

Research Article

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Theorems of Geraghty Type Generalized F -contraction for Dislocated Quasi-metric Spaces

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Abstract

The generalized F -contraction, Geraghty contraction and admissible function are introduced as the most recent generalizations of the Banach contraction. The presence and uniqueness of fixed points for the newly constructed contraction's self-mapping on complete metric spaces were investigated. The findings of this article can be interpreted as an improvement on the key findings of the previous article.

Keywords: Geraghty type contraction mapping, F -contraction, Dislocated quasi-metric space

1. Introduction

The Banach contraction principle has numerous extensions and generalizations (1). By considering an auxiliary function, Geraghty (2) introduced an intriguing contraction and investigated the existence and uniqueness of such mappings in the setting of complete metric spaces. In the context of complete metric spaces, Samet et al. (3) proposed the idea of *new* contractive and admissible mappings and produced a variety of fixed point results for these types of mappings. Chandok (4) extended the concept of admissible mappings by introducing (α, β) -admissible mappings. As a generalization of the Banach contraction principle, Wardowski (5) presented a new contraction known as F -contraction and demonstrated a fixed point result.

Later, Hitzler (6) defined dislocated metric space as showed that the well-known Banach contraction mapping is valid in a space where the self-distance of points is not always zero. Zeyada et al. (7) expanded on the concept

of dislocated metric space and proposed the concept of dislocated quasi-metric space.

In this paper, we define admissible Geraghty type generalized F -contraction and establish some fixed point results for such contractions.

2. Preliminaries

In the sequel, the standard letters \mathbb{R} , \mathbb{R}^+ and \mathbb{N} will represent the set of all real numbers, the set of all nonnegative real numbers and natural numbers, respectively.

Definition 2.1. (7) Let \mathfrak{M} be a nonempty set and let $\theta: \mathfrak{M} \times \mathfrak{M} \rightarrow \mathbb{R}^+$ is a function such that the satisfying the necessary requirements:

- (i) $\theta(h, \omega) = \theta(\omega, h) = 0$ implies that $\omega = h$;
- (ii) $\theta(h, \omega) \leq \theta(h, \varrho) + \theta(\varrho, \omega)$ for all $h, \varrho, \omega \in \mathfrak{M}$.

Then θ is called dislocated quasi-metric on \mathfrak{M} and the pair (\mathfrak{M}, θ) is called a dislocated quasi-metric (DQM) space.

Definition 2.2. (8) Let $Z: \mathfrak{M} \rightarrow \mathfrak{M}$ be mapping and $\alpha: \mathfrak{M} \times \mathfrak{M} \rightarrow \mathbb{R}^+$ be a function. Then Z is said to be α -orbital admissible if $\alpha(h, Z\omega) \geq 1$ implies $\alpha(Zh, Z^2h) \geq 1$.

Definition 2.3. (8) Let \mathfrak{M} be a nonempty set and let $Z: \mathfrak{M} \rightarrow \mathfrak{M}$ be a mapping and $\alpha: \mathfrak{M} \times \mathfrak{M} \rightarrow \mathbb{R}^+$ be a function. Then Z is said to be triangular α -orbital admissible if Z is α -orbital admissible and $\alpha(h, \omega) \geq 1, \alpha(h, Z\omega) \geq 1$ implies $\alpha(\omega, Z\omega) \geq 1$.

Lemma 2.4. (8) Let $Z: \mathfrak{M} \rightarrow \mathfrak{M}$ be triangular α -orbital admissible mapping. Assume that there exists $h_1 \in \mathfrak{M}$ such that $\alpha(h_1, Zh_1) \geq 1$. Define a sequence $\{h_n\}$ by $h_{n+1} = Zh_n$. Then, we have $\alpha(h_n, h_m) \geq 1$ for all $m, n \in \mathbb{N}$ with $n < m$.

Definition 2.5. (9) Let $Z: \mathfrak{M} \rightarrow \mathfrak{M}$ be a mapping on a metric space. For each $h \in \mathfrak{M}$ and for any positive whole number n ,

$$O_Z(h, n) = \{h, Zh, \dots, Z^n h\}$$

and

$$O_Z(h, \infty) = \{h, Zh, \dots, Z^n h, \dots\}.$$

The set $O_Z(h, \infty)$ is called the orbit of Z at h and the metric space \mathfrak{M} is called Z -orbitally complete if every Cauchy sequence in set $O_Z(h, \infty)$ is convergent in \mathfrak{M} .

