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Introducing Climb-Cruise

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Abstract

A climb-cruise is a straight climb with objectives to gain altitude, as well as ground distance. Climb-cruise performance is compared with traditional flight profile consisting of a separate climb and a cruise between the same starting and end points. The flight mechanic analysis of climb-cruise is presented. Examples of performance comparisons between climb-cruise and traditional climb and cruise for 3 example aircraft types are shown. Minimum fuel climb-cruise is the focus of this study, therefore positive fuel saving and time saving are presented to show that climb-cruise is superior over traditional climb and cruise profile.

Keywords: Performance, Climb speed, Fuel saving, Flight mechanics, Climb-cruise

1. Introduction

Traditional flight analysis considers a flight of single segment, such as takeoff, climb, cruise, descent and landing, and formulates an optical flight technique for each segment. Actual flight involves multiple segments, theoretically each segment follows optimal technique, such as climb at maximum rate, cruise at maximum range, etc. Climb-cruise analysis considers 2 segments of climb and cruise together to optimize a technique that produces the best sum of altitude gain and ground distance.

Fig. 1 shows 2 aircraft of the same type, AC1 and AC2, climbing from the same starting point (O) to the same target altitude. AC1 climbs with optimum climb speed (V_1) to point A on target altitude, then turns to fly level with optimum cruise speed (V_2) to point B. AC2 climbs with an arbitrary climb speed (V_3) that is faster than V_1, to the same point B. AC1 and AC2 fly from the same starting point (O) to the same end point (B), covering the same ground distance. AC1 flight profile consists of a climb segment (1) and a cruise segment (2). AC2 combines both segments into one segment (3), defined as a “climb-cruise”. Fuel-used and flight time differences between AC1 and AC2 are the comparison between normal flight profile of separate climb and cruise to a climb-cruise profile.

This author presented similar study named “Far and Fast Climb” in NKRAFA Journal of Science and Technology Vol.17 No.1 [1], which focused mainly on distance extension and fuel saving as a supplement. The current study renames the profile to a “climb-cruise” and focuses mainly on fuel saving, with time saving as a supplement. The analyses are more orderly after gaining more experiences of studying this flight profile. This study is presented in English to offer internationally for discussion.

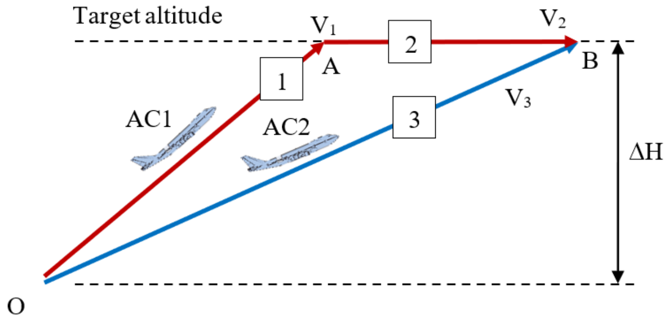

Fig. 1 Flight profile comparison between traditional climb and cruise (AC1) and a climb-cruise (AC2)

2. Flight Mechanics of a Climb-Cruise

On segment 1 in Fig. 1, altitude band (ΔH) is a product of vertical velocity (V_{v1}) and time to climb (t_1),

$$\Delta H = V_{v_1} t_1$$

Ground distance (S_1) is a product of climb speed (V_1) and t_1,

$$S_1 = V_1 t_1 \tag{1}$$

Therefore,

$$\frac{S_1}{\Delta H} = \frac{V_1}{V_{v_1}} \tag{2}$$

Quantity of fuel used (w_f) is the product of specific fuel consumption (SFC = $\dot{w}_{f_v}$) and flight distance, which is approximately equal to ground distance because climb angle is small. Therefore,

$$w_{f_1} = \dot{w}_{fv_1} S_1$$

SFC is a ratio of fuel flow ($\dot{w}_f$) to velocity (V_1), so

$$w_{f_1} = \frac{\dot{w}_{f_1}}{V_1} S_1$$

Divide by ΔH and substitute equation (2) to get

$$\frac{w_{f_1}}{\Delta H} = \frac{\dot{w}_{f_1}}{V_1} \frac{V_1}{V_{v_1}} = \frac{\dot{w}_{f_1}}{V_{v_1}} \tag{3}$$

