-w+1)}{i}}{w^2}}$.

- For the case, $i \ge 2w-1$,

$$LCL = d_2\sigma - 3\sqrt{\frac{d_3^2\sigma^2}{w^2}} = d_2\sigma - 3d_3\sigma\sqrt{\frac{1}{w^2}} = D_{17}^*\sigma \tag{24}$$

$$UCL = d_2\sigma + 3\sqrt{\frac{d_3^2\sigma^2}{w^2}} = d_2\sigma + 3d_3\sigma\sqrt{\frac{1}{w^2}} = D_{18}^*\sigma \tag{25}$$

where $D_{17}^* = \left(d_2 - 3d_3\sqrt{\frac{1}{w^2}}\right)$, and $D_{18}^* = \left(d_2 + 3d_3\sqrt{\frac{1}{w^2}}\right)$

where D_{13}^*, D_{14}^*, D_{15}^*, D_{16}^*, D_{17}^*, D_{18}^* refers to the factor of the $DPMA_R$ chart (for more detail see [13]). The PDMA chart will signal the out-of-control situation when $PDMA_{R_i} < LCL$ or $PDMA_{R_i} > UCL.$

2.3. Nonparametric Control Chart

The sign moving average control chart and the sign double moving average control chart are examples of nonparametric control charts. The following summarizes the process for identifying the signs:

Suppose $X_{jt}, j = 1,2,...,n$ and $t = 1,2,3,...$, denote the t^{th} observation in the j^{th} logical subgroup of size n.

The discrepancy within the findings and the final goal number $X_{jt} - T,$ or among groups, can be represented by (7) as follows if the known target value $T,$ is being tracked:

$$Y_{jt} = X_{jt} - T \text{ , } t = 1,2,3,..., j = 1,2,...,n . \tag{26}$$

The sign statistic S_t can be defined as (8):

$$S_t = \sum_{j=1}^{n} I_{jt} . \tag{27}$$

Equation (8), I_{jt} can be elaborated as (9):

$$I_{jt} = \begin{cases} 1, \ Y_{jt} > 0 \\ 0, \ otherwise \end{cases}. \tag{28}$$

The sign statistic is defined as the total count of observations that adhere to a binomial distribution characterized by the parameter $(n, p_0 = 0.5)$. The value of $p = P(Y > 0)$ is the process proportion which $p = p_0 = P(Y \le T) = P(Y > T) = 0.5$ is in the control process. On the other hand, the process is out of control when $q_0 \ne 0.5.$ Using the method for identifying the specified sign, it is possible to construct and present the moving average control chart for sign, as well as the double moving average control chart for sign, with the following detailed steps and representations.

2.3.1 Nonparametric Moving Average Control Chart (NPMA chart)

Assume that discrete observations are obtained from sign statistics in Eq. (29). The length at the point in the moving average can be described as:

$$NPMA_t = \begin{cases} \frac{1}{i}\sum_{j=1}^{t} S_j & , i < k \\ \frac{1}{k}\sum_{j=t-k+1}^{t} S_j & , i \ge k \end{cases}. \tag{29}$$

The MA characteristics of sign averages when $i < k$ and $i \ge k$ is

$$E(NPMA_t) = \frac{n}{2}. \tag{30}$$

The variance of the NPMA statistics for the two instances of $i < k$ and $i \ge k$ is

$$Var(NPMA_i) = \begin{cases} \frac{n}{4i} & ,i \le k \\ \frac{n}{4k} & ,i > k \end{cases}. \tag{31}$$

The control system restricts of the NPMA measurements are given as follows.

$$= \begin{cases} \frac{n}{2} \pm H_1\sqrt{\frac{n}{4i}} & ,i \le k \\ \frac{n}{2} \pm H_1\sqrt{\frac{n}{4k}} & ,i > k \end{cases} \tag{32}$$

where H_1 includes to a measurement for the coefficient for controlling restriction of the NPMA chart.