Definition 2.6. (5) Let (\mathfrak{M}, θ) be a metric space. The mapping $Z: \mathfrak{M} \rightarrow \mathfrak{M}$ is called an F -contraction, if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $h, \omega \in \mathfrak{M}$,

$$\theta(Zh, Z\omega) > 0 \Rightarrow \tau + F(\theta(Zh, Z\omega)) \leq F(\theta(h, \omega)), \tag{2.1}$$

where $F: (0, \infty) \rightarrow \mathbb{R}$ is strictly increasing $\lim_{n \rightarrow \infty} F(\alpha_n) = -\infty$ if and only if $\lim_{n \rightarrow \infty} \alpha_n = 0$ and there exists a number $\ell \in (0, 1)$ such that $\lim_{\alpha \rightarrow 0^+} \alpha^\ell F(\alpha) = -\infty$.

The family of all functions $F: (0, \infty) \rightarrow \mathbb{R}$ is denoted by \mathcal{F} if F satisfying the necessary requirements:

- (F1) F is strictly increasing;
- (F2) for every sequence $\{\alpha_n\}$ in $(0, \infty)$, we have $\lim_{n \rightarrow \infty} F(\alpha_n) = -\infty$ if and only if $\lim_{n \rightarrow \infty} \alpha_n = 0$;

(F3) there exists a number $\ell \in (0, 1)$ such that $\lim_{\alpha \rightarrow 0^+} \alpha^\ell F(\alpha) = -\infty$.

Definition 2.7. (2) Let Ω be the family of all functions $\phi: [0, \infty) \rightarrow [0, 1)$ which satisfy the condition

$$\lim_{n \rightarrow \infty} \phi(t_n) = 1 \text{ implies } \lim_{n \rightarrow \infty} t_n = 0. \tag{2.2}$$

Theorem 1.1. (2) Let (\mathfrak{M}, θ) be a complete metric space and let Z be a self-mapping on \mathfrak{M} . Suppose that there exists $\phi \in \Omega$ such that, for all $h, \omega \in \mathfrak{M}$,

$$\theta(Zh, Z\omega) \leq \phi(\theta(h, \omega)) \theta(h, \omega), \tag{2.3}$$

then Z has a unique fixed point $\zeta \in \mathfrak{M}$ and $\{Z^n \zeta\}$ converges to ζ for all $\zeta \in \mathfrak{M}$.

3. Main Results

In this section, the main achievements of this article are presented. The existence and uniqueness of fixed points of the self-mappings on Z -orbitally complete DQM spaces satisfying the $(\alpha, \beta) - F - G$ type contraction mapping with suitable assumptions.

Definition 3.1. Let (\mathfrak{M}, θ) be a DQM space, and $\alpha, \beta: \mathfrak{M} \times \mathfrak{M} \rightarrow \mathbb{R}^+$ be a function. A mapping $Z: \mathfrak{M} \rightarrow \mathfrak{M}$ is called an $(\alpha, \beta) - F$ -Geraghty $((\alpha, \beta) - F-G)$ type contraction mapping if there exists $\phi \in \Omega$ and $F \in \mathcal{F}$ such that, for all $h, \omega \in \mathfrak{M}$, with $\tau > 0, \theta(Zh, Z\omega) > 0$ and $\alpha(h, \omega) \geq 1$,

$$\alpha(h, \omega)\beta(h, \omega) \left(\tau + F(\theta(Zh, Z\omega)) \right) \leq \phi(\mathcal{D}_Z(h, \omega))F(\mathcal{D}_Z(h, \omega)), \tag{3.1}$$

where

$$\mathcal{D}_Z(h, \omega) = \max \left\{ \begin{array}{l} \theta(h, \omega), \theta(h, Zh), \theta(\omega, Z\omega), \\ \frac{\theta(Zh, \omega) + \theta(h, Z\omega)}{4}, \\ \frac{(1 + \theta(h, Zh))\theta(\omega, Z\omega)}{1 + \theta(h, \omega)} \end{array} \right\}.$$

Theorem 3.2. Let (\mathfrak{M}, θ) be a Z -orbitally complete DQM space such that $Z: \mathfrak{M} \rightarrow \mathfrak{M}$ is a mapping. $\alpha, \beta: \mathfrak{M} \times \mathfrak{M} \rightarrow \mathbb{R}^+$ is a function satisfying the necessary requirements:

