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Mathematical Tutorial of Discrete-Time Analysis of Sampling Rate Changing Concept for Digital Signal Processing and Digital Communication Prospective

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Abstract

Greatly demanding on digital signals (or Discrete-Time (DT) signals), which are generated from Continuous-Time (CT) signals by a sampling process based on Nyquist-Shannon theorem, for modern digital processing such as Digital Signal Processing (DSP) and digital communication, the concept of sampling rate changing by an integer factor and a non-integer has been extensively investigated for one and a half decade. Thereby, this article introduces the mathematical tutorial of DT analysis of sampling rate changing concept for both an integer factor and a non-integer. This article first algebraically presents the down-sampling concept with an integer factor and later algebraically presents the up-sampling concept with an integer factor. Next, the article algebraically presents the sampling rate changing concept with a non-integer factor for both down-sampling and up-sampling by using the combination of down-sampling concept and the up-sampling concept. In addition, the several examples of the down-sampling with an integer factor, the up-sampling with an integer factor and the sampling rate changing concept with a non-integer factor, which are disclosed in each algebraically detail, are introduced for bring the reader comprehensively recognizing.

Keywords: Down-Sampling Rate, Up-Sampling Rate, Sampling Rate Changing, Aliasing Problem, Digital Signal Processing (DSP)

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1. Conceptual Introduction of Digital Signal Processing

In order to process the Continuous Time (CT) signal $x_c(t)$ by a microprocessor (in computer or microcontroller), we first convert the CT signal $x_c(t)$ to DT signal $x[n]$(by using A/D convertor) and then process the DT signal $x[n]$ by a microprocessor (DT system) as shown in the following figure. (This operation is called as Digital signal Processing or DSP as shown in the following figure). The advantage of the DSP makes the researchers the implementation of advance/complex digital signal processing algorithm (such as speech compression, speech recognition, face recognition, image compression, compressive sensing, video compression, etc.) easily because the modern microprocessors/ microcontrollers have been developed to be more powerful, less expensive, less power consuming and easier for modification.

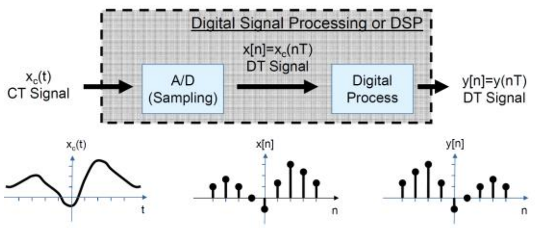

Figure 1 Example of the Digital Signal Processing (DSP) concept.

2. Mathematical Analysis of Sampling Rate Changing Process Concept

The mathematical relationship [4,8] between a continuous-time (CT) signal $x_c(t)$ and a discrete-time (DT) signal $x[n]$ can be mathematically defined as $x[n]=x_c(nT)$. In the bandlimited signal reconstructing and processing concept [9], it can be inferred that it is possible to perfectly reconstruct the continuous-time (CT) signal $x_c(t)$ from discrete-time (DT) signal $x[n]$, which can be mathematically as $x[n]=x_r(nT)=x_c(nT)$ if the reconstructed period is equal to the sampling period [1,5,10].

If the reconstructed period is not equal to the sampling period then the sampling rate of a DT signal is changed (or this process is called resampling) and the mathematical between a continuous-time (CT) signal $x_c(t)$ and a discrete-time (DT) signal $x[n]$ can be mathematically defined as

$$x_1[n]=x_c(nT_1) \text{ where } \quad T_1 \neq T \qquad (1)$$

From the theatrical prospective, a DT signal $x_1[n]$ can be indirectly acquired from a DT signal $x[n]$ by first reconstructing a CT signal $x_c(t)$ from $x[n]$ as $x_r(t)=\sum_{n=-\infty}^{\infty} x[n]\left(\sin\left(\pi(t-nT)/T\right)/\pi(t-nT)/T\right)$ [9]. Later, the $x_1[n]$ can be resampled from the reconstructed CT signal $x_r(t)$ with sampling period T . However, from the practical prospective, a DT signal $x_1[n]$ can be directly acquired from a DT signal $x[n]$, instead.

3. Mathematical Analysis of Down-sampling rate Process Concept with an Integer Factor

The mathematical relationship [11] between a resample DT signal $x_d[n]$ and a discrete-time (DT) signal $x[n]$ can be mathematically defined as

$$x_d[n] = x[nM] = x_c(nMT) \qquad (2)$$

The block diagram of down-sampling rate system (or sampling rate compressor or compressor [4]) for DT signals can be illustrated as following figure.

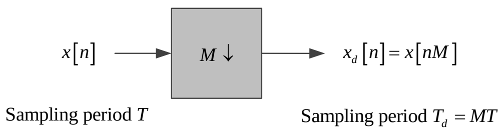

Figure 2 The block diagram of down-sampling rate system for DT signals.

For the band limited signal property [8-9], $X_c(j\Omega) = 0 \text{ for } |\Omega| \geq \Omega_N$, the down sampling DT signal $x_d[n]$ is perfectly description of the original CT signal $x_c(t)$ when $\pi/T_d = \pi/(MT) \geq \Omega_N$. Consequently, the resampling system (so called down-sampling) can decrease the sampling rate from $\pi/(mT)$ up to π/M without aliasing [8] if the original sampling rate is higher than the Nyquist rate [2, 4] at least M times.