Similarly, on segment 3

$$\frac{S_3}{\Delta H} = \frac{V_3}{V_{v_3}} \tag{4}$$

and

$$\frac{w_{f_3}}{\Delta H} = \frac{\dot{w}_{f_3}}{V_{v_3}} \tag{5}$$

On segment 2, ground distance (S_2) is the difference between S_3 and S_1, so

$$\frac{S_2}{\Delta H} = \frac{S_3}{\Delta H} - \frac{S_1}{\Delta H} \quad (6)$$

and
$$\frac{w_{f_2}}{\Delta H} = \frac{\dot{w}_{f_2}}{V_2}\frac{S_2}{\Delta H} = \frac{\dot{w}_{f_2}}{V_2}\left(\frac{S_3}{\Delta H} - \frac{S_1}{\Delta H}\right)$$

Substitute equation (2) and (4) to get
$$\frac{w_{f_2}}{\Delta H} = \frac{\dot{w}_{f_2}}{V_2}\left(\frac{V_3}{V_{v_3}} - \frac{V_1}{V_{v_1}}\right) \quad (7)$$

Fuel-used difference between AC1 and AC2, or fuel saving, is

$$\Delta w_f = \left(w_{f_1} + w_{f_2}\right) - w_{f_3}$$

Substitute equation (3), (5) and (7) to get

$$\frac{\Delta w_f}{\Delta H} = \left[\frac{\dot{w}_{f_1}}{V_{v_1}} + \frac{\dot{w}_{f_2}}{V_2}\left(\frac{V_3}{V_{v_3}} - \frac{V_1}{V_{v_1}}\right)\right] - \frac{\dot{w}_{f_3}}{V_{v_3}} \quad (8)$$

Difference of flight time between AC2 and AC1 is also interesting. Flight time of segment 1 in Fig. 1 can be calculated by equation (1),

$$t_1 = \frac{S_1}{V_1} \quad (9)$$

Divide by ΔH and substitute equation (2) to get

$$\frac{t_1}{\Delta H} = \frac{S_1}{\Delta H}\frac{1}{V_1} = \frac{1}{V_{v_1}} \quad (10)$$

Similarly for segment 3,
$$\frac{t_3}{\Delta H} = \frac{1}{V_{v_3}} \quad (11)$$

For segment 2, apply equation (9)
$$t_2 = \frac{S_2}{V_2}$$

then divide by ΔH and substitute equation (6) to get

$$\frac{t_2}{\Delta H} = \frac{1}{V_2}\left(\frac{S_3}{\Delta H} - \frac{S_1}{\Delta H}\right)$$

Substitute equation (2) and (4) to get
$$\frac{t_2}{\Delta H} = \frac{1}{V_2}\left(\frac{V_3}{V_{v_3}} - \frac{V_1}{V_{v_1}}\right) \quad (12)$$

Flight time difference between AC2 and AC1, or time saving, is calculated from

$$\frac{\Delta t}{\Delta H} = \left(\frac{t_1}{\Delta H} + \frac{t_2}{\Delta H}\right) - \frac{t_3}{\Delta H}$$

Finally, substitute equation (10), (11) and (12) to get

$$\frac{\Delta t}{\Delta H} = \left[\frac{1}{V_{v_1}} + \frac{1}{V_2}\left(\frac{V_3}{V_{v_3}} - \frac{V_1}{V_{v_1}}\right)\right] - \frac{1}{V_{v_3}} \quad (13)$$

Equation (8) and (13) determine fuel saving and time saving (+ differences as + savings) of climb-cruise over separate climb and cruise profile.

3. Minimum Fuel Climb-Cruise

Finding optimal climb-cruise speed depends on the flight objective which would select the objective parameter and the reference flight profile (climb and cruise). For example, if the flight objective is to save fuel, then the objective parameter is fuel saving (Equation (8)) and the reference flight profile consists of minimum fuel climb (V_1 as minimum fuel climb speed, V_{mf}) and maximum range cruise (V_2 as maximum range speed, V_{mr}). If the flight objective is to save time, then the objective parameter is time saving (Equation (13)) and the reference flight profile consists of maximum rate climb (V_1 as maximum rate climb speed, V_y) and maximum ground distance level flight (V_2 as maximum level speed). This research will concentrate on minimum fuel climb-cruise at a specific altitude and demonstrate fuel saving of climb-cruise for example aircraft.

4. Fuel Saving by Climb-Cruise at 10,000 feet, for PA-28R-200 Piper Arrow II

PA-28R-200 Piper Arrow II in Fig. 2, is a light propeller aircraft with a Lycoming IO-360-C1C piston engine. Following data on the aircraft, engine and propeller are from McCormick [2] and PA-28R-200 Flight Manual [3].