- (i) \mathcal{Z} is an $(\alpha, \beta) - F-G$ type contraction mapping;
 - (ii) \mathcal{Z} is triangular (α, β) -orbital admissible mapping;
 - (iii) There exists $h_1 \in \mathfrak{M}$ such that $\alpha(h_1, \mathcal{Z}h_1) \geq 1$ and $\beta(h_1, \mathcal{Z}h_1) \geq 1$.
- Then \mathcal{Z} has a fixed point $\varsigma \in \mathfrak{M}$ and $\{\mathcal{Z}^n h_1\}$ converges to ς .

Proof. Let $h_1 \in \mathfrak{M}$ such that $\alpha(h_1, \mathcal{Z}h_1) \geq 1$ and $\beta(h_1, \mathcal{Z}h_1) \geq 1$. Define a sequence $\{h_n\}$ by $h_{n+1} = \mathcal{Z}^n h$, for $n \geq 1$. If $h_n = h_{n+1}$ for some n , then obviously \mathcal{Z} has a fixed point. Consequently, throughout the proof, we suppose that $h_n \neq h_{n+1}$ for all $n \geq 1$. By Lemma 2.3, used recursively, we have

$$\alpha(h_n, h_{n+1}) \geq 1 \text{ and } \beta(h_n, h_{n+1}) \geq 1 \quad (3.2)$$

By (3.1), we get

$$\begin{aligned} & \tau + F(\theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)) \\ & \leq \tau + F(\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)) \\ & \leq \alpha(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h) \beta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h) \\ & \quad \times (\tau + F(\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h))) \\ & \leq \phi(\mathfrak{D}_{\mathcal{Z}}(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)) F(\mathfrak{D}_{\mathcal{Z}}(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)), \end{aligned} \quad (3.3)$$

where

$$\begin{aligned} & \mathfrak{D}_{\mathcal{Z}}(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h) \\ & = \max \left\{ \frac{\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h), \theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^{n+1} h), \theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)}{\theta(\mathcal{Z}^n h, \mathcal{Z}^n h) + \theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^{n+1} h)}, \frac{(1 + \theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)) \theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)}{1 + \theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)} \right\} \\ & = \max \left\{ \frac{\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h), \theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)}{\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^{n+1} h)}, \frac{\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h), \theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)}{\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h) + \theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)} \right\} \\ & = \max\{\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h), \theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)\}. \end{aligned}$$

The assertion $\mathfrak{D}_{\mathcal{Z}}(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h) = \theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)$ is not true. This is because,

$$\begin{aligned} & \tau + F(\theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)) \\ & \leq \phi(\theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)) F(\theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)) \\ & \leq F(\theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)) \end{aligned} \quad (3.4)$$

is a contradiction. Consequently, $\theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h) < \theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)$. Thus,

$$\begin{aligned} & \tau + F(\theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)) \\ & \leq \phi(\theta(\mathcal{Z}^n h, \mathcal{Z}^{n-1} h)) F(\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)) \\ & \leq F(\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)) \end{aligned} \quad (3.5)$$

or

$$\begin{aligned} & F(\theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)) \\ & \leq F(\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)) - \tau. \end{aligned} \quad (3.6)$$

In general, one can get

$$\begin{aligned} & F(\theta(\mathcal{Z}^n h, \mathcal{Z}^{n+1} h)) \\ & \leq F(\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)) - n\tau. \end{aligned} \quad (3.7)$$

Letting $n \rightarrow \infty$ in (2.10) that shows $\lim_{n \rightarrow \infty} F(\theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h)) = -\infty$, hence

$$\lim_{n \rightarrow \infty} \theta(\mathcal{Z}^{n-1} h, \mathcal{Z}^n h) = 0. \quad (3.8)$$

