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A Novel Mixed Attribute-Variable Sampling Plan Based on a Multiple Dependent State Repetitive Framework

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Abstract

This paper presents a novel mixed acceptance sampling plan, termed the attribute-variable sampling plan based on a multiple dependent state repetitive framework (AVS-MDSR), designed to enhance efficiency and accuracy in modern quality inspection. AVS-MDSR integrates attribute, variable, and multiple dependent state repetitive sampling, using the process capability index as the primary decision statistic for normally distributed quality characteristics. A key innovation is the inclusion of a repetitive sampling stage, allowing re-inspection when initial results are inconclusive, thereby reducing the risks of accepting nonconforming lots and rejecting conforming ones. To ensure optimal performance, a genetic algorithm determines the plan's design parameters, minimizing the average sample number while controlling producer's and consumer's risks. Numerical studies, considering both symmetric and asymmetric nonconforming proportions, demonstrate the superiority of AVS-MDSR over existing mixed acceptance sampling plans, consistently reducing the required sample size. Its practical applicability is further validated with real industrial data. The findings confirm that AVS-MDSR provides an accurate and flexible solution for stringent quality control environments.

Keywords: Average Sample Number, Mixed Sampling Plan, Process Capability Index, Repetitive Sampling Plan

1. Introduction

In the realm of industrial manufacturing and quality assurance, the maintenance of rigorous control over production processes is fundamental to ensuring product consistency and reliability. An acceptance sampling plan serves as a pivotal statistical methodology that facilitates the decision to accept or reject a product lot based on the inspection of representative samples. Over time, various acceptance sampling plans—such as single, double, and multiple sampling plans—have been developed to achieve an optimal balance between inspection efficiency and quality assurance. Among these, the single sampling plan (SSP) remains the most prevalent due to its simplicity and ease of implementation. However, the SSP often lacks flexibility, particularly when rejected lots are not reconsidered, which may lead to inefficiencies and an increased risk burden for producers. To address these shortcomings, Wortham and Baker (1) introduced the multiple dependent state (MDS) sampling plan, which utilizes information from previously inspected lots to inform current decision-making. This dependency among lots enables a reduction in sample size without compromising decision accuracy. Subsequent studies, including those by (2) and (3), refined the MDS framework to minimize producer's and consumer's risks, while (4) further enhanced its efficiency by incorporating the process capability index (PCI). Collectively, these contributions mark a significant evolution toward adaptive and data-driven sampling methodologies capable of addressing the increasing complexity and quality demands of modern manufacturing systems.

Building upon these advancements, researchers have integrated repetitive sampling with the MDS framework, resulting in the multiple dependent state repetitive (MDSR) sampling plan. This hybrid approach offers notable reductions in both inspection time and cost by allowing iterative decision-making across consecutive production lots. Pioneering efforts in this area include (5), who developed the MDSR sampling plan based on the yield index for linear profiles, and (6), who extended the approach to variable sampling plan under normal distributions to minimize average sample size. Subsequent enhancements by (7) incorporated the PCI into the MDSR framework. More recent studies—such as those by (8), (9), (10), and (11)—have expanded the application of MDSR sampling plans to various lifetime and reliability models, thereby demonstrating their adaptability and practical relevance across diverse industrial contexts.

At a theoretical level, acceptance sampling plans are broadly classified into two principal categories based on data characteristics: attribute sampling, which deals with qualitative, dichotomous classifications (e.g., conforming/nonconforming), and variable sampling, which relies on quantitative, continuous measurements (e.g., length, weight, or temperature). Recognizing the complementary strengths of both methods, numerous researchers have focused on developing mixed acceptance sampling plans, which integrate attribute and variable sampling plans to achieve greater precision and cost efficiency. For instance, Aslam et al. (12) introduced a mixed repetitive sampling plan based on the PCI for normally distributed characteristics, effectively balancing producer's and consumer's risks. Subsequent extensions include the mixed MDS plan by (13) and the EWMA-based mixed acceptance sampling plan proposed by (14). Balamurali (15) later developed a chain sampling plan integrating attribute and variable components, while (16) proposed a two-stage mixed plan optimized for both producer and consumer protection. Continuing this trajectory, (17) and (18) formulated PCI-based mixed sampling frameworks for resubmitted lots to enhance inspection efficiency and reduce quality control costs. Most recently, Marques et al. (19) proposed a mixed repetitive sampling plan that leverages repetitive inspection to improve decision-making accuracy when initial sample results are inconclusive.