2.3.2 Nonparametric Double Moving Average Control Chart (NPDMA chart)

Assessments about NPDMA measurements are the obtained double-moving average of the NPMA statistic. The NPDMA of span k at the time i is defined as

$$NPDMA_i = \begin{cases} \dfrac{NPMA_i + NPMA_{i-1} + ...}{i} & , i \le k \\ \dfrac{NPMA_i + ... + NPMA_{i-w+1}}{k} & , k < i < 2k-1 \\ \dfrac{NPMA_i + ... + NPMA_{i-w+1}}{k} & , i \ge 2k-1 \end{cases} \tag{33}$$

where $NPMA_i$ indicates for the measurement of the NPMA chart. That represents a time-weighted running chart with control based on an elementary unweighted codes average of movements. Assume $NPMA_1, NPMA_2, ...$ are obtained from normal distribution. The NPMA statistic of span k at a time i

$$NPMA_i = \frac{S_i + S_{i-1} + ... + S_{i-k+1}}{k}, \text{ for } i > k. \tag{34}$$

During the period $i < k$, there are not enough k measurements available to calculate a moving average with a span of k. Regarding these time frames, the NPMA is defined as the average of all available measurements up to period i. The mean derived from an management procedure for the NPDMA chart is:

$$E(NPDMA_i) = \frac{n}{2} \tag{35}$$

and variance of the NPDMA statistics are

$$Var(NPDMA_i) = \begin{cases} \displaystyle\sum_{j=1}^{i} \frac{1}{j} \frac{n}{4i^2} & ; i \le k \\ \displaystyle\sum_{j=i-k+1}^{k-1} \frac{1}{j} + (j-k+1)\left(\frac{1}{k}\right)\frac{n}{4k^2} & ; k < i < 2k-1 \\ \dfrac{n}{4k^2} & ; i \ge 2k-1. \end{cases} \tag{36}$$

From Eq. (15) and (16), the control limits of the NPDMA chart can be established as follows:

$$= \begin{cases} \dfrac{n}{2} \pm H_2 \sqrt{\displaystyle\sum_{j=1}^{i} \frac{1}{j} \frac{n}{4i^2}} & , i \le k \\ \dfrac{n}{2} \pm H_2 \sqrt{\displaystyle\sum_{j=i-k+1}^{k-1} \frac{1}{j} + (j-k+1)\left(\frac{1}{k}\right)\frac{n}{4k^2}} & , k < i < 2k-1 \\ \dfrac{n}{2} \pm H_2 \sqrt{\dfrac{n}{4k^2}} & , i \ge 2k-1 \end{cases} \tag{37}$$

where H_2 serves as the control limit coefficient, calculated based on the desired in-control Average Run Length (ARL_0). The NPDMA chart signals an out-of-control situation when $NPDMA_i < LCL$ or $NPDMA_i > UCL$ breaches the established control limits.

2.4 The Performance of Control Chart

Average Run Length (ARL) is the expected number of observations before a control chart signals out-of-control. Phase I (ARL_0) measures this in an in-control state, while Phase II (ARL_1) reflects detection in an out-of-control state. The ARL can be determined as follows:

$$ARL = \sum_{i=1}^{T} RL_i \Big/ T. \tag{38}$$

In this scenario, RL_i represents the sample being inspected before the process exceeds the control limits for the first time. T, which is set to 100,000, is the number of experiment repetitions in the simulation's round i.

The criteria for selecting control charts based on the Average Run Length (ARL) in practical applications include the following:

1) Detection sensitivity**:** The ability of the control chart to detect small shifts in the process mean or variation, with a focus on minimizing the ARL for detecting such shifts.

2) Robustness**:** The control chart's ability to maintain an acceptable ARL even under different operational conditions or in the presence of noise and outliers.

3) Ease of Implementation**:** The complexity of the control chart, including data requirements and computational demands, should be considered in practical settings. A chart that requires less data manipulation or complex calculations may be preferred.

3. Analytical Results and Utilization

The results of this research can be divided into two parts, as follows:

3.1 Performance of Control Chart

The purpose of this study is to compare the performance of non-parameterized control charts, such as the moving average control chart for markers (NPMA) and the double moving average control chart for markers (NPDMA), with parameterized control charts, such as the PDA and PMA. The effectiveness of these charts is examined in the study under three distinct data distributions: the Laplace distribution (2,1), the lognormal distribution (0,1), and the normal distribution (0,1). The Average Run Length (ARL_0) is set to 200 and 500 when the process is under control. The ARL_0 is used to evaluate the performance of the control charts when no shift has occurred in the process. A shift in the process is defined by the change in the mean (μ) of the distribution, represented by δ, where δ is specified as: $\sigma_1 = \delta\sigma_0$. Here, δ varies across multiple values: 1.025, 1.05, 1.075, 1.1, 1.2, 1.3, 1.5, 1.75, and 2. This allows the study to analyze how the control charts perform under different shifts in the process. The moving average parameter for the control charts is set to a window size (*k*) of 5, meaning the charts will average over the past five data points. The sample size (*n*) is fixed at 5, which refers to the number of individual observations in each sample taken during the study.