Fig. 2 PA-28R-200 Piper Arrow II

Table 1. PA-28R-200 Piper Arrow II and engine performance data

Weight	2,650 lbs
Wing area	169 sqf
Zero-lift drag coeff. (C_{D0})	0.026627
Induced drag factor (K)	0.074119
Climb power @ 10,000 ft	130 hp (65% power)
Climb fuel flow @ 10,000 ft	9.16 gal/hr (@ 65% power)
Cruise power @ 10,000 ft	55% (100% = 200 hp)
Cruise speed @ 10,000 ft	147 mi/hr (@55% power)
Cruise fuel flow @ 10,000 ft	8 gal/hr (@55% power)
Propeller diameter	6.17 ft
Propeller speed	2,500 rpm

For a propeller aircraft, rate of climb (V_v) is calculated from

$$V_v = \frac{P_a - P_r}{W} \tag{14}$$

P_a is available power and calculated from

$$P_a = \eta_p P_e \tag{15}$$

P_e is engine power and η_p is propeller efficiency. For a climb at 10,000 ft, engine climb power (P_e) is at 130 hp. Data for PA-28R η_p is given in [2, pp 361-362]. Fig. 3 presents η_p as a function of advance ratio (J) at power coefficient (C_p) of 0.063 which corresponds to climb power of 130 hp, propeller diameter of 6.17 ft and propeller speed of 2,500 rpm.

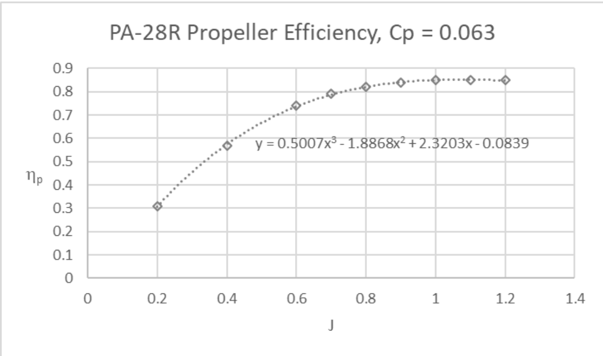

Fig. 3 Propeller efficiency (η_p) for PA-28R Piper Arrow at C_p of 0.063

P_r in equation (13), is required power and calculated from [4, pp 294-299, 363-367]

$$P_r = DV = (C_D q S)V \tag{16}$$

D is a drag and V is the climb speed. D is calculated from drag coefficient (C_D), dynamic pressure ($q = ½ \rho V^2$) and wing area (S). C_D is calculated from drag polar relation,

$$C_D = C_{D_0} + K C_L^2 \tag{17}$$

C_{D0} is the zero-lift drag coefficient and K is the induced drag factor. C_{D0} and K can be calculated from data given in [2, pp 435]. C_L is lift coefficient which can be calculated from

$$C_L = \frac{L}{qS} = \frac{W}{qS} \tag{18}$$

L is lift and assumed to equal weight (W) during a climb because climb angle is small.

For aircraft with piston engine, fuel flow is constant at the same power setting and altitude. Therefore, for a climb at 10,000 ft with 130 hp (65% power), fuel flow of PA-28R-200 is 9.16 gal/hr for all airspeeds.

For climb (segment 1) and cruise (segment 2) segments in Fig. 1, V_1 is minimum fuel climb speed (V_{mf}) and V_2 is maximum range speed (V_{mr}) to achieve minimum fuel used. V_{mr} is given in the Flight Manual [3, pp 36] at 10,000 ft and 55% power as 147 mi/hr (216 ft/s), and fuel flow of 8 gal/hr [3, pp 37]. Fig. 4 shows rate of climb (ROC) and specific rate of climb (SRC) at 10,000 ft, calculated from Equation (14). SRC is the ratio of ROC to fuel flow and V_{mf} is the speed at maximum SRC, which is 157 ft/s in Fig. 4. As shown in Fig. 4, V_{mf} is the same as V_y, which is the maximum climb rate speed. V_y and maximum ROC is given in the Flight Manual [3, pp 26, 35] as 95 mi/hr (139 ft/s) and 400 ft/min (6.67 ft/s). Comparison of maximum ROC from the Flight Manual (6.67 ft/s) with value in Fig. 4 from calculations (6.74 ft/s) shows the difference of 1%, confirming the correspondence of calculations to actual performance.