Although extensive research has been conducted on acceptance sampling plans—particularly on MDS and MDSR frameworks that integrate dependency and repetition to enhance decision accuracy—most existing studies have focused on either attribute-based or variable-based plans independently. Moreover, prior works have seldom addressed the challenge of integrating both data types within a single repetitive sampling framework that simultaneously minimizes inspection cost and maintains acceptable producer's and consumer's risks. In addition, while PCI has proven valuable for assessing process performance, its potential within a mixed repetitive context remains underexplored. To bridge this gap, the present study proposes a novel mixed acceptance sampling plan based on the PCI. The proposed plan integrates the strengths of attribute and variable sampling within an MDSR inspection structure, enabling more flexible and precise decision-making when initial results are inconclusive. This design aims to minimize classification errors and reduce the average sample number. The remainder of this paper is organized as follows. Section 2 presents the methodological formulation and optimization framework. Section 3 discusses numerical analyses, comparative results, and an empirical case study using real industrial data. Section 4 concludes the paper with key findings and directions for future research.

2. Materials and Experiment

2.1 Design of a new mixed acceptance sampling plan

In this section, we propose a new mixed sampling plan that integrates attribute, variable, and MDSR strategies, termed the Attribute-Variable Sampling Plan based on a Multiple Dependent State Repetitive framework (AVS-MDSR). This sampling plan presents an approach aimed at enhancing the precision and efficiency of product or process quality inspection. The attribute sampling component focuses on the inspection of defects or product non-conformance based on enumeration. Meanwhile, the variable sampling component evaluates quantitative data that is continuously measurable, such as weight or length. The MDSR sampling component serves to minimize decision errors and improve the likelihood of a correct disposition by allowing for additional data collection when initial sample results are inconclusive. Typically, variable sampling plans utilize the PCI, denoted as C_{pk}, as the basis for making acceptance decisions. This index is applicable when the data is assumed to follow a normal distribution with a mean of μ and a variance of σ^2. Under these conditions, the value C_{pk} can be calculated as shown in equation (2.1).

$$C_{pk} = min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right) \quad (2.1)$$

In cases where the population mean and variance are unknown, these parameters can be estimated using the sample mean, denoted as $\bar{X}$, and the sample variance, denoted as S^2. Therefore, the estimated value of C_{pk} is shown in equation (2.2).

$$\hat{C}_{pk} = min\left(\frac{USL-\bar{X}}{3S}, \frac{\bar{X}-LSL}{3S}\right) \quad (2.2)$$

Assume a quality characteristic is modeled by a normal distribution with an unknown mean, μ, and an unknown standard deviation, σ. The manufacturing process is defined by two-sided specification limits: an upper specification limit (USL) and a lower specification limit (LSL). An item is classified as a nonconforming item if the value of its quality characteristic falls outside the range set by the USL or LSL. The operating procedure for the AVS-MDSR, which is based on the estimated PCI, $\hat{C}_{pk}$, is detailed as follows and shown in Figure 1.

Step 1: (Attribute Inspection) Take a random sample of size n_1 and count the number of nonconforming items, d. If $d \le c$, accept the current lot immediately. Otherwise, proceed to Step 2.

Step 2: (Variable Inspection) Take a second random sample of size n_2 and then calculate the $\hat{C}_{pk}$. If $\hat{C}_{pk} \ge k_a$, accept the current lot. Otherwise, proceed to Step 3.

Step 3: The decision in this step depends on the following conditions:

Acceptance: Accept the current lot if $k_d \le \hat{C}_{pk} < k_a$ and m previous lots are accepted as $\hat{C}_{pk} \ge k_a$.

Repetition: If $k_r < \hat{C}_{pk} \le k_d$, the inspection process is repeated, starting again from Step 2. If any of the conditions for acceptance or repetition are not met, the current lot will be rejected.

This proposed sampling plan is defined by the parameters $n_1, n_2, d, c, m, k_r, k_d, k_a$, subject to the condition that $k_a > k_d > k_r$.
Where the parameters are defined as

- n_1: The sample size for the first sampling stage.
- n_2: The sample size for the second sampling stage.
- d: The number of nonconforming items found in the first sample.
- c: The maximum allowable number of nonconforming items in the first sample for immediate acceptance.
- m: The number of previous lots considered.
- k_r: The rejection threshold; if the process estimate falls below this value, the lot is rejected.
- k_d: The decision threshold; if the process estimate falls between this value and k_r, the sampling is repeated.
- k_a: The acceptance threshold; if the process estimate is equal to or greater than this value, the lot is accepted.