From the down-sampling rate system for DT signals as shown in the above figure, the frequency domain relationship between a DT input signal $x[n]$ can be mathematically defined as

$$x[n] = x_c(nT)$$

$$\xrightarrow{DTFT} X\left(e^{j\omega}\right) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right] \qquad (3)$$

The frequency domain relationship of a down-sampling DT output signal $x_d[n]$ can be mathematically defined as

$$x_d[n] = x_c(nT_d), T_d = MT$$

$$\xrightarrow{DTFT} X_d\left(e^{j\omega}\right) = \frac{1}{T_d}\sum_{r=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{T_d} - \frac{2\pi r}{T_d}\right)\right] \qquad (4)$$

Due to $T_d = MT$, the above equation can be mathematically simplified as

$$\rightarrow X_d\left(e^{j\omega}\right) = \frac{1}{MT}\sum_{r=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{MT} - \frac{2\pi r}{MT}\right)\right] \qquad (5)$$

From Eq. (5), the summation index r (where r is an integer with $-\infty < r < +\infty$) can be mathematically defined as $r = i + kM$ where k is an integer with $-\infty < k < +\infty$ and i is an integer with $0 \leq i \leq M-1$.

$$X_d\left(e^{j\omega}\right) = \frac{1}{MT}\sum_{r=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{MT} - \frac{2\pi r}{MT}\right)\right]$$

$$\rightarrow$$

$$X_d\left(e^{j\omega}\right) = \frac{1}{M}\sum_{i=0}^{M-1}\left[\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{MT} - \frac{2\pi k}{T} - \frac{2\pi i}{MT}\right)\right]\right] \qquad (6.1)$$

$$\rightarrow$$

$$X_d\left(e^{j\frac{(\omega-2\pi i)}{M}}\right) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega - 2\pi i}{MT} - \frac{2\pi k}{T}\right)\right] \qquad (6.2)$$

$$\rightarrow X_d\left(e^{j\omega}\right) = \frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi i}{M}\right)}\right) \qquad (6.3)$$

From mathematical prospective, Eq. (3) and Eq. (6.3) has a vigorous similarity relationship because the Eq. (3) mathematically defines the frequency domain of a sampling DT signal $x[n]$ in the term of the frequency domain of a CT input signal $x_c(t)$ and the Eq. (6.3) mathematically defines the frequency domain of a sampling DT down-sampling signal $x_d[n]$ in the term of the frequency domain of a DT input signal $x[n]$. From the DTFT property, $X_d(e^{j\omega})$ is periodic with period 2π and if $X(e^{j\omega})$ is a bandlimited signal ($X(e^{j\omega}) = 0, \omega_N \le |\omega| \le \pi$ with $2\pi/M \ge 2\omega_N$).

The example of a down-sampling process (for $M = 2$ or $T_d = 2T$) with the higher sampling rate than the Nyquist rate at twice times ($2\pi/T = 4\Omega_N$) can be illustrated as figure 3. From this frequency domain analysis [4, 11], the frequency domain of continuous-time (CT) bandlimited input signal $x_c(t)$, the sampling signal of a CT bandlimited input signal $x_s(t)$, the DT sampling input signal $x[n]$, the DT down-sampling input signal $x_d[n]$ (plotted as a function of DT or ω) and the DT down-sampling input signal $x_d[n]$ (plotted as a function of CT or Ω) can be illustrated in the figure 3(a), figure 3(b), figure 3(c), figure 3(d) and figure 3(e), respectively. From the figures, because the DT sampling input signal $x[n]$ is created by sampling rate at twice times of the Nyquist rate, subsequently, the DT sampling input signal $x[n]$ is downsampled by $M = 2$ then the DT down-sampling input signal $x_d[n]$ does not have aliasing problem (there is no overlapped of the signal spectrum as shown in figure 3(d) and figure 3(e)). However, if the DT sampling input signal $x[n]$ is downsampled at rate $M > 2$ then the DT down-sampling input signal $x_d[n]$ will have aliasing problem.

The another example of a down-sampling process (for $M = 3$ or $T_d = 3T$) with the higher sampling rate than the Nyquist rate at twice times ($2\pi/T = 4\Omega_N$) can be illustrated as figure 4. From this frequency domain analysis [4, 11], the frequency domain of continuous-time (CT) bandlimited input signal $x_c(t)$, the sampling signal of a CT bandlimited input signal $x_s(t)$, the DT sampling input signal $x[n]$ and the DT down-sampling input signal $x_d[n]$ (plotted as a function of DT or ω) can be illustrated in the figure 4(a), figure 4(b), figure 4(c) and figure 4(d), respectively. From the figures, because the DT sampling input signal $x[n]$ is created by sampling rate at twice times of the Nyquist rate, subsequently, the DT sampling input signal $x[n]$ is downsampled by $M = 3$ then the DT down-sampling input signal $x_d[n]$ have the aliasing problem (there is overlapped of the signal spectrum as shown in figure 4(d)).

For preventing the aliasing problem, the DT sampling input signal $x[n]$ is reduced the bandwidth by using the ideal lowpass filter with cutoff frequency π/M as illustrated in the figure 4(e). The frequency domain of the DT output signal $x[n]$ from the ideal lowpass filter and the DT down-sampling input signal $x_d[n]$ (plotted as a function of DT or ω) can be illustrated in the figure 4(f) and figure 4(g), respectively. From the theoretical prospective, in order to present aliasing caused by a down-sampling process with a factor of M, the a factor of M must be mathematically defined as $\omega_N M \le \pi$ or $\omega_N \le \pi/M$. In another point of view, the aliasing problem can be prevented in a down-sampling process if the DT signal $x[n]$ is reduced its bandwidth (by using the ideal lowpass filter with cutoff frequency π/M). The general block diagram of a down-sampling system with a factor M can be illustrated as the figure 5.

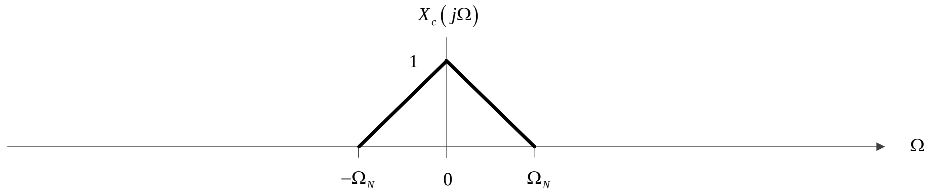

Figure 3 (a) The FT of a continuous-time (CT) bandlimited input signal $x_c(t)$

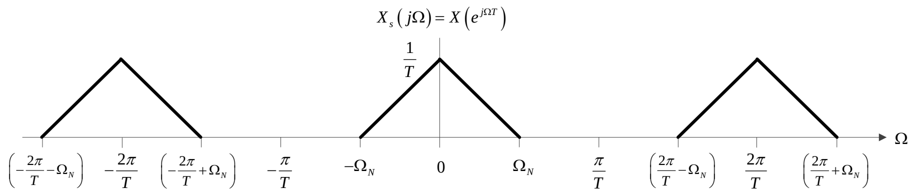

Figure 3 (b) The FT of the sampling signal of a CT bandlimited input signal $x_s(t)$