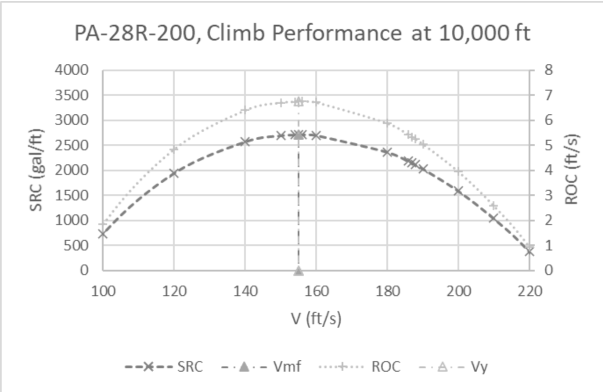

Fig. 4 PA-28R-200 Climb Performance at 10,000 ft

Fig. 5 shows results of fuel saving, calculated with Equation (8), and time saving, calculated with Equation (13). Positive fuel saving exists over the 157 - 208 ft/s speed range and maximum fuel saving occurs at speed of 187 ft/s (127.5 mi/hr). The maximum fuel saving is 0.13 gal/mi which accounts for 4.9% of the total fuel used by the reference flight profile (separate minimum fuel climb and maximum range cruise). It is interesting to note that the Flight Manual also recommends en route climb speed of 110 mi/hr [3, pp 26] which is higher than best rate climb speed (95 mi/hr). Minimum fuel climb-cruise speed is 127.5 mi/hr which is higher than both best rate climb speed and en route climb speed. Time saving at minimum fuel climb-cruise speed is also positive at 0.0114 s/ft, which accounts for 8.2% of total time used for the reference flight profile (separate minimum fuel climb and maximum range cruise). Positive time saving shows continuous increase from the beginning of positive fuel saving speed at 157 ft/s onwards.

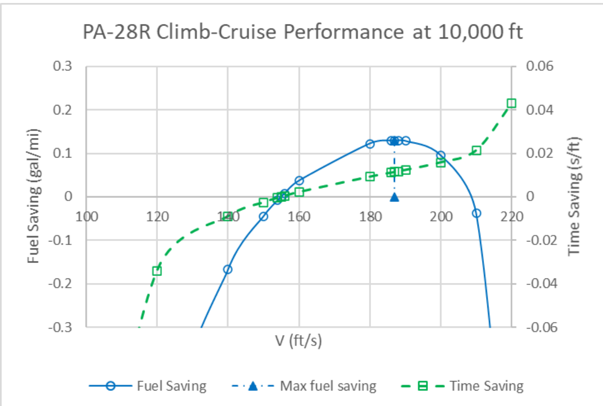

Fig. 5 PA-28R-200 Climb-Cruise Performance at 10,000 ft

Fig. 5 confirms that a PA-28R-200 aircraft travels from O to B in Fig. 1, using climb-cruise profile with V_3 = 127.5 mi/hr (187 ft/s), uses less fuel and takes less time than climbs with V_1 = 107 mi/hr (157 ft/s), then cruises with V_2 = 147 mi/hr (215.6 ft/s), as recommended by the Flight Manual.

5. Fuel Saving by Climb-Cruise at 15,000 feet, for C-130H

C-130H, Fig. 6, is a medium military transport. Propulsion system consists of (4) T56-A-15 turboprop engines from Allison Rolls-Royce, and constant-speed propellers 54H60 from Hamilton Standard. Fuselage is pressurized to enable cruising at altitudes up to 30,000 ft. Therefore, fuel saving with climb-cruise will be demonstrated at 15,000 ft altitude and 155,000 lb weight, with following data.

Fig. 6 C-130H with (4) T56-A-15 turboprop engines

Table 2. C-130H and engine performance data

Weight	155,000 lbs [5, pp 1-4]
Wing area	1,745 sqf [6]
Zero-lift drag coeff. (C_{D0})	0.0227 [7]
Induced drag factor (K)	0.0291 [7]
Engine at TIT 1,010 ^{o}C - Power available - Fuel Flow	 [8, pp 2-4] [8, pp 2-6]
Specific Range	[8, pp 5-19]
Propeller diameter	13.5 ft [9, pp 1-7]
Propeller speed	1,020 rpm [9, pp 1-8]

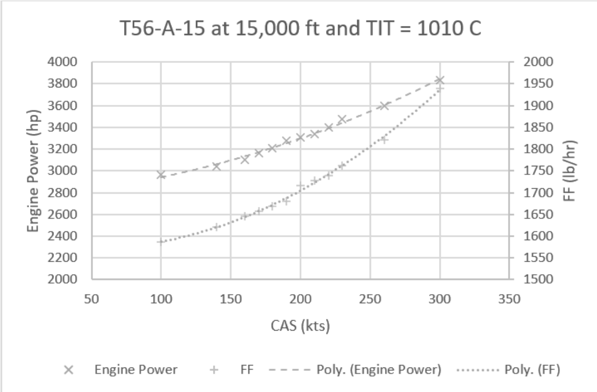

Fig. 7 T56-A-15 power available and fuel flow at 15,000 ft and TIT at 1,010 °C

C-130H is a propeller aircraft, similar to PA-28R-200, but turboprop engines and constant-speed propellers are more affected by airspeeds. In a climb at 15,000 ft, T56-A-15 engine TIT is set at 1,010 °C and power available and fuel flow for each engine are obtained from SMP 777 Performance Data Flight Manual [8, pp 2-4, 2-6] and shown in Fig. 7.