Suppose that the sequence $\{h_n\}$ is not a Cauchy, then there exists $\epsilon > 0$ and we can define two subsequences $\{\mathcal{Z}^{m_l} h\}$ and $\{\mathcal{Z}^{n_l} h\}$ of the sequence $\{\mathcal{Z}^n h\}$ such that, for any $n_l > m_l > l$, $\theta(\mathcal{Z}^{m_l} h, \mathcal{Z}^{n_l} h) > \epsilon$, but $\theta(\mathcal{Z}^{m_l} h, \mathcal{Z}^{n_l-1} h) < \epsilon$, we observe that

$$\begin{aligned} \epsilon & \leq \theta(\mathcal{Z}^{m_l} h, \mathcal{Z}^{n_l} h) \\ & \leq \theta(\mathcal{Z}^{m_l} h, \mathcal{Z}^{n_l-1} h) + \theta(\mathcal{Z}^{n_l-1} h, \mathcal{Z}^{n_l} h) \\ & \leq \theta(\mathcal{Z}^{m_l} h, \mathcal{Z}^{m_l-1} h) + \theta(\mathcal{Z}^{m_l-1} h, \mathcal{Z}^{n_l-1} h) \\ & \quad + \theta(\mathcal{Z}^{n_l-1} h, \mathcal{Z}^{n_l} h) \\ & \leq \theta(\mathcal{Z}^{m_l} h, \mathcal{Z}^{m_l-1} h) + \theta(\mathcal{Z}^{m_l-1} h, \mathcal{Z}^{n_l-1} h) \\ & \quad + 2\theta(\mathcal{Z}^{n_l-1} h, \mathcal{Z}^{n_l} h) \\ & \leq \theta(\mathcal{Z}^{m_l} h, \mathcal{Z}^{m_l-1} h) + \epsilon 2\theta(\mathcal{Z}^{n_l-1} h, \mathcal{Z}^{n_l} h). \end{aligned} \quad (3.9)$$

Since $\theta(\mathcal{Z}^n h, \mathcal{Z}^{n-1} h) \neq 0$, we get

$$\begin{aligned} \lim_{l \rightarrow \infty} \theta(\mathcal{Z}^{m_l} h, \mathcal{Z}^{n_l} h) & = \lim_{l \rightarrow \infty} \theta(\mathcal{Z}^{m_l} h, \mathcal{Z}^{n_l-1} h) \\ & = \lim_{l \rightarrow \infty} \theta(\mathcal{Z}^{m_l-1} h, \mathcal{Z}^{n_l-1} h) \\ & = \lim_{l \rightarrow \infty} \theta(\mathcal{Z}^{m_l-1} h, \mathcal{Z}^{n_l} h) \\ & = \epsilon. \end{aligned} \quad (3.10)$$

From \mathcal{Z} is an $(\alpha, \beta) - F-G$ type contraction mapping and $\alpha(h, \omega) \geq 1$ and $\beta(h, \omega) \geq 1$, we obtain

$$\begin{aligned} & \tau + F(\theta(Z^{m_l-1}h, Z^{n_l-1}h)) \\ & \leq \alpha(Z^{m_l-1}h, Z^{n_l-1}h)\beta(Z^{m_l-1}h, Z^{n_l-1}h) \\ & \quad \times (\tau + F(\theta(Z^{m-1}h, Z^{n-1}h))) \\ & \leq \phi(\mathcal{D}_Z(Z^{m_l-1}h, Z^{n_l-1}h)) \\ & \quad \times F(\mathcal{D}_Z(Z^{m_l-1}h, Z^{n_l-1}h)), \end{aligned} \tag{3.11}$$

where

$$\mathcal{D}_Z(Z^{m_l-1}h, Z^{n_l-1}h) = \max \left\{ \begin{aligned} & \theta(Z^{m_l-1}h, Z^{n_l-1}h), \theta(Z^{m_l-1}h, Z^{m_l}h), \\ & \theta(Z^{n_l-1}h, Z^{n_l}h), \\ & \frac{\theta(Z^{m_l}h, Z^{n_l-1}h) + \theta(Z^{m_l-1}h, Z^{n_l}h)}{4}, \\ & \frac{(1 + \theta(Z^{m_l-1}h, Z^{m_l}h))\theta(Z^{n_l-1}h, Z^{n_l}h)}{1 + \theta(Z^{m_l-1}h, Z^{n_l-1}h)} \end{aligned} \right\}. \tag{3.12}$$

Letting $l \rightarrow \infty$ in (2.15) and using (2.13), we obtain

$$\lim_{l \rightarrow \infty} \theta(Z^{m_l-1}h, Z^{n_l-1}h) = \epsilon. \tag{3.13}$$