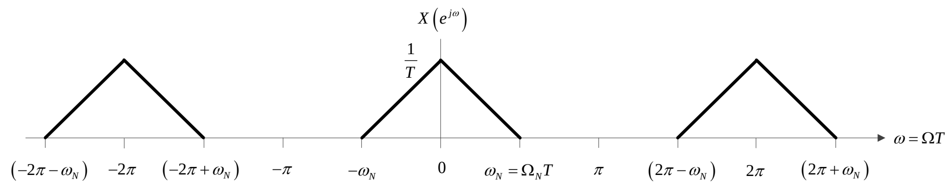

Figure 3 (c) The FT of the DT sampling input signal $x[n]$

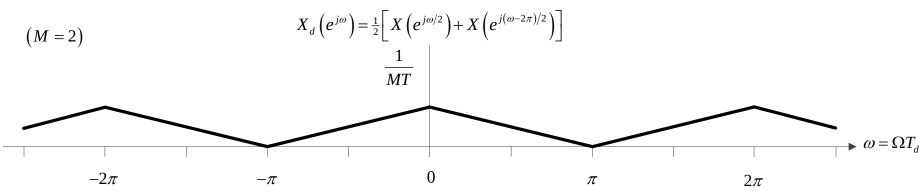

Figure 3 (d) The FT of the DT down-sampling input signal $x_d[n]$ (plotted as a function of DT or ω)

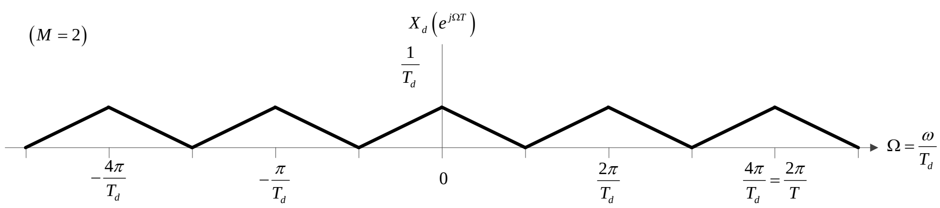

Figure 3 (e) The FT of the DT output signal $x_d[n] = x_c(nT_d)$ (plotted as a function of CT or Ω)

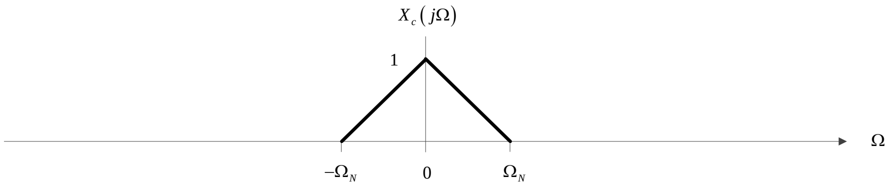

Figure 4 (a) The FT of a continuous-time (CT) bandlimited input signal $x_c(t)$

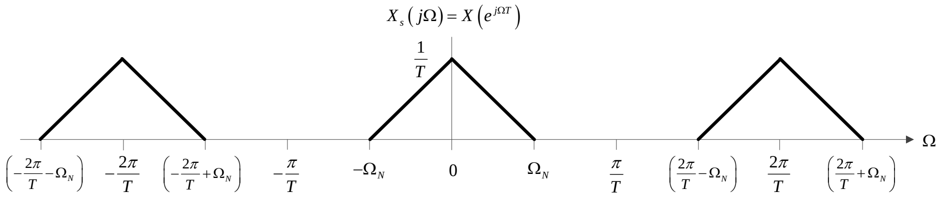

Figure 4 (b) The FT of the sampling signal of a CT bandlimited input signal $x_s(t)$

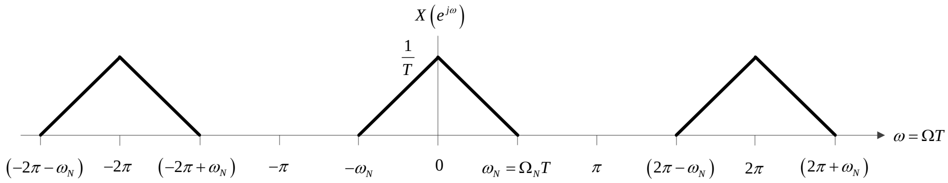

Figure 4 (c) The FT of the DT sampling input signal $x[n]$

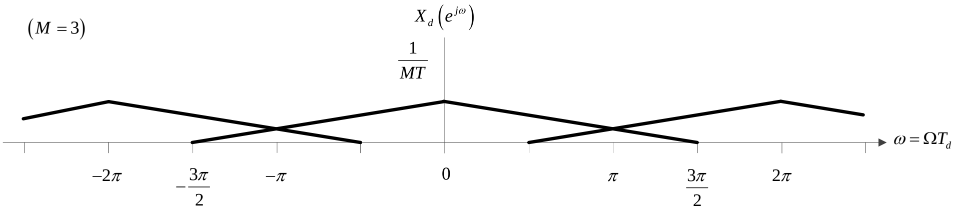

Figure 4 (d) The FT of the DT down-sampling input signal $x_d[n]$ (plotted as a function of DT or ω)