Propeller efficiency (η_p) can be calculated from a cruise performance chart which displays engine power and true airspeed during a cruise. At constant speed and constant altitude, power available equals power required (DV).

$$P_a = P_r$$

$$\eta_p P_e = DV$$

and $$\eta_p = \frac{DV}{P_e} \qquad (19)$$

C-130H specific range chart at 15,000 ft and 155,000 lb weight [8, pp 5-9], displays engine torque (Tq) against true airspeed (V), as shown in Fig. 8. Engine power (P_e) relates to torque as $P_e = 0.2139$ Tq [8, pp 2-1] and shows in Fig. 8 also.

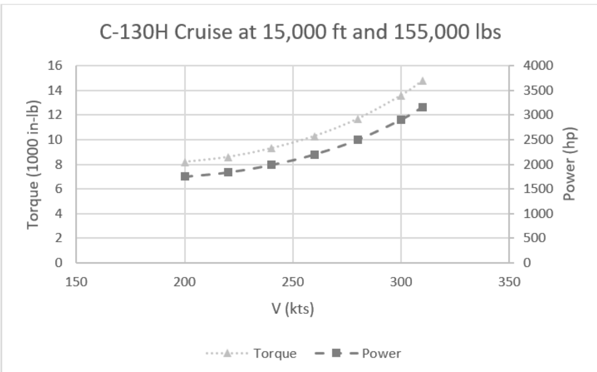

Fig. 8 C-130H power required in a cruise at 15,000 ft and 155,000 lbs

Drag (D) in Equation (19), can be obtained from drag polar, using Equation (16), (17) and (18). Propeller efficiency (η_p) is calculated from Equation (19) and plotted against advance ratio, $J = \frac{V}{nD}$, in Fig. 9.

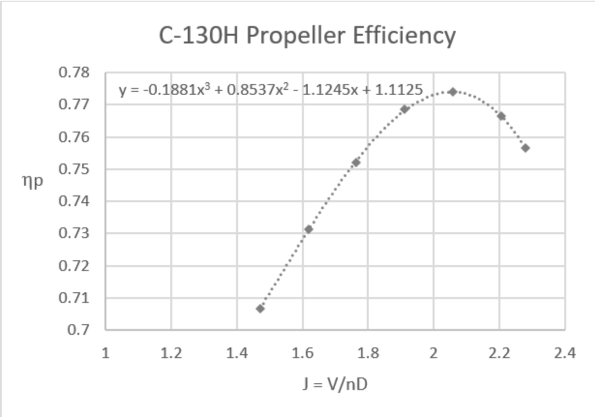

Fig. 9 C-130H propeller efficiency

Climb performance at 15,000 ft and 155,000 lbs weight is shown in Fig. 10. Rate of climb (ROC) varies with the difference between power available (P_a) and power required (P_r), as shown in Equation (14). P_a is calculated by Equation (15) with η_p from Fig. 9 and P_e from Fig. 7. P_r is obtained from drag polar using (16), (17) and (18), and C_{D0} and K from [7]. Specific rate of climb (SRC) is the ratio of climb rate and fuel flow and also shown in Fig. 10. V_y is the maximum ROC speed and V_{mf} is the maximum SRC speed. Both V_y and V_{mf} occur at 170 kts CAS.