The ability of the various control charts to identify process changes and their resilience to diverse data distributions and shift magnitudes will be compared in order to assess their effectiveness. The usefulness of each chart in various real-world quality control settings will be revealed by this thorough comparison.

The results of the comparison of the performance of the control charts can be presented in terms of ARL (Average Run Length) and EARL (Expected Average Run Length) as follows:

When ARL_0 is set to 200, Table 1 displays control charts for the conventional normal distribution (0,1). Based on the research findings, the parameterized double moving average control chart (PDMA) is the most effective tool for spotting process changes that result in an increase in the mean. Afterwards, when ARL_0 is set to 500, Table 2 displays the outcomes of the comparison of control charts for the standard normal distribution (0,1). The results of the study demonstrate that the parameterized double moving average control chart (PDMA) continues to be the most successful in identifying changes when there is a change in the process that raises the mean, as shown in Table 1.

Table 1 Comparative ARL_1 of parametric and nonparametric charts when ARL_0=200, *k*=5, *n*=5 for Normal (0,1)

δ	Parametric		Nonparametric-Sign	
	MA	DMA	MA	DMA
0	200.01	200.60	200.85	201.24
1.025	158.02	**112.36**	198.91	197.4
1.05	105.97	**49.91**	187.38	165.08
1.075	68.25	**25.35**	163.02	106.24
1.1	44.81	**15.55**	92.13	72.21
1.2	12.68	**7.13**	28.43	26.97
1.3	6.33	**5.55**	14.89	16.23
1.5	3.48	**3.54**	5.16	7.78
1.75	2.50	**2.32**	3.39	4.66
2	1.26	**1.22**	2.44	2.99
EARL	44.81	**24.77**	77.31	66.62

The bold value indicates the minimum of ARL and EARL.

Table 2 Comparative ARL_1 of parametric and nonparametric charts when ARL_0=500, *k*=5, *n*=5 for Normal (0,1)

δ	Parametric		Nonparametric-Sign	
	MA	DMA	MA	DMA
0	500.45	500.03	503.58	497.25
1.025	374.93	**358.40**	472.9	476.72
1.05	232.3	**233.19**	342.24	366.88
1.075	138.40	**147.53**	312.39	222.49
1.1	114.75	**94.46**	185.39	181.72
1.2	28.44	**22.22**	57.58	50.9
1.3	11.23	**8.79**	24.93	21.53
1.5	4.16	**3.52**	7.49	8.38
1.75	2.48	**2.26**	3.44	5.24
2	1.31	**1.81**	2.59	3.61
EARL	100.89	**96.91**	156.55	148.61

The bold value indicates the minimum of ARL and EARL.

ARL_0 is set to 200 in Table 3, which compares control charts for the lognormal distribution (0,1). According to research findings, the PDMA control chart is the most effective tool for identifying process changes when the mean moves from 0.025 to 0.2. However, it is discovered that the PMA control chart is the most successful in identifying changes when the process has a shift in the mean from 0.3 onwards. ARL_0 is then set to 500, and control charts for the lognormal distribution (0,1) are compared in Table 4. The results of the study show that the PDMA control chart is very good at identifying these slow changes when the process undergoes a mean change around 0.025 to 0.3. It is especially appropriate for this range because of its sensitivity to slight to moderate changes in the process mean. As opposed to this, the PMA control chart works best when the process mean changes more significantly, surpassing 0.5. Larger shifts are best detected by this chart, which enables quick identification and reaction to significant variations in the process mean.