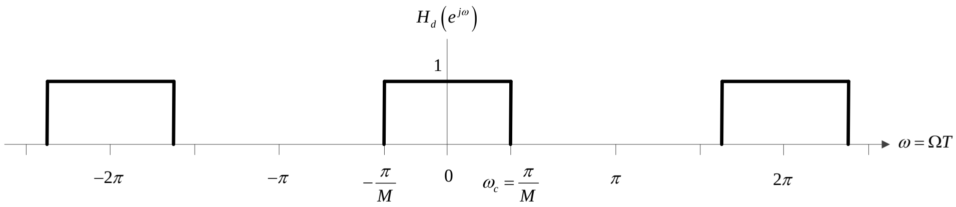

Figure 4 (e) The FT of the ideal lowpass filter with cutoff frequency π/M

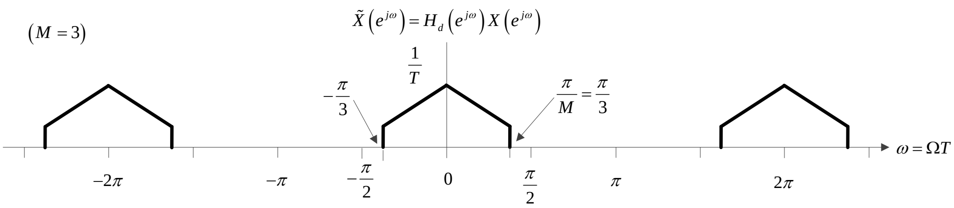

Figure 4 (f) The FT of the DT output signal $x[n]$ from the ideal lowpass filter with cutoff frequency π/M

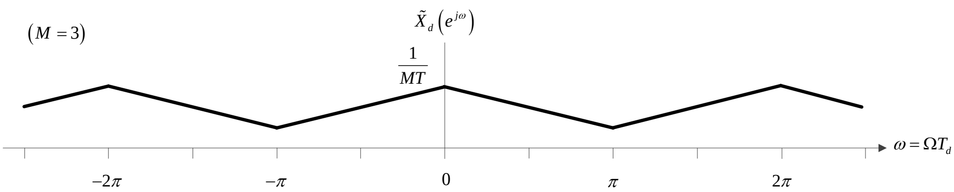

Figure 4 (g) The FT of the DT down-sampling input signal $x_d[n]$ when $M = 3$ (plotted as a function of DT or ω)

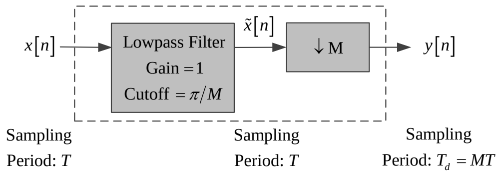

Figure 5 The general block diagram of a down-sampling system with a factor M

4. Mathematical Analysis of Up-Sampling Rate Process Concept with an Integer Factor

This section presents the mathematical analysis of an up-sampling rate process (so called an expander or interpolation [4]) of a discrete-time (DT) signal $x[n]$ with a factor L. The mathematical between a continuous-time (CT) signal $x_c(t)$ and a discrete-time (DT) signal $x[n]$ can be mathematically defined as

$$x[n]=x_c(nT)\rightarrow x_i[n]=x_c(nT_i)\text{ where }$$

$$T_i = T/L \tag{7.1}$$

From the above equation, the up-sampling DT signal $x_i[n]$ can be mathematically defined as

$$x_i[n]=x[n/L]=x_c(nT/L)\quad ,n=0,\pm L,\pm 2L,\ldots \tag{7.2}$$

The general block diagram of an up-sampling system with a factor L can be illustrated as the figure 6.

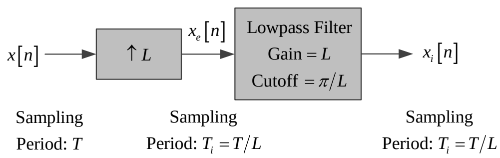

Figure 6 The general block diagram of an up-sampling system with a factor L

From an up-sampling system with a factor L as shown in the above figure, the mathematical between a DT up-sampling signal $x_e(t)$ and a discrete-time (DT) signal $x[n]$ can be mathematically defined as

$$x_e[n]=\begin{cases} x[n/L] & ,n=0,\pm L,\pm 2L,\ldots \\ 0 & ,\text{otherwise}\end{cases} \tag{8}$$

$$\rightarrow x_e[n]=\sum_{k=-\infty}^{\infty} x[k]\delta(n-kL) \tag{9}$$

The low-pass DT filter with cutoff frequency π/L and gain L performs as the D/C converter [9]. The Fourier transform of DT signal $x_e[n]$ can be mathematically defined as

$$X_e\left(e^{j\omega}\right)=\sum_{n=-\infty}^{\infty}\left(\sum_{k=-\infty}^{\infty} x[k]\delta[n-kL]\right)e^{-j\omega n}$$

$$\rightarrow \qquad X_e\left(e^{j\omega}\right)=\sum_{k=-\infty}^{\infty} x[k]e^{-j\omega Lk}$$

$$\rightarrow \qquad X_e\left(e^{j\omega}\right)=X\left(e^{j\omega L}\right) \tag{10}$$

Consequently, the Fourier transform of DT signal $x_e[n]$ is a frequency-scaled form of the Fourier transform of DT signal $x[n]$ (where ω is relieved by ωL) and ω is normalized by $\omega = \Omega T_i$.

From this frequency domain analysis [4, 11], the frequency domain of continuous-time (CT) bandlimited input signal $x_c(t)$, the DT sampling input signal $x[n]=x_c(nT)$, the DT up-sampling signal $x_e[n]$, the up-sampling low-pass filter (with cutoff frequency π/L and gain L) and the DT up-sampling output signal $x_i[n]$ can be illustrated in the figure 7(a), figure 7(b), figure 7(c), figure 7(d) and figure 7(e), respectively.