The probability of acceptance for each case is given as follows:

Acceptance type 1 (attribute inspection)

This occurs when the condition from step 1 of the procedure is met, where $d \le c$. The lot is accepted immediately, and the probability of this event, denoted as P_{a1}, is given by the cumulative binomial probability in equation (2.3):

$$P_{a1} = P(d \le c) = \sum_{d=0}^{c} \binom{n}{d} p^d (1-p)^{n-d} \quad (2.3)$$

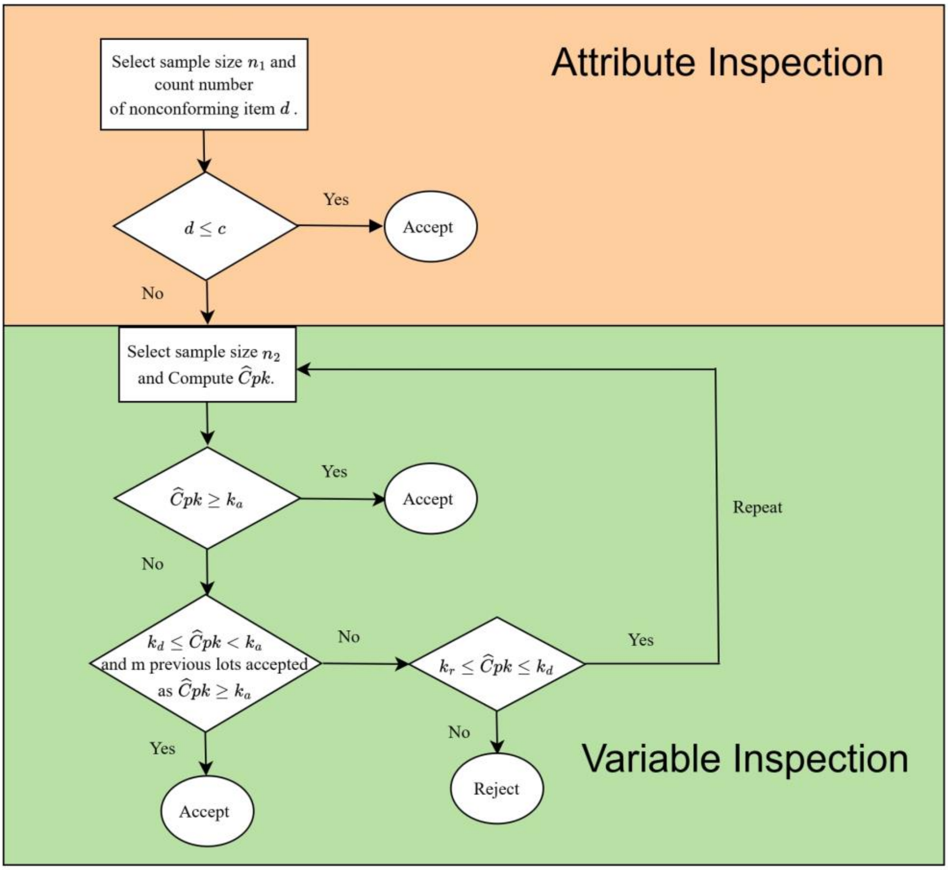

Figure 1 Operation procedure of the AVS-MDSR

Acceptance type 2 (variable inspection)

This occurs when the condition from step 2 of the procedure is met, where $\hat{C}_{pk} \geq k_a$. The lot is accepted based on the variable inspection, and the probability of this event, denoted as P_{a2}, is given by equation (2.4):

$$P_{a2} = P(\hat{C}_{pk} \geq k_a) \quad (2.4)$$

Acceptance type 3 (variable inspection)

This acceptance occurs under the condition from step 3 of the procedure, where $k_d \leq \hat{C}_{pk} < k_a$, and additionally, the $m$previous lots were accepted with $\hat{C}_{pk} \geq k_a$. The lot is then accepted, and the probability of this event, denoted as P_{a3}, is given by equation (2.5):

$$P_{a3} = P(k_d \leq \hat{C}_{pk} < k_a)\left[P(\hat{C}_{pk} \geq k_a)\right]^m \quad (2.5)$$

where

$$P\left(k_d \leq \hat{C}_{pk} < k_a\right) = P\left(\hat{C}_{pk} \leq k_a\right) - P\left(\hat{C}_{pk} \leq k_d\right) = P(\hat{C}_{pk} > k_d) - P(\hat{C}_{pk} > k_a).$$

Acceptance type 4 (variable inspection and repetitive sampling)