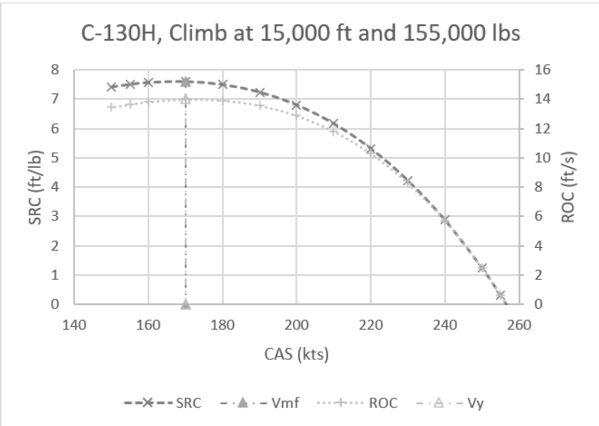

Fig. 10 C-130H climb performance at 15,000 ft and 155,000 lbs

Fig. 11 demonstrates climb-cruise performance of C-130H at 15,000 ft and 155,000 lb weight. The speed in climb segment of Fig. 1 (V_1) is flown with V_{mf} (maximum SRC speed) of 170 kt CAS to minimize fuel used. The speed in cruise segment of Fig. 1 (V_2) is flown with V_{mr} (maximum range cruise speed) of 260 kt TAS (206 kt CAS). V_{mr} is selected at the point of maximum specific range (SR of 52 nm/1,000 lb of fuel) from the SR chart at 15,000 ft and 155,000 lbs [8, pp 5-19]. Fuel saving in Fig. 11 is calculated with Equation (8) and time saving with Equation (13). Positive fuel saving exists over the 170 - 230 kt CAS range, and maximum fuel saving occurs at speed of 210 kt CAS. The maximum fuel saving is 44.9 lb/nm which accounts for 4.36% of the total fuel used by the reference flight profile (separate minimum fuel climb and maximum range cruise).

Time saving at minimum fuel climb-cruise speed is also positive at 0.0238 hr/nm, which accounts for 14.3% of total time used by the reference flight profile (separate minimum fuel climb and maximum range cruise). Positive time saving shows continuous increase from the beginning of positive fuel saving speed at 170 kts CAS onwards.

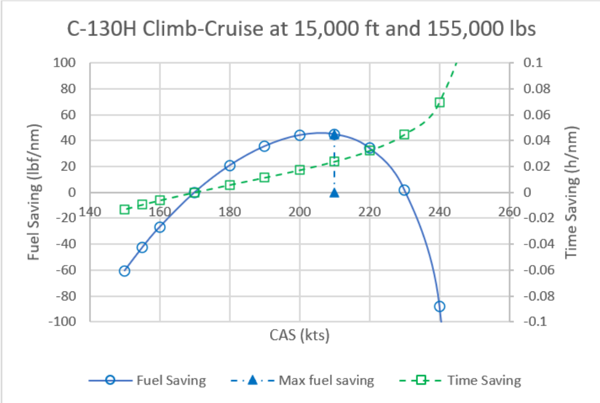

Fig. 11 C-130H climb-cruise performance at 15,000 ft and 155,000 lbs

Similar to the findings with PA-28R-200 in Fig. 5, Fig. 11 confirms that C-130H aircraft travels from O to B in Fig. 1, using climb-cruise profile with V_3 = 210 kt CAS, uses less fuel and takes less time than climbs with V_1 = 170 kt CAS, then cruises with V_2 = 206 kt CAS, as recommended by the Flight Manual.

6. Fuel Saving by Climb-Cruise at 20,000 feet, for B747-100

Fig. 12 B747-100 aircraft with 4 JT9D-7A turbofan engines

B747-100 in Fig. 12, is a large passenger aircraft with (4) JT9D-7A turbofan engines. Following data on aircraft and engine are from [2, pp 432, 400-401].

Table 3. B747-100 and engine performance data

Weight	3,260 kN
Wing area	511 sqm
Zero-lift drag coeff. (C_{D0})	0.01818
Induced drag factor (K)	0.06543
Engine maximum climb performance	Fig. 13 [2, pp 400]
Engine maximum cruise TSFC	Fig. 14 [2, pp 401]

For a turbofan aircraft, rate of climb (V_v) is calculated from [4, pp 479-489]

$$V_v = V\left(\frac{T-D}{W}\right) \tag{20}$$

T is thrust and varies with Mach number (M) as shown in Fig. 13. D is drag and calculated with Equation (16), (17) and (18) at 6,000 m (19,685 ft) altitude and fuel flow ($\dot{W}_f$) is calculated from