The impulse response of the up-sampling low-pass filter shown in figure 6 can be mathematically defined as

$$h_i[n] = \sin(\pi n/L)/(\pi n/L) \tag{10.1}$$

$$\rightarrow \quad h_i[0] = 1 \tag{10.2}$$

$$\rightarrow \quad h_i[n] = 0 \quad , n = \pm L, \pm 2L, \ldots \tag{10.3}$$

Then the DT up-sampling output signal $x_i[n]$ can be mathematically defined as

$$x_i[n] = \sum_{k=-\infty}^{\infty} x[k]\left(\sin\left(\pi(n-kT)/L\right)/\left(\pi(n-kT)/L\right)\right) \tag{11}$$

$$\rightarrow \quad x_i[n] = x[n/L] = x_c[nT/L] = x_c[nT_i] \quad , n = 0, \pm L, \pm 2L, \ldots \tag{12}$$

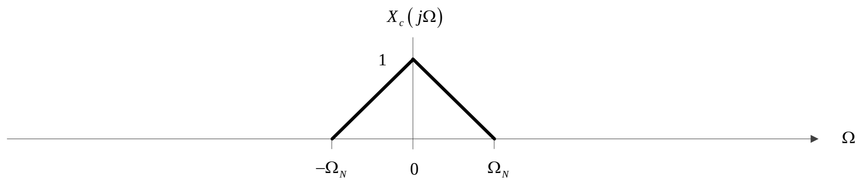

Figure 7 (a) The FT of a continuous-time (CT) bandlimited input signal $x_c(t)$

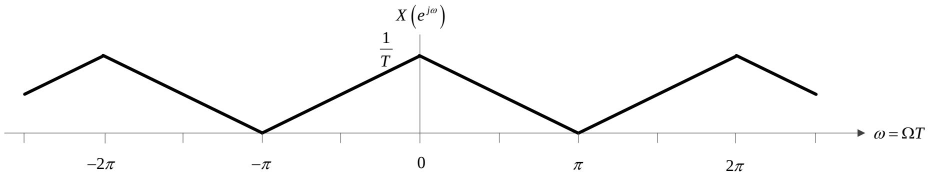

Figure 7 (b) The FT of the DT sampling input signal $x[n] = x_c(nT)$ where $\pi/T = \Omega_N$

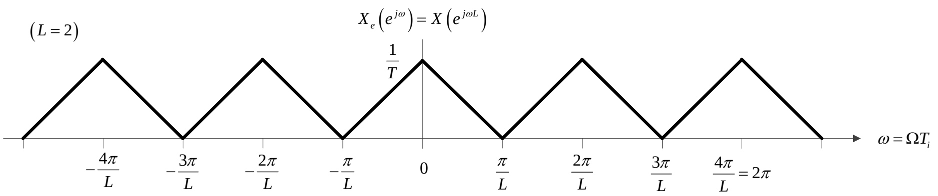

Figure 7 (c) The FT of the DT up-sampling DT signal $x_e[n]$ where $L = 2$

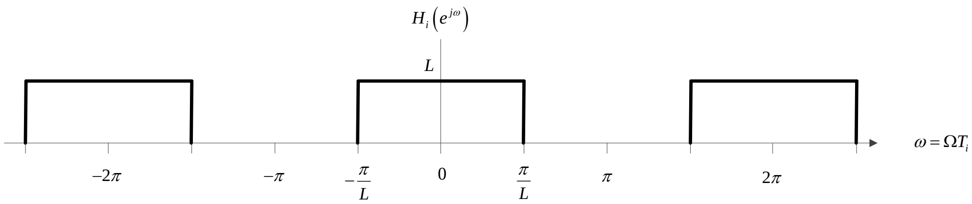

Figure 7 (d) The FT of the low-pass filter for a DT signal with cutoff frequency $\pi/2$ and gain 2

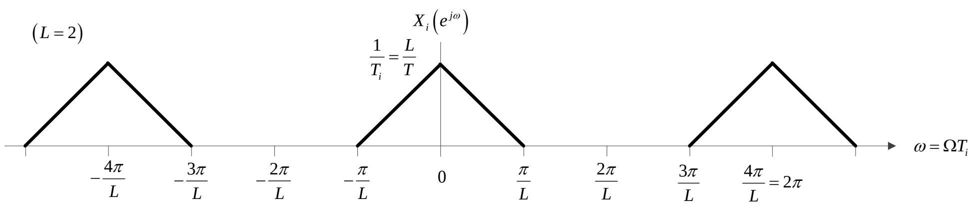

Figure 7 (e) The FT of the DT up-sampling DT signal $x_i[n]$ where $L = 2$

5. Mathematical Analysis of Efficient and Uncomplicated Filtering Concept

One of the most useful and efficient up-sampling process is the linear interpolation therefore this section present the mathematical analysis of the linear interpolation concept, which directly correlates to the interpolation (the interpolated signals between two original sampled signals depend on a straight line linking the two original sampled signals). The impulse response of the linear interpolation filter, which is the triangularly shape, can be mathematically defined as following equation and can be illustrated as figure 8.

$$h_{\text{lin}}[n] = \begin{cases} 1 - |n|/L & |n| \le L \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

$$\rightarrow \qquad H_{\text{lin}}\left(e^{j\omega}\right) = \frac{1}{L}\left[\frac{\sin(\omega L/2)}{\sin(\omega/2)}\right]^2 \tag{14}$$

From the above equation of an impulse response of the linear interpolation filter, the original sampled signals are maintain and protect because $h_{\text{lin}}[0] = 1$ and $h_{\text{lin}}[n] = 0, |n| \ge L$. Using this above linear interpolation filter, which is comprised to be a low-pass filter of a general system for up-sampling process as shown in figure 6, the output of the up-sampling process $x_{\text{lin}}[n]$ can be mathematically defined as

$$x_{\text{lin}}[n] = \sum_{k=n-L+1}^{n+L-1} x_e[k]\, h_{\text{lin}}[n-k] \tag{15}$$

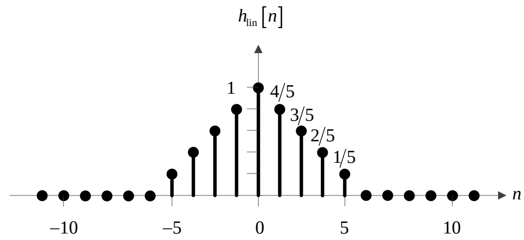

Figure 8 The impulse response of the linear interpolation filter where $L = 5$

Due to the bandlimited signal reconstructing and processing concept [9], if the original sampled signal is created by using the sampling process at the Nyquist rate [8], the reconstructed output of the linear interpolation filter is not precise because, in the frequency domain analysis, the reconstructed output comprises of the input signals in the frequency band $\pi/L < |\omega| \le \pi$ because the linear interpolation filter, as shown in the figure 7, cannot eliminate the signals in the frequency band $\pi/L < |\omega| \le \pi$ perfectly. Nevertheless, the reconstructed output (as shown in fig. 9(a) and 9(b)) of the linear interpolation filter can be precise if the original sampled signal is created by using the sampling process at the much higher than Nyquist rate because the main component of the input signals is contributed in the frequency band $|\omega| < \pi/L$ and only few main component of the input signals is contributed in the frequency band $\pi/L < |\omega| \le \pi$.