This case occurs when the condition for resampling is met, as defined in Step 3 of the procedure $(k_r \leq \hat{C}_{pk} < k_d)$. In this scenario, the lot is not immediately accepted or rejected; instead, the process enters a repetitive sampling phase. The probability of eventually accepting the lot through this iterative process, denoted as P_{a4}, is given by equation (2.6):

$$P_{a4} = P(k_r \le \hat{C}_{pk} < k_d)\left[\frac{P(\hat{C}_{pk} \ge k_a)}{P(\hat{C}_{pk} \ge k_a) + P(\hat{C}_{pk} < k_r)}\right] \quad (2.6)$$

where the probability of entering the repetitive phase is calculated as $P(k_r \le \hat{C}_{pk} < k_d) = P(\hat{C}_{pk} > k_r) - P(\hat{C}_{pk} > k_d)$. Therefore, the overall probability of lot acceptance for the AVS-MDSR, denoted as $P_a(p)$, is presented in equations (2.7) and (2.8).

$$P_a(p) = P_{a1} + (1 - P_{a1})P_V\ ; \quad P_V = P_{a2} + P_{a3} + P_{a4} \quad (2.7)$$

$$P_a(p) = P(d \le c) + P(\hat{C}_{pk} \ge k_a) + P(k_d \le \hat{C}_{pk} < k_a)\left[P(\hat{C}_{pk} \ge k_a)\right]^m + P(k_r \le \hat{C}_{pk} < k_d)\left[\frac{P(\hat{C}_{pk} \ge k_a)}{P(\hat{C}_{pk} \ge k_a) + P(\hat{C}_{pk} < k_r)}\right] \quad (2.8)$$

The average sample number (ASN) for this sampling plan, which represents the average number of items inspected per lot, is given by equation (2.9):

$$ASN = n_1 + n_2(1 - P_{a1}) = n_1 + n_2(1 - P(d \le c)) \quad (2.9)$$

2.2 Probability of acceptance for asymmetric fraction nonconforming case

For cases where the fraction of nonconforming items is asymmetric, it is assumed that $P(\bar{X} < LSL) = p_L$ and $P(\bar{X} > USL) = p_U$. Here, p_L represents the fraction nonconforming relative to the LSL, and p_U represents the fraction nonconforming relative to the USL, with the total fraction nonconforming being $p_L + p_U = p$. Assuming that the product lifetime follows a normal distribution with a mean of μ and a variance of σ^2, the key probabilities of lot acceptance for the AVS-MDSR can be determined as follows:

$$P(\hat{C}_{pk} \ge k_a) = \Phi\left((z_{pU} - 3k_a)\sqrt{\frac{n_1}{(1+\frac{9k_a^2}{2})}}\right) - \Phi\left(-(z_{pL} - 3k_a)\sqrt{\frac{n_1}{(1+\frac{9k_a^2}{2})}}\right) \quad (2.10)$$

$$P(k_d \le \hat{C}_{pk} < k_a) = \Phi\left((z_{pU} - 3k_d)\sqrt{\frac{n_1}{(1+\frac{9k_d^2}{2})}}\right) - \Phi\left(-(z_{pL} - 3k_d)\sqrt{\frac{n_1}{(1+\frac{9k_d^2}{2})}}\right) - \Phi\left((z_{pU} - 3k_a)\sqrt{\frac{n_1}{(1+\frac{9k_a^2}{2})}}\right) + \Phi\left(-(z_{pL} - 3k_a)\sqrt{\frac{n_1}{(1+\frac{9k_a^2}{2})}}\right) \quad (2.11)$$

$$P(k_r \le \hat{C}_{pk} < k_d) = \Phi\left((z_{pU} - 3k_r)\sqrt{\frac{n_1}{(1+\frac{9k_d^2}{2})}}\right) - \Phi\left(-(z_{pL} - 3k_r)\sqrt{\frac{n_1}{(1+\frac{9k_r^2}{2})}}\right) - \Phi\left((z_{pU} - 3k_d)\sqrt{\frac{n_1}{(1+\frac{9k_d^2}{2})}}\right) + \Phi\left(-(z_{pL} - 3k_d)\sqrt{\frac{n_1}{(1+\frac{9k_d^2}{2})}}\right) \quad (2.12)$$

where $\Phi(\cdot)$ represents the cumulative distribution function (CDF) of the standard normal distribution. $z_p$represents the standard score (z-score) corresponding to a cumulative probability of p.