Table 3 Comparative ARL_1 of parametric and nonparametric charts when ARL_0=200, k=5, n=5 for Lognormal (0,1)

δ	Parametric		Nonparametric-Sign	
	MA	DMA	MA	DMA
0	200.03	200.03	199.59	202.87
1.025	157.60	**112.06**	150.31	143.21
1.05	105.72	**49.80**	115.16	107.87
1.075	68.10	**25.31**	90.03	84.56
1.1	44.72	**15.53**	59.66	57.54
1.2	12.66	**7.13**	24.05	20.84
1.3	**6.33**	5.55	10.12	11.65
1.5	**3.48**	3.54	4.14	8.00
1.75	**2.50**	2.32	4.00	8.00
2	**1.99**	1.80	4.00	8.00
EARL	44.79	**24.78**	51.27	49.96

The bold value indicates the minimum of ARL and EARL.

The comparison of control charts for the lognormal distribution (0,1) with ARL_0 = 200 is displayed in Table 5. The findings indicate that the PMA control chart is superior at spotting greater mean changes (above 1.5), whereas the PDMA control chart is best at detecting small to moderate mean changes (1.025 to 1.3).

Control charts for the lognormal distribution (0,1) with ARL_0 equal to 500 are compared in Table 6. The findings show that the parameterized double moving average control chart (PDMA) is the best tool for detecting changes in processes when they result in an increase in the mean.

According to Table 7, the PDMA control chart is the best at identifying changes and typically provides the shortest expectation of average run length (EARL). In particular, it functions best under the normal distribution when ARL_0 is 500 and under the lognormal and Laplace distributions for all ARL_0 values. When ARL_0 is 200, the PMA chart performs best with a normal distribution.

Table 4 Comparative ARL_1 of parametric and nonparametric charts when ARL_0=500, k=5, n=5 for Lognormal (0,1)

δ	Parametric		Nonparametric-Sign	
	MA	DMA	MA	DMA
0	500.451	500.451	500.31	498.19
1.025	374.932	**250.53**	447.52	339.15
1.05	232.30	**95.98**	401.57	275.81
1.075	138.40	**42.22**	329.74	215.44
1.1	84.67	**22.68**	205.21	163.32
1.2	19.04	**7.97**	77.74	48.25
1.3	8.12	**6.03**	25.17	17.73
1.5	**3.92**	3.90	5.29	8.12
1.75	**2.72**	2.52	4.00	8.00
2	**2.14**	1.93	4.00	8.00
EARL	96.25	**48.20**	166.69	120.42

The bold value indicates the minimum of ARL and EARL.

Table 5 Comparative ARL_1 of parametric and nonparametric charts when ARL_0=200, k=5, n=5 for Laplace (2,1)

δ	Parametric		Nonparametric-Sign	
	MA	DMA	MA	DMA
0	200.03	200.03	201.53	202.86
1.025	158.02	**112.36**	189.89	171.68
1.05	105.97	**49.91**	147.65	113.17
1.075	68.25	**25.35**	116.61	92.76
1.1	44.81	**15.55**	95.44	61.21
1.2	12.68	**7.13**	26.65	22.12
1.3	6.33	**5.55**	13.72	12.29
1.5	**3.48**	3.54	5.25	7.21
1.75	**2.50**	2.32	3.15	4.69
2	**2.00**	1.80	2.47	3.89
EARL	44.89	**24.83**	66.76	54.34

The bold value indicates the minimum of ARL and EARL.

Table 6 Comparative ARL_1 of parametric and nonparametric charts when ARL_0=500, k=5, n=5 for Laplace(2,1)

δ	Parametric		Nonparametric-Sign	
	MA	DMA	MA	DMA
0	500.02	500.02	503.05	502.82
1.025	452.12	**250.53**	481.61	476.78
1.05	342.10	**99.99**	360.07	349.22
1.075	206.54	**42.22**	243.07	210.49
1.1	127.88	**22.68**	147.6	131.82
1.2	40.27	**7.97**	55.32	41.76
1.3	16.86	**6.03**	19.35	15.37
1.5	6.98	**3.90**	7.11	8.62
1.75	3.43	**2.52**	3.94	6.57
2	2.56	**1.93**	2.93	4.77
EARL	133.19	**48.64**	146.78	138.38

The bold value indicates the minimum of ARL and EARL.