Because, $\lim_{l \rightarrow \infty} \phi(\mathcal{D}_Z(Z^{m_l-1}h, Z^{n_l-1}h)) \leq 1$, we conclude that

$$\tau + F(\epsilon) \leq \phi(\epsilon)F(\epsilon) \leq F(\epsilon), \tag{3.14}$$

a contradiction since $\tau > 0$. Therefore

$$\lim_{l \rightarrow \infty} \theta(Z^{m_l}h, Z^{n_l}h) = 0. \tag{3.15}$$

So, it follows that $\{Z^n h\}$ is a Cauchy sequence. From Z -orbitally complete, there exists $\zeta \in \mathfrak{M}$ such that $Z^n h \rightarrow \zeta$ as $n \rightarrow \infty$. To show that $Z\zeta = \zeta$, suppose that

$$\theta(\zeta, Z\zeta) = \lim_{n \rightarrow \infty} \theta(Z^n h, Z\zeta) > 0.$$

We have

$$\begin{aligned} & \tau + F(\theta(Z^{n+1}h, Z\zeta)) \\ & \leq \tau + F(\theta(Z^n h, Z\zeta)) \\ & \leq \alpha(Z^{n-1}h, \zeta)\beta(Z^{n-1}h, \zeta) \\ & \quad \times (\tau + F(\theta(Z^n h, Z\zeta))) \\ & \leq \phi(\mathcal{D}_Z(Z^{n-1}h, \zeta))F(\mathcal{D}_Z(Z^{n-1}h, \zeta)), \end{aligned} \tag{3.16}$$

where

$$\mathcal{D}_Z(Z^{n-1}h, \zeta) = \max \left\{ \begin{aligned} & \theta(Z^{n-1}h, \zeta), \theta(Z^{n-1}h, Z^n h), \theta(\zeta, Z\zeta), \\ & \frac{\theta(Z^n h, \zeta) + \theta(Z^{n-1}h, Z\zeta)}{4}, \\ & \frac{(1 + \theta(Z^{n-1}h, Z^n h))\theta(\zeta, Z\zeta)}{1 + \theta(Z^{n-1}h, \zeta)} \end{aligned} \right\}.$$

Letting $n \rightarrow \infty$, we get

$$\begin{aligned} & \lim_{n \rightarrow \infty} \mathcal{D}_Z(Z^{n-1}h, \zeta) \\ & = \max \left\{ \begin{aligned} & \theta(\zeta, \zeta), \theta(\zeta, \zeta), \theta(\zeta, Z\zeta), \frac{\theta(\zeta, \zeta) + \theta(\zeta, Z\zeta)}{4}, \\ & \frac{(1 + \theta(\zeta, \zeta))\theta(\zeta, Z\zeta)}{1 + \theta(\zeta, \zeta)} \end{aligned} \right\} \\ & = \max \left\{ \theta(\zeta, Z\zeta), \frac{\theta(\zeta, Z\zeta)}{4} \right\} \\ & = \theta(\zeta, Z\zeta). \end{aligned}$$

Hence, by letting the limits as $n \rightarrow \infty$ in (2.19), we get

$$\begin{aligned} F(\theta(\zeta, Z\zeta)) & \leq \phi(\theta(\zeta, Z\zeta))F(\theta(\zeta, Z\zeta)) - \tau \\ & \leq F(\theta(\zeta, Z\zeta)) - \tau \end{aligned}$$

a contradiction. Therefore, we obtain $\theta(\zeta, Z\zeta) = 0$. Similarly, $\theta(Z\zeta, \zeta) = 0$. That is, $\zeta = Z\zeta$ and the fixed point of Z is ζ .

Theorem 3.3. All the conditions of Theorem 2.2, we find that ζ is a unique fixed point of Z .

Proof. From the proof of Theorem 3.2, ζ is a fixed point of Z . Assume that ζ and δ are distinct fixed points of Z . By condition (ii) in Theorem 3.2, we get

$$\begin{aligned} & \tau + F(\theta(\zeta, \delta)) \\ & = \tau + F(\theta(Z\zeta, Z\delta)) \\ & \leq \alpha(\zeta, \delta)\beta(\zeta, \delta)(\tau + F(\theta(Z\zeta, Z\delta))) \\ & \leq \phi(\mathcal{D}_Z(\zeta, \delta))F(\mathcal{D}_Z(\zeta, \delta)), \end{aligned}$$

where

$$\begin{aligned} \mathcal{D}_Z(\zeta, \delta) & = \max \left\{ \begin{aligned} & \theta(\zeta, \delta), \theta(\zeta, Z\zeta), \theta(\delta, Z\delta), \\ & \frac{\theta(Z\zeta, \delta) + \theta(\zeta, Z\delta)}{4}, \\ & \frac{(1 + \theta(\zeta, Z\delta))\theta(\delta, Z\delta)}{1 + \theta(\zeta, \delta)} \end{aligned} \right\} \\ & = \theta(\zeta, \delta). \end{aligned}$$