2.3 Probability of lot acceptance for symmetric fraction nonconforming case

When the fraction nonconforming is symmetric, it is assumed that the probabilities of falling outside the specification limits are equal:$P(\bar{X} < LSL) = P(\bar{X} > USL) = \frac{p}{2}$. Consequently, this means that$p_U = p_L = \frac{p}{2}$. For a symmetrically distributed fraction nonconforming, the probability of lot rejection under the AVS-MDSR is given as follows:

$$P(\hat{C}_{pk} < k_r) = 2\Phi\left((z_{\frac{p}{2}} - 3k_a)\sqrt{\frac{n_1}{1+\frac{9k_a^2}{2}}}\right) - 1 \quad (2.13)$$

$$P\left(k_r \le \hat{C}_{pk} < k_d\right) = 2\Phi\left((z_{\frac{p}{2}} - 3k_r)\sqrt{\frac{n_1}{(1+\frac{9k_r^2}{2})}}\right) - 2\Phi\left((z_{\frac{p}{2}} - 3k_d)\sqrt{\frac{n_1}{(1+\frac{9k_d^2}{2})}}\right) \quad (2.14)$$

$$P\left(k_d \le \hat{C}_{pk} < k_a\right) = 2\Phi\left((z_{\frac{p}{2}} - 3k_d)\sqrt{\frac{n_1}{(1+\frac{9k_r^2}{2})}}\right) - 2\Phi\left((z_{\frac{p}{2}} - 3k_a)\sqrt{\frac{n_1}{(1+\frac{9k_a^2}{2})}}\right) \quad (2.15)$$

$$P_a(p) = P(d \le c) + 2\Phi\left((z_{\frac{p}{2}} - 3k_a)\sqrt{\frac{n_1}{1+\frac{9k_a^2}{2}}}\right) - 1 + 2\Phi\left((z_{\frac{p}{2}} - 3k_r)\sqrt{\frac{n_1}{(1+\frac{9k_r^2}{2})}}\right) - 2\Phi\left((z_{\frac{p}{2}} - 3k_d)\sqrt{\frac{n_1}{(1+\frac{9k_a^2}{2})}}\right) + 2\Phi\left((z_{\frac{p}{2}} - 3k_d)\sqrt{\frac{n_1}{(1+\frac{9k_r^2}{2})}}\right) - 2\Phi\left((z_{\frac{p}{2}} - 3k_a)\sqrt{\frac{n_1}{(1+\frac{9k_a^2}{2})}}\right) \quad (2.16)$$

2.4 Optimization of the AVS-MDSR using a genetic algorithm

This section focuses on determining the optimal parameters for the AVS-MDSR by employing a genetic algorithm (GA) optimization. To determine the optimal parameters of the AVS-MDSR$(n_1, n_2, c, m, k_a, k_d, k_r)$, the attribute component is modeled with a binomial distribution, while the variable component is represented by a normal distribution, ensuring accurate and efficient sampling decisions. The objective is to satisfy both the producer's risk (α)at a specified acceptable quality level (AQL or p_1) and the consumer's risk (β)at a specified rejectable quality level (RQL or p_2). The implementation of this AVS-MDSR is defined by studying two key points on its operating characteristic (OC) curve: the producer's risk point, $(AQL, 1-\alpha)$, and the consumer's risk point, (RQL, β). The producer requires that the probability of accepting a lot at the $AQL(p_1)$ must be greater than or equal to$1-\alpha$. Conversely, the consumer requires that the probability of accepting a lot at the $RQL(p_2)$ must be less than or equal to β. This research employs the GA, a non-linear optimization technique, to determine the optimal set of plan parameters using the R program. The optimization problem is formulated with the following objective function and constraints:

Objective function

$$\text{Minimize } \frac{1}{2}[ASN(p_1) + ASN(p_2)]$$

Subject to

$$P_a(p_1) \ge 1-\alpha$$
$$P_a(p_2) \le \beta$$
$$n_1 > n_2 \ge 1, c \ge 0, k_a > k_d > k_r > 0, m \ge 1.$$

3. Results

3.1 Numerical Illustration of the AVS-MDSR

From Tables 1 and 2, it is observed that when p_1 is held constant, an increase in the value of p_2 leads to a decrease in the$ASN$for both symmetric and asymmetric cases. A comparison between the tables reveals that for the same given values of p_1 and p_2, the ASN is consistently greater in the case of a symmetric nonconforming fraction compared to the asymmetric case. For instance, with $p_1 = 0.001$ and $p_2 = 0.002$, the ASN for the symmetric case is 178.679, while the ASN for the asymmetric case is 162.575. This demonstrates that while the symmetric case yields a higher ASN, the general trend for both scenarios is that an increase in p_2 (while p_1 is constant) leads to a reduction in the required ASN.