$$\dot{w}_f = (TSFC)T \quad (21)$$

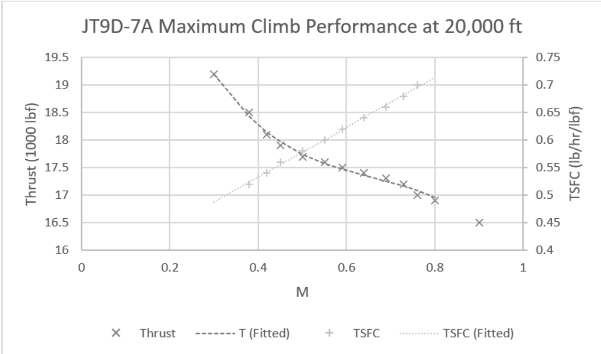

Fig. 13 JT9D-7A turbofan engine, maximum climb performance at 20,000 ft

TSFC is thrust specific fuel consumption, a ratio of fuel flow to thrust, and also obtained from Fig. 13. For minimum fuel used in the climb segment of Fig. 1, V_1 is the minimum fuel climb speed (V_{mf}). Fig. 14 shows rate of climb (ROC) and specific rate of climb (SRC) at 6,000 m (19,685 ft), calculated from Equation (20). SRC is the ratio of ROC to fuel flow and V_{mf} is the speed at maximum SRC. V_{mf} is at 198 m/s in Fig. 15 and slightly less than V_y, the maximum climb rate speed of 204 m/s. Reference [10] shows indicated climb speed of 290 kts (true airspeed of 390 kts, or 201 m/s) produces rate of climb of 1,000 ft/min (5.08 m/s) at 20,000 ft, which are compatible with maximum rate climb speed (V_y at 204 m/s) and maximum ROC (5.1 m/s) in Fig. 14.

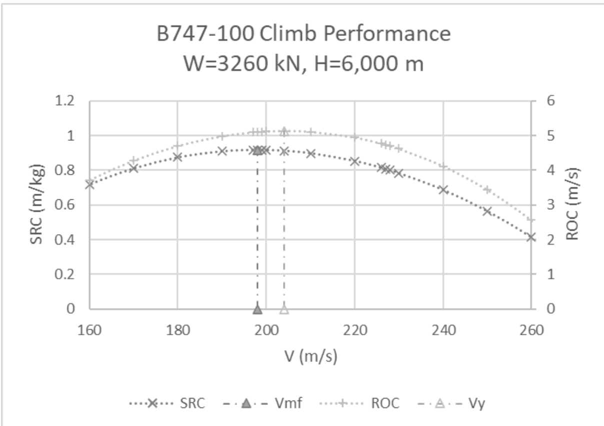

Fig. 14 B747-100 Climb Performance at 3,260 kN weight and 6,000 m altitude

For minimum fuel used in the cruise segment of Fig. 1, V_2 is the maximum range speed (V_{mr}). V_{mr} occurs at minimum specific fuel consumption (SFC) [4, pp 508-516] which is the ratio of fuel flow ($\dot{w}_f$) to airspeed (V). Fuel flow ($\dot{w}_f$) is calculated from Equation (21) and TSFC is obtained from Fig. 15. Thrust (T) equals to drag (D) which is calculated from Equation (16), (17) and (18) at 6,000 m (19,685 ft). SFC is presented in Fig. 16 and V_{mr}, at minimum SFC, is at 218.5 m/s.

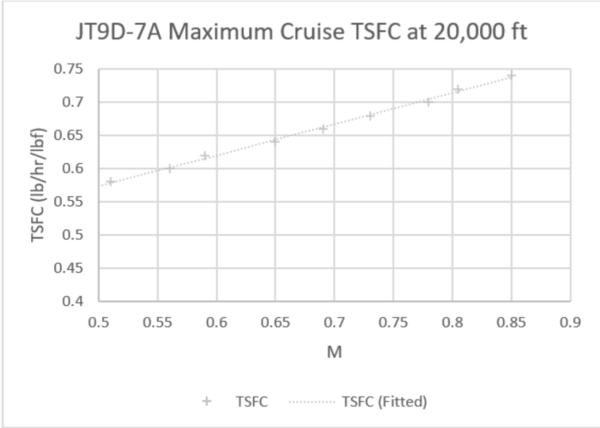

Fig. 15 JT9D-7A turbofan engine, maximum cruise TSFC at 20,000 ft

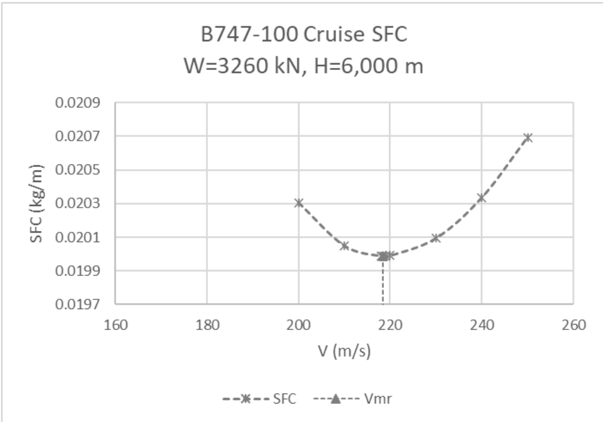

Fig. 16 B747-100 Cruise SFC at 3,260 kN weight and 6,000 m altitude

Fig. 17 shows results of fuel saving, calculated with Equation (8), and time saving, calculated with Equation (13). Positive fuel saving exists over the 198 - 251 m/s speed range, and maximum fuel saving occurs at speed of 227 m/s (0.72 M). The maximum fuel saving is 0.0373 kg/m which accounts for 2.9% of the total fuel used by the reference flight profile (separate minimum fuel climb and maximum range cruise). Time saving at minimum fuel climb-cruise speed is also positive at 0.0275 s/m, which accounts for 11.5% of total time used by the reference flight profile (separate minimum fuel climb and maximum range cruise). Positive time saving shows continuous increase from the beginning of positive fuel saving speed at 198 m/s onwards.

