ประมวลองค์ความรู้การรวมข้อมูลแบบฟัซซี

A Brief Review of Fuzzy Aggregation

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บทคัดย่อ

หลักการและการประยุกต์ใช้การรวมข้อมูลแบบฟัซซีได้รับการยอมรับในช่วงหลายสิบปีที่ผ่านมา ครอบคลุมการใช้งานควบคุมระบบการตัดสินตกลงใจ และการเรียนรู้ของเครื่องตราบจนปัจจุบัน การพัฒนาเชิงทฤษฎียังคงมีมาอย่างต่อเนื่อง เช่น โมเดลการรวมข้อมูลด้วยตัวดำเนินการโอดับบลิวเอ รวมถึงการประยุกต์ใช้งานกับปัญหาอื่น ๆ จากเหตุผลที่กล่าวมา การประมวลความรู้ตามหลักการนี้จึงมีความสำคัญต่อการสรุปกรอบการวิจัยที่ผ่านมา และทิศทางการพัฒนาในห้วงต่อไป อีกทั้งจะเป็นประโยชน์ต่อทั้งการศึกษาวิทยาการพื้นฐานและการนำไปแก้ไขปัญหาในงานวิจัย

คำสำคัญ : ฟัซซี, การรวมข้อมูล, การตัดสินตกลงใจ

Abstract

The concept and applications of fuzzy aggregation have been witnessed over the past decades, spanning from system control, decision-making as well as machine learning. Even now, the theoretical development of several models like OWA still continues, with further exploitation in many new problem domains. Given this insight, it is important to provide a review of landscape for fuzzy aggregation, with respect to both types and future challenges. The paper is to be useful for those who are interested in this subject in general, and others that are keen to employ a fuzzy aggregator in their research studies.

Keywords: fuzzy, aggregation, decision making

1. Introduction

Aggregation denotes the integration process of values, specified by numeric or non-numeric terms, such that a group representative outcome takes into account all the individual values [36]. It increasingly involves in a current digital society where effective data integration tools are required to handle ever more data being exchanged and stored at inclining rates. To improve data quality and summarization, new and existing techniques for information fusion and aggregation operation have to comply with such challenges [69].

Due to the continuous success with fuzzy set theory over the past 30 years, fuzzy oriented techniques have been incorporated into the main stream of research on aggregation operators [6][14] [70]. Fuzzy approaches to aggregation provide several advantages as there are numerous ways of combining fuzzy sets in addition to union (maximum) and intersection (minimum). Moreover, fuzzy set intuition allows for modeling imprecision appropriately and later permits reasoning in imprecise terms [32].

2. Triangular norms and conorms as aggregation tools

At the outset, aggregation was inherently studied in terms of fuzzy logical connectives, which are appropriate extensions of logical connectives AND and OR in the case when the valuation set is the unit interval [0, 1] rather than {0, 1}. Fuzzy connectives modeling AND and OR are called triangular norms (t-norms for short) and triangular conorms (t-conorms), respectively [3][20][42][45][63]. Bonissone [9] investigated the properties of these operators with the goal of using them in the development of intelligent systems. Good overviews and classifications of these operators can be found in [21][47][48].

Definition1. A t-norm is a function $T : [0, 1] \times [0, 1] \longrightarrow [0, 1]$ with the following properties:

- Commutativity, $T(x, y) = T(y, x)$
- Monotonicity, $T(x, y) \leq T(u, v)$ if $x \leq u$ and $y \leq v$
- Associativity, $T(x, T(y, z)) = T(T(x, y), z)$
- One is a neutral element, $T(x, 1) = x$
- $T(x, y) \leq \min(x, y)$

Definition2. A t-conorm is a function $S : [0, 1] \times [0, 1] \longrightarrow [0, 1]$ with the following properties:

- Commutativity, $S(x, y) = S(y, x)$
- Monotonicity, $S(x, y) \leq S(u, v)$ if $x \leq u$ and $y \leq v$
- Associativity, $S(x, S(y, z)) = S(S(x, y), z)$
- Zero is a neutral element, $S(x, 0) = x$
- $S(x, y) \geq \max(x, y)$

Despite their notable roles in fuzzy logic domain, t-norm and t-conorm do not admit a compensating behavior [91]. Accordingly, the family of uniform aggregation operators (uninorm) was introduced as a generalization of both t-norm and t-conorm [27][90]. This operator has a neutral element lying anywhere in the unit interval rather than at one or zero as for the t-norms and t-conorms, respectively.