Example 1 (Symmetric case)

Assume that the quality characteristic of the data follows a normal distribution. A quality inspector needs to select the optimal parameter of the AVS-MDSR for a given p_1 at a producer's risk level of $\alpha = 0.05$ and for a given p_2 at a consumer's risk level of $\beta = 0.01$. To implement the AVS-MDSR, assume the quality levels are specified as $p_1 = 0.001$ and $p_2 = 0.008$, respectively. From Table 1, the quality inspector can find the optimal parameters for the AVS-MDSR as follows. $n_1 = 132, n_2 = 17, c = 1, m = 1, k_a = 1.8930, k_d = 0.7577, k_r = 0.2528$. For the optimal AVS-MDSR, the probability of acceptance at p_1 is $P_a(p_1) = 0.9692$, the probability of acceptance at p_2 is $P_a(p_2) = 0.0961$, and the ASN is 134.489, respectively.

Example 2 (Asymmetric case)

In the asymmetric case, assume that the quality characteristic follows a normal distribution as defined in Table 2. Using repetitive sampling, the optimal acceptance sampling plan can be determined to meet the specified requirements. Specifically, a quality inspector needs to select a plan for a given p_1 at a producer's risk level of $\alpha = 0.05$ for a given p_2 at a consumer's risk level of $\beta = 0.01$. To implement the AVS-MDSR, assume the quality levels are specified as $p_1 = 0.0025$ and $p_2 = 0.015$, respectively. From Table 2, the quality inspector can find the optimal parameters for the AVS-MDSR as follows: $n_1 = 62, n_2 = 15, c = 1, m = 1, k_a = 1.8926, k_d = 0.5492, k_r = 0.3458$. For the optimal AVS-MDSR, the probability of acceptance at p_1 is $P_a(p_1) = 0.9578$, the probability of acceptance at p_2 is $P_a(p_2) = 0.0746$, and the ASN is 63.86.

3.2 Application of the AVS-MDSR to real data

The data (amplified pressure sensor) was studied by (20). The dataset contains 128 values as follows: 1.9422 1.9651 2.0230 1.9712 1.9975 2.0164 1.9927 1.9566 1.9738 1.9541 1.9800 1.9596 1.9811 2.0088 1.9858 1.9677 2.0001 1.9659 1.9955 1.9842 1.9909 1.9829 1.9684 1.9942 1.9897 1.9836 1.9891 1.9608 2.0109 1.9912 2.0077 1.9803 2.0106 1.9885 1.9704 1.9882 1.9689 1.9553 1.9741 1.9825 1.9640 2.0187 1.9616 1.9865 1.9556 1.9817 1.9774 1.9316 1.9841 1.9919 1.9737 1.9958 2.0121 2.0021 1.9665 1.9773 1.9841 1.9570 1.9610 2.0015 1.9750 1.9825 1.9758 1.9682 1.9668 1.9696 2.0334 1.9656 1.9819 2.0116 1.9754 1.9986 2.0114 1.9861 1.9743 1.9594 1.9712 1.9849 1.9711 1.9486 1.9837 1.9424 1.9744 1.9605 1.9719 1.9656 1.9549 2.0174 1.9779 2.0072 1.9875 1.9781 1.9834 1.9893 1.9276 1.9513 1.9971 1.9963 1.9375 1.9941 1.9763 2.0108 1.9687 1.9559 1.9611 1.9729 1.9992 1.9925 2.0073 1.9742 1.9557 1.9726 1.9964 1.9614 1.9768 1.9991 1.9832 1.9847 1.9849 1.9918 1.9748 1.9664 2.0035 1.9822 1.9882 1.9809 1.9920 1.9994

Given that USL = 2.1 V and LSL = 1.9 V, where $p_U = p_L = \frac{p}{2}$. Assume $p_1 = 0.001$ and $p_2 = 0.01$ with $\alpha = 0.05$ and $\beta = 0.10$, from Table 1, it is found that the optimal parameters are $n_1 = 95, n_2 = 13, c = 1, m = 1, k_a = 1.7191, k_d = 0.6871$ and $k_r = 0.6013$.

It is also found that the ASN value is 98.24944, respectively. The proposed sampling plan can be applied to the data as follows:

Step 1 (attribute inspection): Take the first sample of size $n_1 = 95$ and count the number of defective items, d. If $d \leq 1$, the lot is accepted. Otherwise, proceed to Step 2.

Step 2 (variable inspection): Take a second sample of size $n_2 = 13$.