Table 7 EARL of control charts

Dist.	ARL_0	Parametric		Nonparametric-Sign	
		PMA	PDMA	NPMA	NPDMA
N(0,1)	200	44.81	**24.77**	77.31	66.62
	500	100.89	**96.91**	156.55	148.61
LN(0,1)	200	44.79	**24.78**	51.27	49.96
	500	96.25	**48.20**	166.69	120.42
Laplace (2,1)	200	44.89	**24.83**	66.76	54.34
	500	133.19	**48.64**	146.78	138.38

The bold value indicates the minimum of EARL.

3.2 Research Utilization

Tensile measurements for carbon fiber bundles and single carbon fibers (-1,000) made up the dataset. A length-measuring device was used to evaluate carbon fiber bundles (-1,000) at 20, 50, 150, and 200 mm and single carbon fibers at 1, 10, 20, and 50 mm under stress [19]. As seen in Figure 1, this study concentrated on 69 data sets of measurements of one carbon fiber at 20 mm. The following is a summary of the findings from the assessment of parametric and nonparametric control charts' effectiveness:

The performance evaluation of the PMA control chart is shown in Figure 2, which shows that it cannot identify changes in the data mean.

Figure 3 illustrates the performance evaluation of the PDMA control chart, showing that the PDMA control chart successfully detects changes in the mean of data points 2, 3, 4, 5, 6, 7, 67, 68, and 69.

Figure 4 shows that the NPMA control chart is ineffective in detecting changes in the mean of the data, highlighting its limitations in identifying shifts in the process mean.

Figure 5 demonstrates that the NPDMA control chart is not effective in detecting changes in the mean of the data, underscoring its limitations in monitoring process variations.