Definition3. A uninorm is a function $U : [0, 1] \times [0, 1] \longrightarrow [0, 1]$ with the following properties:

- Commutativity, $U(x, y) = U(y, x)$
- Monotonicity, $U(x, y) \leq U(u, v)$ if $x \leq u$ and $y \leq v$
- Associativity, $U(x, U(y, z)) = U(U(x, y), z)$
- Neutral element e, $\exists e \in [0,1], \forall x \in [0, 1], U(x, e) = x$

Uninorms are frequently used in fuzzy systems modeling [92], such as MYCIN's aggregation operator [19][72]. Specifically, Beliakov, Pradera and Calvo extensively emphasized neural elements and absorbent behavior upon a variety of aggregation operators [7] [8]. In addition to aforementioned techniques, several other aggregation operators have been similarly developed to obtain a compromise between two extremes of t-norms and t-conorms: for instance, nullnorms or t-operators [13][54][55], averaging operators [21][33], γ-operators [95], exponential compensatory operators [73], associative compensatory operator [46] and convex-linear compensatory operators [52][73].

3. Fuzzy integrals as aggregation tools

As mentioned by the end of Section 2 that the generation of cascaded classifier requires a set of training samples. This is composed of Next to fuzzy logical connectives, Choquet [17] and Sugeno integrals [66], as the most representative of fuzzy integral, have been broadly used as aggregation tools [33] in many diverse domains such as subjective evaluation [12][38] [49][74], pattern classification [37][44], image processing [35][44], information fusion [4][30][76], and regression analysis [75].

Contrary to the weighted arithmetic means, fuzzy integrals are able to represent a certain kind of interaction between criteria, ranging from redundancy (negative interaction) to synergy (positive interaction). For this reason they have been thoroughly studied in the context of multi-criteria decision problems [34] [58]. Extensive details of their mathematical properties as aggregation functions can be found in [24][32][53] [59].

Essentially, many classical aggregation operators are particular cases of these so-called fuzzy integrals [14], [69], for instance, the weighted arithmetic mean, ordered weighted averages (OWA) [84], weighted minimum and maximum [22], [23]. In particular, links between OWA operators and fuzzy integrals were investigated in [31], [43].

Difnition4. An OWA operator of dimension n is a mapping $\varphi : R^n \longrightarrow R$ with an associated weight vector $w = (w_1, . . . , w_n)^T$, where $w_i \in [0, 1]$ and $w_1 + . . . + w_n = 1$. Given $x = (x_1, . . . , x_n) \in R^n$, φ is defined as follows:

$$\varphi(x) = \varphi(x_1, \ldots, x_n) = \sum_{i=1}^{n} w_i x_{\sigma(i)}$$

where σ is a permutation function that orders the elements such that $x_{\sigma(i)} \geq x_{\sigma(i+1)}, \forall i = 1, . . . , n - 1$.

Since their introduction in 1988, OWA operators have been applied to many fields as neural networks [82][86], data base systems [83][87], fuzzy logic controllers [25][85], market analysis [88], image compression [56], query system [50][78], service quality evaluation [16], feature selection [10], and decision making [11][18][84]. The OWA operators can also be used in decision-making under uncertainty [61] [64].

Along with their applications to diverse problems, many authors have concentrated on different algorithms for determining the weight vector: maximum- entropy [28][60], quantifier guided method [89], Gaussian distribution [79][81], centering functions [93], cluster-based reliability measure [10], recursive formulation [71], weight learning [26], minimum variance [28], majority rule [51], and minimax disparity [77]. Furthermore, the weighted OWA (WOWA) operator was proposed in [67] with combined advantages of both the OWA operator and the weighted mean, and the dynamic fuzzy OWA model is introduced in [15] for multi-criteria decision making with fuzzy and incomplete information.

Some extensions of OWA operators have been developed to aggregate linguistic information, especially in group decision making, such as linguistic OWA (LOWA) operator [11][39], induced LOWA (I-LOWA) operator [57], induced uncertain LOWA (IULOWA) operator [80], two-tuple OWA (TOWA) operator [40][41] and expanded TOWA (ETOWA) operator [94].

4. Research challenges

The primary challenge in fuzzy aggregation is the selection of an appropriate aggregation operator. This troublesome process can be intuitively achieved through matching the aggregation behavior required for a specific problem with properties of operators [1][2]. In addition, a well-specified classification of operators can also ease such barrier according to the assumption that an inadequate operator can be replaced by more generalized ones within the same family [69]. Hence, a methodology or software should be developed for such task [5].

Another crucial burden arises with parameterized aggregation functions, such as fuzzy integrals and OWA operators. A bad selection of parameters implies a bad performance. Traditionally, parameter values are dictated by experts' knowledge, for instance, in the Analytic Hierarchical Process [62] and as the orness for OWA operators [84]. However, quality of such knowledge is greatly subjected to communication, personal bias, experience, physical and emotional status. In contrary, the supervised learning approach [96] was thus taken by many authors [26][68], to extract parameter settings from training examples. It is challenging to explore the possibility with other learning methods like unsupervised (see [65] for example) and reinforcement.

5. References

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