For precisely estimating the ideal bandlimited interpolation, the interpolation filter must have the longer impulse response therefore the FIR (Finite Impulse Response) filter is meet this requirement. Subsequently, the impulse response

$\tilde{h}[n]$ of an FIR filter, which is designed to be an interpolation filter with a factor L, will have mathematical properties as

$$\tilde{h}_i[n]=0 \quad ,|n|\geq KL \qquad (16\text{-}1)$$

$$\tilde{h}_i[n]=\tilde{h}_i[-n] \quad ,|n|\leq KL \qquad (16\text{-}2)$$

$$\tilde{h}_i[n]=1 \quad ,n=0 \qquad (16\text{-}3)$$

$$\tilde{h}_i[n]=0 \quad ,n=\pm L,\pm 2L,\ldots \qquad (16\text{-}4)$$

Therefore, the interpolated output signal $x_{\text{lin}}[n]$ can be mathematically defined as

$$\tilde{x}_i[n]=\sum_{k=n-KL+1}^{n+KL-1} x_e[k]\tilde{h}_i[n-k] \qquad (16\text{-}5)$$

The example of the interpolation calculation using the above FIR filter where $L=5$ and $K=2$ can be illustrated as figure 10.

6. Mathematical Analysis of Sampling Rate Changing Process Concept with a Non-Integer Factor

For designing the sampling rate changing process with a non-integer factor, the sampling rate changing process must comprise of both up-sampling process and down-sampling process as shown in the following figure. At first, the DT input signal $x[n]$ is up-sampled by a factor L (or the sampling period is reduced from T to T/L) in order to produce the $x_i[n]$ and, then, the up-sampled signal $x_i[n]$ is down-sampled by a factor M (or the sampling period is increased from T/L to TM/L) in order to produce the output signal $\tilde{x}_d[n]$, which has the sampling period TM/L. Therefore, if $M>L$ then the overall sampling rate changing process is down-sampling process but if $M<L$ then the overall sampling rate changing process is up-sampling process as shown in figure 11.

The example of a sampling rate changing process with a non-integer factor M/L (for $M=3$ and $L=2$) for the bandlimited input with the sampling rate equal to the Nyquist rate ($2\pi/T=2\Omega_N$) can be illustrated as figure 12.

From this frequency domain analysis [4,11], the frequency domain of continuous-time (CT) bandlimited input signal $x_c(t)$, the DT sampling input signal $x[n]$ (or $X(e^{j\omega})=(1/T)\sum_{k=-\infty}^{\infty} X_c[j((\omega/T)-(2\pi k/T))]$), the DT up-sampling signal $x_e[n]$ where $L=2$, the impulse response of a low-pass filter for a DT signal with cutoff frequency $\pi/3$ and gain 2, the output signal of the low-pass filter $x_i[n]$ and the DT sampling rate changing signal $\tilde{x}_d[n]$ can be illustrated in the figure 12(a), figure 12(b), figure 12(c), figure 12(d)), figure 12(e) and figure 12(f), respectively.

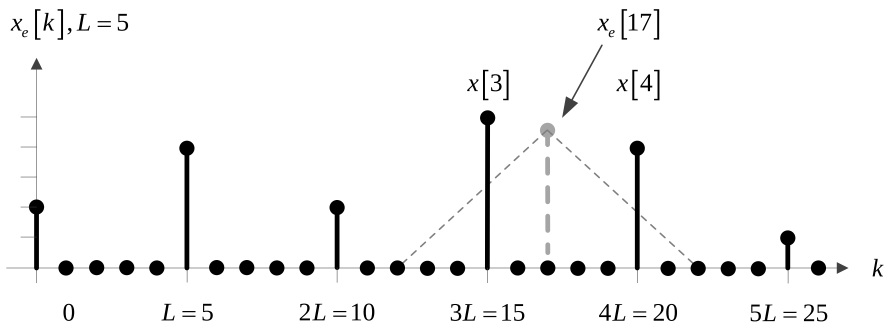

Figure 9 (a) The example of the linear interpolation calculation where $L=5$

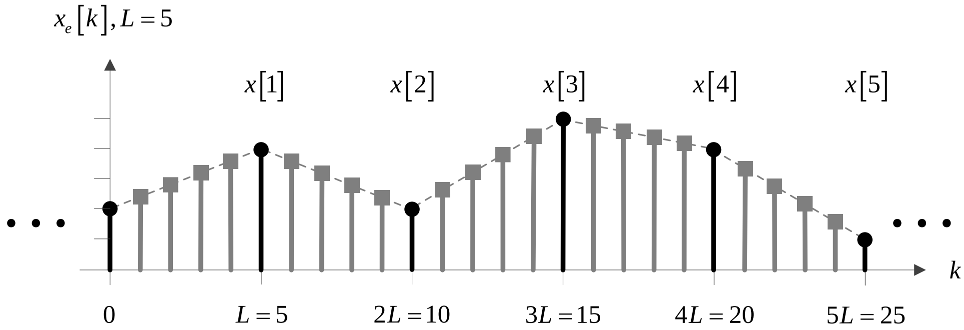

Figure 9 (b) The example of the result of the linear interpolation calculation where $L=5$