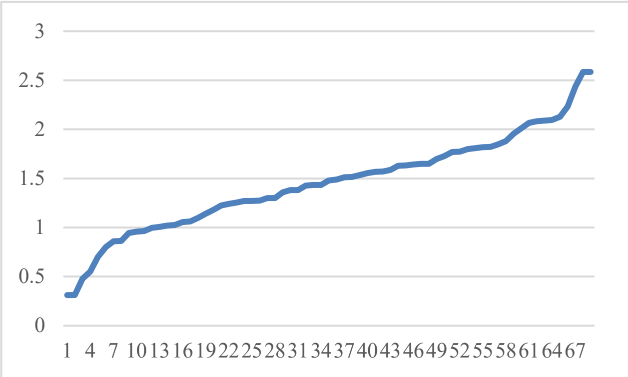

Figure 1. Data set

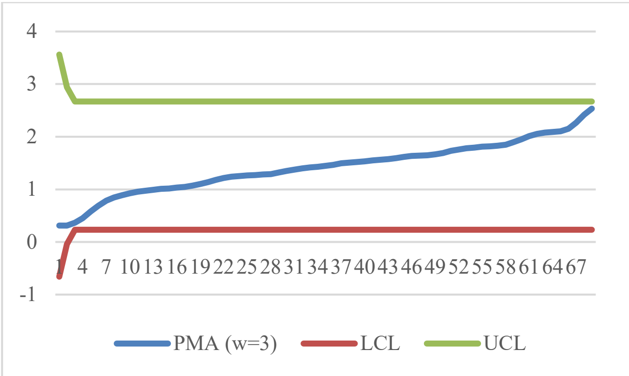

Figure 2. PMA chart

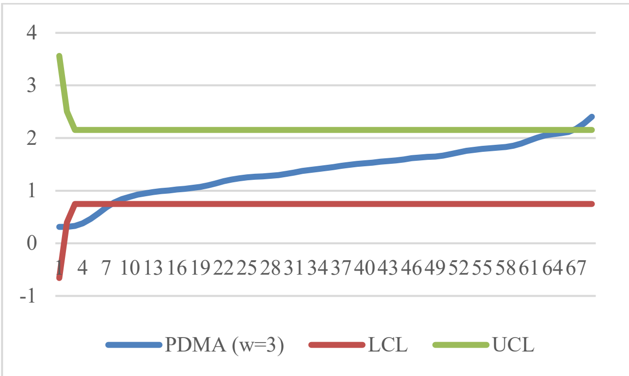

Figure 3. PDMA chart

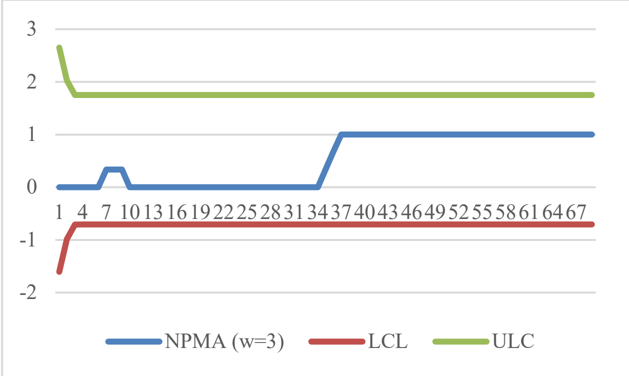

Figure 4. NPMA chart

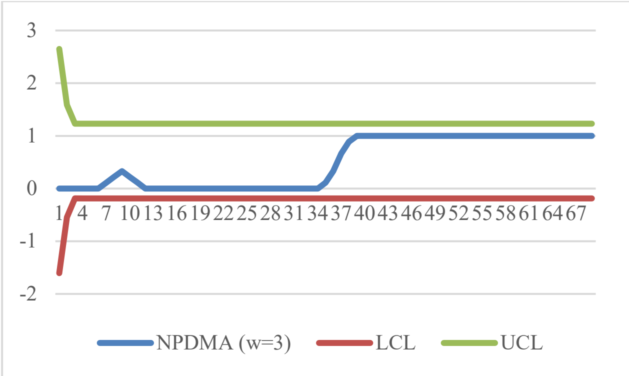

Figure 5. NPDMA chart

4. Discussion

The performance of parametric and nonparametric control charts utilizing single and double moving averages is the main emphasis of this study. Data from three different distributions are evaluated in the study: Laplace (2,1), Lognormal (0,1), and Normal (0,1). Assuming statistical control, the process's starting Average Run Lengths (ARL_0) are set at 200 and 500. Both the subgroup size and the moving average length (*k*) are fixed at 5.

The analysis reveals that when there is a shift in the process mean, parametric control charts outperform nonparametric control charts in detecting the change. Specifically, the parametric charts demonstrate superior sensitivity to mean shifts, which is critical in maintaining process quality.

In addition to simulated data, the study incorporates real-world data to validate these findings. The analysis of actual data confirms that parametric control charts consistently exhibit the highest efficiency in identifying process mean shifts compared to their nonparametric counterparts, which underscores the robustness of parametric methods in various industrial applications, such as aerospace manufacturing, semiconductor production, pharmaceutical quality assurance, and chemical processing, where precise detection of process deviations is crucial for maintaining product integrity, safety, and operational efficiency.

The highlights of this research are that parametric moving average control charts generally outperform nonparametric counterparts in detecting process variability and mean shifts when data follow a known distribution. To apply these findings effectively, organizations should select parametric charts for stable, normally distributed processes and opt for nonparametric methods when data are non-normal or unpredictable. A preliminary assessment of data characteristics is essential before choosing the chart type. These insights are broadly applicable across various industries, including manufacturing, healthcare, energy, and logistics, where the accurate and timely detection of process changes is crucial for quality control, operational efficiency, and risk mitigation.

Several limitations of research exist, which assume known data distributions for parametric charts, potentially failing **to** reflect real-world variability. The scope of the process may not cover complex or abrupt changes that are tested. The study also considers a limited set of variable types or measurement scales, which may constrain the applicability of results to broader or more heterogeneous datasets. While simulations offer controlled conditions, they may not fully capture the complexity of real industrial processes. Additionally, limited case studies may limit generalizability across different industries, and fixed chart parameters may not be suitable for all applications. Lastly, the evaluation primarily focuses on detection performance, with less emphasis on false alarms, ease of use, and implementation costs, which are important in practical settings.

Finally, this study uses Average Run Length (ARL) to evaluate control chart performance, acknowledging its limitations in fully capturing practical performance, especially in complex or noisy environments. While parametric charts are preferred in stable, well-understood data distributions, such as high-precision manufacturing, nonparametric charts offer greater robustness in industries with skewed or noisy data, like healthcare and food processing. In sectors where false alarms are costly, such as pharmaceuticals and aerospace, charts with higher ARL under stable conditions may be more appropriate. The study suggests that future research should develop a decision-support framework to guide chart selection based on process characteristics and risk tolerance, incorporating additional metrics such as Type I and Type II error rates, Median Run Length (MRL), and robustness analysis to provide a more comprehensive and practical evaluation.