[List of 13 numerical data points]

1.9422 1.9651 2.0230 1.9712
1.9975 2.0164 1.9927 1.9566
2.0088 1.9858 1.9677 2.0001
1.9659

As illustrated in Figure 2, the sampled data closely follows a normal distribution, as confirmed by the Kolmogorov–Smirnov (K–S) test with a test statistic of 0.1605 and a corresponding p-value of 0.843, indicating no significant deviation from normality. The sample mean ($\bar{X}$) and sample standard deviation (S) are then calculated: $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = 1.9841$ and $S = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}} = 0.0247$

From these values, the estimated $\hat{C}_{pk}$ is calculated:

$$\hat{C}_{pk} = min\left\{\frac{USL - \bar{X}}{3S}, \frac{\bar{X} - LSL}{3S}\right\}$$
$$= min\{1.1350, 1.5641\}$$
$$= 1.1350$$

It is found that $0.6871 \leq \hat{C}_{pk} < 1.7191$ which corresponds to the condition $(k_d \leq \hat{C}_{pk} < k_a)$. Since the one preceding lot ($m = 1$) was also accepted, the decision is to accept the current lot. However, in the asymmetric case, the same method can be used as in the symmetric case. We provide dedicated tables for an asymmetric case where $P_L = \frac{p}{3}, P_U = \frac{2p}{3}$ (Table 2), and another where $P_L = \frac{p}{4}, P_U = \frac{3p}{4}$ (Table 3).

Table 1 Optimal parameters of the AVS-MDSR for the symmetric case where α = 0.05 and β = 0.1.

p_1	p_2	n_1	n_2	c	m	k_a	k_d	k_r	$P(p_1)$	$P(p_2)$	ASN
0.001	**0.002**	**177**	**53**	**1**	**3**	**1.6781**	**1.0365**	**1.0305**	**0.9539**	**0.0978**	**178.679**
	0.003	177	15	1	3	1.8733	1.0066	0.9889	0.9582	0.0766	177.850
	0.004	173	12	1	3	1.8703	1.0624	0.9763	0.9723	0.1004	173.997
	0.006	160	25	1	1	1.9857	0.4816	0.3106	0.9543	0.0023	163.262
	0.008	132	17	1	1	1.8930	0.7577	0.2528	0.9692	0.0961	134.489
	0.01	124	37	1	1	1.7136	0.8110	0.3948	0.9721	0.0579	130.646
	0.015	78	20	1	1	1.9537	0.7434	0.5562	0.9887	0.0999	81.298
	0.02	74	24	1	1	1.6165	1.2603	1.1035	0.9715	0.0996	79.276
0.0025	0.005	166	24	1	1	1.7637	1.0501	0.9686	0.9513	0.0921	169.208
	0.01	161	23	2	1	1.3887	1.0880	0.9275	0.9977	0.2507	163.607
	0.015	159	13	2	2	1.8526	1.1597	1.0252	0.9540	0.0994	161.826
	0.02	149	29	2	1	1.5482	0.7916	0.6795	0.9755	0.0102	157.424
	0.03	116	22	2	3	1.8522	0.7808	0.5233	0.9879	0.0953	123.511
	0.05	104	23	2	1	1.7373	0.7196	0.4956	0.9907	0.0313	114.345
0.005	0.01	153	34	2	1	1.6153	0.9949	0.8842	0.9744	0.0809	157.084
	0.015	131	35	3	1	1.7092	0.9336	0.8271	0.9910	0.0207	133.444
	0.02	128	25	3	1	1.6226	0.8109	0.7934	0.9856	0.0573	131.230
	0.03	113	24	3	2	1.6452	1.4010	1.0590	0.9928	0.0992	118.320
	0.04	64	22	3	1	1.9154	0.4925	0.4151	0.9988	0.0169	66.790
	0.05	45	26	2	1	1.6358	0.6332	0.4717	0.9940	0.0977	50.120
0.01	0.02	149	49	4	1	1.4374	0.8941	0.7963	0.9794	0.0097	153.831
	0.03	145	27	4	1	1.0973	0.7027	0.2563	0.9507	0.0237	151.156
	0.04	143	20	4	3	1.4661	0.7388	0.5076	0.9531	0.0812	149.958
	0.05	144	5	3	2	0.8063	0.4513	0.3995	0.9531	0.0985	146.476
	0.1	140	10	4	3	1.7572	0.6634	0.3723	0.9554	0.0697	145.063
	0.15	136	3	3	2	0.8693	0.2433	0.1942	0.9507	0.0911	137.573
	0.2	127	18	4	3	1.5921	0.5011	0.3101	0.9634	0.0924	136.084
0.03	0.06	107	28	5	2	1.8449	0.7217	0.7196	0.9557	0.0049	117.208
	0.09	71	50	2	1	0.6544	0.5778	0.3050	0.9519	0.0079	103.976
	0.12	64	63	5	1	1.7724	0.5477	0.1874	0.9506	0.0300	89.437
	0.15	54	29	3	3	0.6745	0.4945	0.4435	0.9517	0.0993	69.212
	0.3	12	9	3	2	1.7406	0.9544	0.7202	0.9513	0.0341	14.286
0.05	0.1	72	34	6	0	0.7051	0.6147	0.6128	0.9521	0.0947	83.170
	0.15	67	17	3	3	0.5044	0.3604	0.2599	0.9554	0.0872	79.123
	0.2	41	6	5	5	1.8377	0.5198	0.2155	0.9578	0.0057	43.614
	0.25	18	18	4	1	1.6905	0.9724	0.6609	0.9666	0.0172	22.346
	0.5	3	1	1	2	1.6661	0.5877	0.4332	0.9563	0.0980	3.254