Thus,

$$\tau + F(\theta(\zeta, \delta)) \leq \phi(\theta(\zeta, \delta))F(\theta(\zeta, \delta)) \leq F(\theta(\zeta, \delta)),$$

which is a contradiction, as $\tau > 0$. So, $\zeta = \delta$. Hence, \mathcal{Z} has a unique fixed point.

Corollary 3.4. Let (\mathfrak{M}, θ) be a \mathcal{Z} -orbitally complete DQM space and $\alpha, \beta : \mathfrak{M} \times \mathfrak{M} \rightarrow \mathbb{R}^+$ is a function. Suppose there exist $\phi \in \Omega$ and $F \in \mathcal{F}$ with $\tau > 0$ such that, for all $h, \omega \in \mathfrak{M}$ with $\theta(\mathcal{Z}h, \mathcal{Z}\omega) > 0$ and $\alpha(h, \omega) \geq 1$,

$$\alpha(h, \omega)\beta(h, \omega) \left(\tau + F(\theta(\mathcal{Z}h, \mathcal{Z}\omega)) \right) \leq \phi(\mathfrak{D}_{\mathcal{Z}}(h, \omega))F(\mathfrak{D}_{\mathcal{Z}}(h, \omega)),$$

where

$$\mathfrak{D}_{\mathcal{Z}}(h, \omega) = \max\{\theta(h, \omega), \theta(h, \mathcal{Z}h), \theta(\omega, \mathcal{Z}\omega)\}$$

and satisfying the necessary requirements:

(i) \mathcal{Z} is an (α, β) -F-G type contraction mapping;

(ii) There exists $h_1 \in \mathfrak{M}$ such that $\alpha(h_1, \mathcal{Z}h_1) \geq 1$ and $\beta(h_1, \mathcal{Z}h_1) \geq 1$.

Then \mathcal{Z} has a fixed point $\zeta \in \mathfrak{M}$ and $\{\mathcal{Z}^n h_1\}$ converges to ζ .

Proof. We obtain the proof by following the proof in Theorem 3.2.

Corollary 3.5. Let (\mathfrak{M}, θ) be a \mathcal{Z} -orbitally complete DQM space and $\alpha, \beta : \mathfrak{M} \times \mathfrak{M} \rightarrow \mathbb{R}^+$ is a function. Suppose there exist $\phi \in \Omega$ and $\tau > 0$ such that, for all $h, \omega \in \mathfrak{M}$ with $\theta(\mathcal{Z}h, \mathcal{Z}\omega) > 0$ and $\alpha(h, \omega) \geq 1$,

$$\alpha(h, \omega)\beta(h, \omega) \left(\tau + F(\theta(\mathcal{Z}h, \mathcal{Z}\omega)) \right) \leq \phi(\theta(h, \omega))F(\theta(h, \omega)),$$

and satisfying the necessary requirements:

(i) \mathcal{Z} is triangular (α, β) -orbital admissible mapping.

(ii) There exists $h_1 \in \mathfrak{M}$ such that $\alpha(h_1, \mathcal{Z}h_1) \geq 1$ and $\beta(h_1, \mathcal{Z}h_1) \geq 1$.

Then \mathcal{Z} has a fixed point $\zeta \in \mathfrak{M}$ and $\{\mathcal{Z}^n h_1\}$ converges to ζ .

Proof. We obtain the proof by following the proof in Theorem 3.2.