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References

[1] E. S. Page, "Continuous inspection schemes," *Biometrika*, vol. 42, pp. 243-54, 1954.

[2] S. W. Roberts, "Control chart tests based on geometric moving average," *Technometrics*. vol. 42, pp. 239-250, 1959.

[3] D. C. Montgomery, Introduction to Statistical Quality Control, 6th ed., New York: John Wiley & Sons. 2009**.**

[4] B. C. Khoo, "A moving average control chart for monitoring the fraction non-conforming," *Quality and Reliability Engineering International*, vol. 20, pp. 617-635, 2004. DOI: 10.1002/qre.576.

[5] B. C. Khoo and V. H. Wong, "A double moving average control chart," *Communications in Statistics - Simulation and Computation*, vol. 37, no. 8, pp. 1696-1708, 2007, DOI: 10.1080/03610910701832459.

[6] F. Alemi, "Tukey's control chart," *Quality Management in Health Care*, vol. 13, no. 4, pp. 216-221, 2004.

[7] S. F. Yang, and S.W. Cheng, "A new nonparametric CUSUM mean chart," *Quality and Reliability Engineering International,* vol. 27, no. 6, pp. 867–875, 2011. DOI: 10.1002/qre.1171.

[8] S. F. Yang, J. S. Lin, and S. W. Cheng, "A new nonparametric EWMA sign control chart," *Expert Systems with Applications*, vol. 38, no. 5, pp. 6239-6243, 2011, DOI: 10.1016/j.eswa.2010.11.044

[9] S. Sukparungsee, "A comparison of performance of CUSUM and EWMA with parametric and nonparametric for exponential data," *Far East Journal of Mathematical Sciences*, vol. 95, no. 2, pp. 249–258, 2014.

[10] S. Y. Li, L. C. Tang, and S. H. Ng, "Nonparametric CUSUM and EWMA Control Charts for Detecting Mean Shifts," *Journal of Quality Technology*, vol. 42, no. 2, pp. 209–226, 2010, DOI: 10.1080/00224065.2010.11917817.

[11] Y. Areepong, and S. Sukparungsee, "Novel of statistical quality control development and econometric applications," *Asian Journal of Economics and Banking*, 2024, DOI: 10.1108/AJEB-12-2023-0139.

[12] Z. Wang, Q. Wu, P. Qiu, "Novel nonparametric control charts for monitoring dispersion of count data," *Quality and Reliability Engineering International,* vol. 40, no. 4, pp. 1636-1651, 2024, DOI: 10.1002/qre.3479.

[13] P. Suganya, A. Yupaporn and S. Sukparungsee, "Sensitivity of the Modified Exponentially Weighted Moving Average Sign-Rank Control Chart to Process Changes in Counted Data," *WSEAS Transactions on Mathematics*, vol.23, pp.331-339, 2024, DOI: 10.37394/23206.2024.23.36.

[14] C. Chananet, Y. Areepong, S. Sukparungsee, "On design a moving average-range control chart for enhancing a process variation detection," *Applied Science and Engineering Progress,* vol. 17, no. 1, pp. 1-15, Jan.-Mar. 2024, DOI: 10.14416/j.asep.2023.06.001.

[15] S. Phantu, C. Chananet, Y. Areepong, S. Sukparungsee, "Development of DMA control charts within range to enhance detection of changes in process variation," *Engineering Letters*, vol. 32, no. 7, pp.1465-1475, 2024.

[16] A. Papoulis, S. U. Pillai, *Probability, random variables, and stochastic processes*, 4th ed., McGraw-Hill., 2002.

[17] J. Aitchison, J. A. C. Brown, *The lognormal distribution*, Cambridge University Press, 1957.

[18] S. Kotz, T. J. Kozubowski, K. Podgórski, *The Laplace distribution and generalizations: A revisit with applications to communications, economics, engineering, and finance*, Birkhäuser, 2001.

[19] M.G. Badar and A.M. Priest, Statistical aspects of fiber and bundle strength in hybrid composites, Progress in Science and Engineering Composites, ICCM-IV, Japan, 1982.