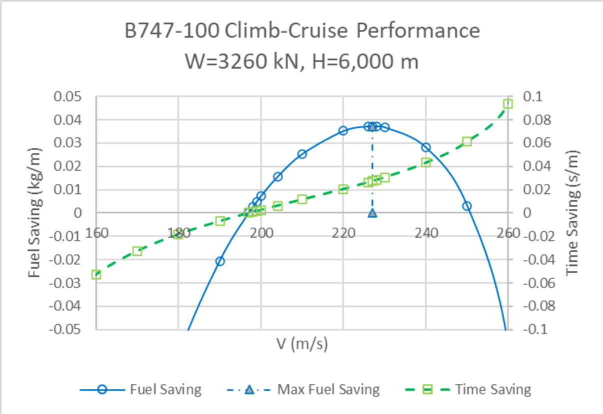

Fig. 17 B747-100 Climb-Cruise Performance at 3,260 kN weight and 6,000 m altitude

Similar to the findings with PA-28R-200 in Fig. 5 and C-130H in Fig. 11, Fig. 17 confirms that B747-100 aircraft travels from O to B in Fig. 1, using climb-cruise profile with V_3 = 227 m/s, uses less fuel and takes less time than climbs with V_1 = 198 m/s, then cruises with V_2 = 218.5 m/s, as recommended by theory.

7. Conclusions and Recommendations

7.1 Climb-cruise provides fuel and time savings over separate climb and cruise segments. The results are confirmed by light propeller aircraft (PA-28R-200), medium military transport (C-130H) and large passenger aircraft (B747-100).

7.2 Optimal climb-cruise speed depends on the flight objective. This study chooses minimum fuel-used as the objective and chooses the reference profile to consist of minimum fuel climb and maximum range cruise. The optimal climb-cruise speed is found to be appreciably higher than minimum fuel climb speed.

7.3 Fuel saving is not very high, 4.9% for light propeller aircraft, 4.4% for medium military transport and 2.9% for large passenger aircraft. Time saving is higher but still not very high, 8.2% for light propeller aircraft, 14.3% for medium military transport and 11.5% for large passenger aircraft.

7.4 Although fuel and time savings are not high, climb-cruise is more comfortable to aircraft pilots and passengers. Higher climb speed of climb-cruise allows for lower climb angle and better visibility for pilots. Lower climb rate reduces aircraft cabin pressurization rate of adjustment and less passenger discomfort.

7.5 Further studies should be conducted on:

- A full climb through an altitude band from lower altitude to maximum altitude,
- Other flight objective, such as minimum time climb-cruise, and
- Other flight segments combination, such as cruise-descent, climb-descent and climb-cruise-descent.

8. Nomenclature

ρ	= Air density	Tq	= Torque
η_p	= Propeller efficiency	TSFC	= Thrust specific fuel consumption
C_D	= Drag coefficient	V	= True airspeed
C_{D0}	= Zero-lift drag coefficient	V_{mf}	= Minimum fuel climb speed
C_L	= Lift coefficient	V_{mr}	= Maximum range cruise speed

C_p = Power coefficient
CAS = Calibrated airspeed
d = Propeller diameter
D = Drag
FF = Fuel flow
ΔH = Altitude band
J = Advance ratio
K = Induced drag factor
L = Lift
M = Mach number
n = Propeller rotational speed
P_a = Power available
P_e = Engine power
P_r = Power required
q = Dynamic pressure
ROC = Rate of climb
S = Wing area
SFC = Specific fuel consumption
SR = Specific range
SRC = Specific rate of climb
t = Flight time
Δt = Time saving
T = Engine thrust
TAS = True airspeed
TIT = Turbine inlet temperature
V_v = Vertical velocity, or rate of climb
V_y = Maximum rate of climb speed
w_f = Fuel used
Δw_f = Fuel saving
$\dot{w}_f$ = Fuel flow
$\dot{w}_{fv}$ = Specific fuel consumption
W = Aircraft weight
C = Celsius
ft = foot
gal = gallon
hp = horse power
hr = hour
in = Inch
kN = kilonewton
kt = knot (nautical mile/hr)
lb = pound
lb_f = pound force
m = meter
mi = mile
nm = nautical mile
rpm = revolution per minute
s = second
sqf = square foot
sqm = square meter

9. References

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