Corollary 3.6. Let (\mathfrak{M}, θ) be a \mathcal{Z} -orbitally complete DQM space. Suppose there exist $\phi \in \Omega$ and $F \in \mathcal{F}$ with $\tau > 0$ such that, for all $h, \omega \in \mathfrak{M}$ with $\theta(\mathcal{Z}h, \mathcal{Z}\omega) > 0$ and $\alpha(h, \omega) \geq 1$,

$$\tau + F(\theta(\mathcal{Z}h, \mathcal{Z}\omega)) \leq \phi(\mathfrak{D}_{\mathcal{Z}}(h, \omega))F(\mathfrak{D}_{\mathcal{Z}}(h, \omega)),$$

where

$$\mathfrak{D}_{\mathcal{Z}}(h, \omega) = \max \left\{ \begin{array}{l} \theta(h, \omega), \theta(h, \mathcal{Z}h), \theta(\omega, \mathcal{Z}\omega), \\ \frac{\theta(\mathcal{Z}h, \omega) + \theta(h, \mathcal{Z}\omega)}{4}, \\ \frac{(1 + \theta(h, \mathcal{Z}h))\theta(\omega, \mathcal{Z}\omega)}{1 + \theta(h, \omega)} \end{array} \right\},$$

and satisfying the necessary requirements:

(i) \mathcal{Z} is triangular (α, β) -orbital admissible mapping.

(ii) There exists $h_1 \in \mathfrak{M}$ such that $\alpha(h_1, \mathcal{Z}h_1) \geq 1$ and $\beta(h_1, \mathcal{Z}h_1) \geq 1$.

Then \mathcal{Z} has a fixed point $\zeta \in \mathfrak{M}$ and $\{\mathcal{Z}^n h_1\}$ converges to ζ .

Proof. Letting $\alpha(h, \omega) = 1$ and $\beta(h, \omega) = 1$, we obtain the proof by following the proof in Theorem 3.2.

Corollary 3.7. Let (\mathfrak{M}, θ) be a \mathcal{Z} -orbitally complete DQM space. Suppose there exist $\phi \in \Omega$ and $F \in \mathcal{F}$ with $\tau > 0$ such that, for all $h, \omega \in \mathfrak{M}$ with $\theta(\mathcal{Z}h, \mathcal{Z}\omega) > 0$ and $\alpha(h, \omega) \geq 1$,

$$\tau + F(\theta(\mathcal{Z}h, \mathcal{Z}\omega)) \leq \phi(\mathfrak{D}_{\mathcal{Z}}(h, \omega))F(\mathfrak{D}_{\mathcal{Z}}(h, \omega)),$$

where

$$\mathfrak{D}_{\mathcal{Z}}(h, \omega) = \max\{\theta(h, \omega), \theta(h, \mathcal{Z}h), \theta(\omega, \mathcal{Z}\omega)\}$$

and satisfying the necessary requirements:

(i) \mathcal{Z} is triangular (α, β) -orbital admissible mapping.

(ii) There exists $h_1 \in \mathfrak{M}$ such that $\alpha(h_1, \mathcal{Z}h_1) \geq 1$ and $\beta(h_1, \mathcal{Z}h_1) \geq 1$.

Then \mathcal{Z} has a fixed point $\zeta \in \mathfrak{M}$ and $\{\mathcal{Z}^n h_1\}$ converges to ζ .

Proof. Letting $\alpha(h, \omega) = 1$ and $\beta(h, \omega) = 1$, we obtain the proof by following the proof in Theorem 3.2.

Corollary 3.8. Let (\mathfrak{M}, θ) be a \mathcal{Z} -orbitally complete dislocated quasi-metric space. Suppose there exist $\phi \in \Omega$ and $F \in \mathcal{F}$ with $\tau > 0$ such that, for all $h, \omega \in \mathfrak{M}$ with $\theta(\mathcal{Z}h, \mathcal{Z}\omega) > 0$ and $\alpha(h, \omega) \geq 1$,

$$\tau + F(d(\mathcal{Z}h, \mathcal{Z}\omega)) \leq \beta(d(h, \omega))F(d(h, \omega)),$$

and satisfying the necessary requirements:

(i) \mathcal{Z} is triangular (α, β) -orbital admissible mapping.

(ii) There exists $h_1 \in \mathfrak{M}$ such that $\alpha(h_1, \mathcal{Z}h_1) \geq 1$ and $\beta(h_1, \mathcal{Z}h_1) \geq 1$.

Then \mathcal{Z} has a fixed point $\varsigma \in \mathfrak{M}$ and $\{\mathcal{Z}^n h_1\}$ converges to ς .

Proof. Letting $\alpha(h, \omega) = 1$ and $\beta(h, \omega) = 1$, we obtain the proof by following the proof in Theorem 3.2.

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