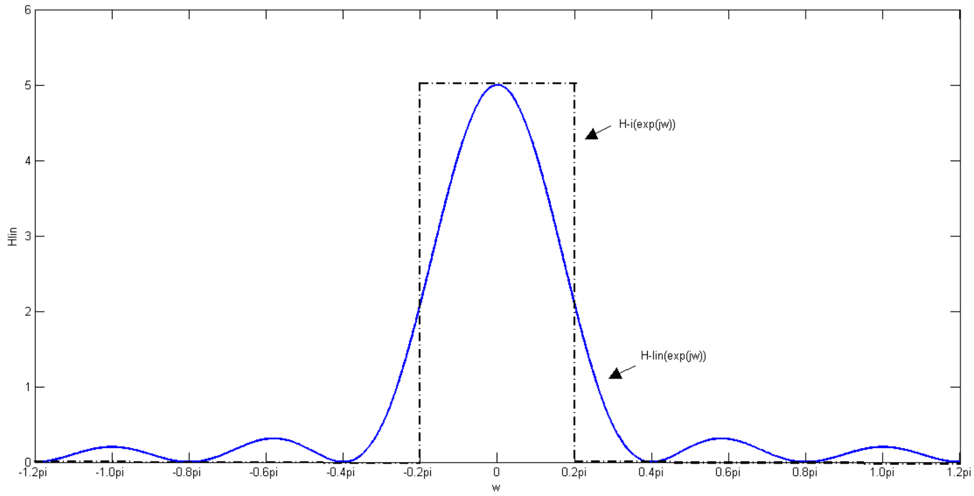

Figure 9 (c) The frequency response of the linear interpolation filter,

which is compared with the ideal low-pass interpolation filter

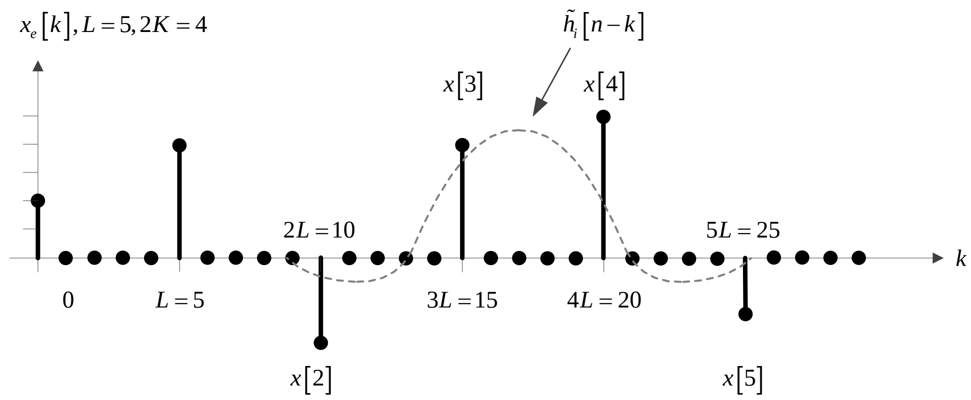

Figure 10 The example of the interpolation calculation using the above FIR filter where $L=5$ and $K=2$

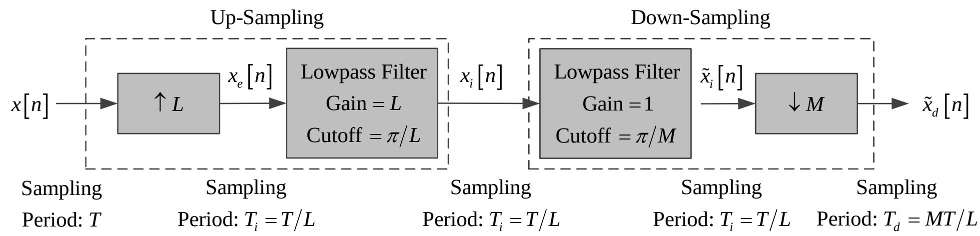

Figure 11 (a) The general block diagram of sampling rate changing process with a non-integer factor M/L

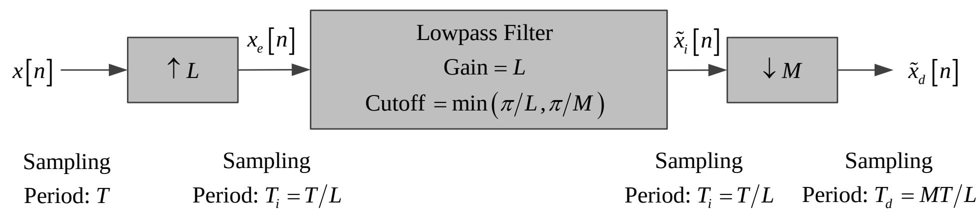

Figure 11 (b) The simplification of a general block diagram of sampling rate changing process with a non-integer factor M/L

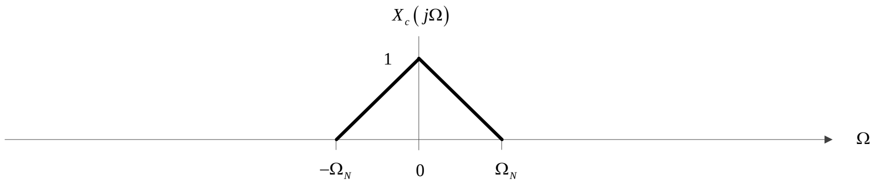

Figure 12 (a) The FT of a continuous-time (CT) bandlimited input signal $x_c(t)$

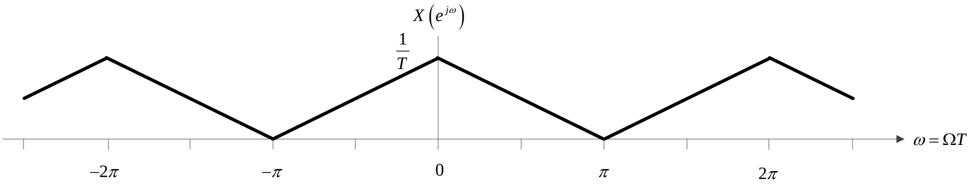

Figure 12 (b) The FT of the DT sampling input signal $x[n] = x_c(nT)$ where $\pi/T = \Omega_N$

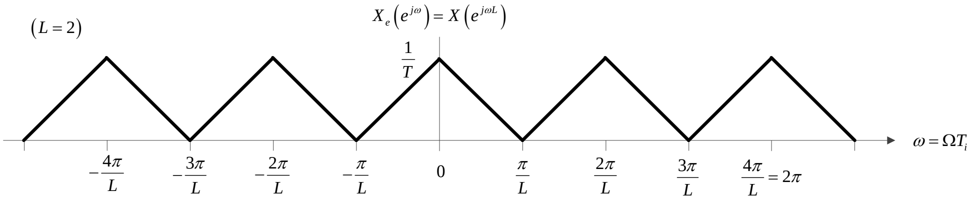

Figure 12 (c) The FT of the DT up-sampling DT signal $x_e[n]$ where $L = 2$

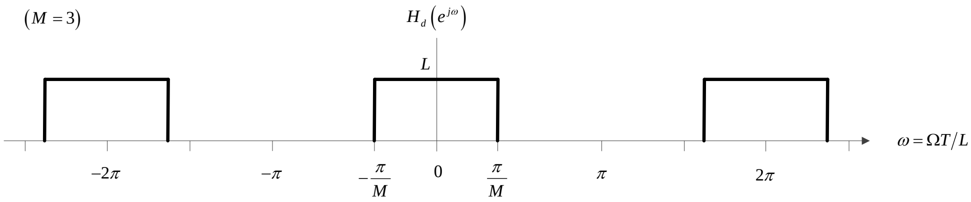

Figure 12 (d) The FT of the low-pass filter for a DT signal with cutoff frequency $\pi/3$ and gain 2

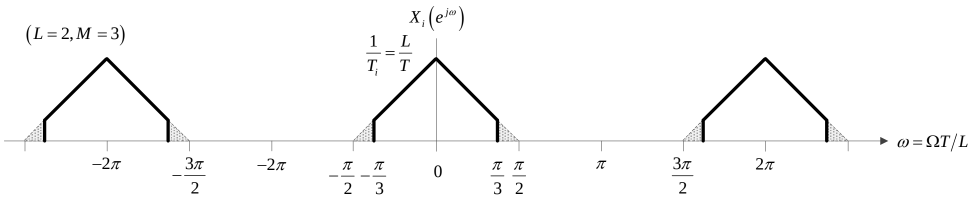

Figure 12 (e) The FT of the result of the low-pass filter for a DT signal with cutoff frequency $\pi/3$ and gain 2

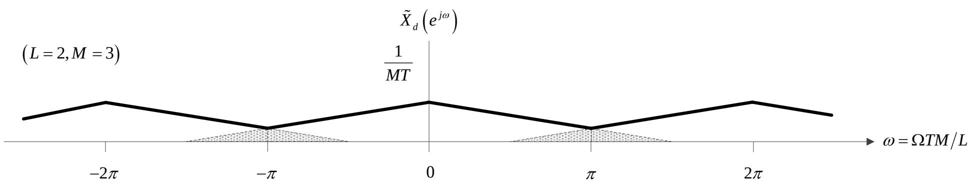

Figure 12 (f) The FT of the DT sampling rate changing signal $\tilde{x}_d[n]$ where $L = 2$ and $M = 3$

7. Conclusion

The imperative propose of this article was to devote the sampling rate changing concept of a DT bandlimited signal for the reader and its mathematical scrutiny for forthcoming researching because the sampling rate changing concept is usually applied for signal/video/speech standard format changing in Digital Signal Processing (DSP) [4] and digital communication [11] during the last fifteen years. Consequently, a lot of mathematical equations, mathematical explanations and pictures are devoted in this article.

Furthermore, authors have also applied sampling rate changing concept of a DT bandlimited signal in the research field of the Super Resolution Reconstruction (SRR) for up-sampling the lower resolution signal to be the higher resolution by using DT up-sampled process.

8. Acknowledgement

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9. References

[1] Haykin, S. & Veen, B. V. (2003). Signals and Systems. John Wiley & Sons, Inc., 2nd Edition.

[2] Ingle, Vinay K. & Proakis, John G. (2000), Digital Signal Processing using Matlab, Brooks/Cole Thomson Learning.

[3] Kreyszig, E. (2011). Advanced Engineering Mathematics. John Wiley & Sons, Inc., 10th Edition.

[4] Oppenheim, A.V. & Schafer, R.W. (2009). Discrete-Time Signal Processing. Prentice Hall, 3rd Edition.

[5] Oppenhiem, A. V., Willsky, A. S. & Nawab, S. H. (1997). Signals and Systems. Prentice-Hall, 2nd Edition.

[6] Patanavijit, V. (2011). The empirical performance study of a super resolution reconstruction based on frequency domain from aliased multi-low resolution images. In Proceedings of The 34th Electrical Engineering Conference (EECON-34), Ambassador City Jomtien Hotel, Pataya, Chonburi, Thailand, Dec., pp. 645-648.

[7] Patanavijit, V. (2016a). Conceptual framework of super resolution reconstruction based on frequency domain from Aliased multi-low resolution images: theory part. 14, Panyapiwat Journal, Panyapiwat Institute of Management (PIM), Thailand, Vol. 8, No. 2, May. – Aug. (indexed by TCI Group 1)

[8] Patanavijit, V. (2016b). Mathematical tutorial of discrete-time analysis of aliasing and non-aliasing periodic sampling concept for digital signal processing and digital communication prospective, SDU Research Journal Sciences and Technology, Suan Dusit Rajabhat University. Vol. 9, No. 3, Sep. – Dec. (indexed by TCI Group 1)

[9] Patanavijit, V. (2016c). Mathematical Tutorial of Discrete-Time Analysis of Bandlimited Signal Reconstructing and

Processing Concept for Digital Signal Processing and Digital Communication Prospective, SDU Research Journal Sciences and Technology, Suan Dusit Rajabhat University. (Submitted)

[10] Phillips, L., Parr, J. M. & Riskin, E. A. (2007). Signals, Systems, and Transforms. Prentice-Hall, 4th Edition.

[11] Proakis , J. & Salehi M. (2007), Digital Communications, McGraw-Hill Companies, 5th Edition

[12] Wylie, C. R. & Barrett, L. C. (1995). Advanced Engineering Mathematics. McGraw-Hill Companies, Inc., 6th Edition.