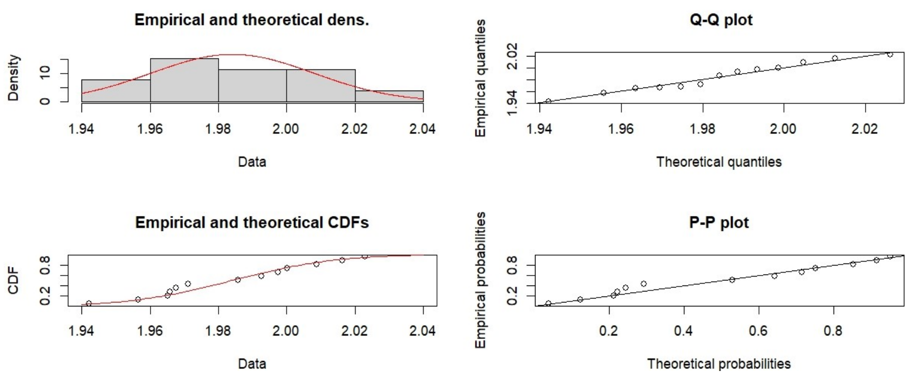

Figure 2 Test for Normal Distribution

Table 2 Optimal parameters of the AVS-MDSR for the asymmetric case $P_L = \frac{p}{3}, P_U = \frac{2p}{3}$ where α = 0.05 and β = 0.1.

p_1	p_2	n_1	n_2	c	m	k_a	k_d	k_r	$P(p_1)$	$P(p_2)$	ASN
0.001	**0.002**	**161**	**59**	**1**	**1**	1.8811	1.0401	1.0375	0.9607	0.0992	162.575
	0.003	124	23	1	1	1.7151	1.0954	1.0026	0.9859	0.0717	124.701
	0.004	122	13	1	1	1.8906	0.9701	0.9695	0.9774	0.0660	122.605
	0.006	119	37	2	3	1.7334	0.9541	0.9265	0.9991	0.0999	119.659
	0.008	113	21	1	3	1.7114	0.4662	0.1669	0.9777	0.0901	115.465
	0.01	98	46	1	1	1.6792	0.7482	0.5789	0.9822	0.0999	104.010
	0.015	62	11	1	1	1.9650	0.4661	0.0778	0.9931	0.0625	63.321
	0.02	57	16	1	1	1.8948	0.6926	0.3024	0.9940	0.0933	59.541
0.0025	0.005	142	40	1	1	1.7372	1.0619	0.9733	0.9714	0.0956	146.175
	0.01	84	16	1	1	1.8897	0.9700	0.9259	0.9501	0.0918	85.795
	0.015	62	15	1	1	1.8926	0.5492	0.3458	0.9578	0.0746	63.868
	0.02	52	24	1	1	1.8325	0.6080	0.3688	0.9696	0.0354	55.441
	0.03	40	11	1	2	1.8009	1.4033	1.0225	0.9502	0.0991	41.887
	0.05	15	5	1	1	1.9519	0.7222	0.7118	0.9980	0.0999	15.429
0.005	0.01	121	25	2	1	1.6153	0.9692	0.8867	0.9688	0.0992	122.814
	0.015	111	28	2	1	1.7926	0.9189	0.8836	0.9534	0.0983	114.521
	0.02	93	37	2	1	1.7456	0.6081	0.4571	0.9536	0.0950	98.489
	0.03	33	12	1	1	1.6604	0.5149	0.3152	0.9544	0.0454	34.634
	0.04	24	6	1	1	1.3999	0.1277	0.0273	0.9811	0.0993	24.767
	0.05	19	4	1	1	1.1317	1.0494	0.6759	0.9531	0.0257	19.499
0.01	0.02	115	24	3	3	1.6112	0.8754	0.8140	0.9504	0.0960	117.737
	0.03	37	12	1	2	1.6923	0.8413	0.7976	0.9867	0.0991	39.149
	0.04	23	3	1	3	1.5614	0.7253	0.6753	0.9692	0.0919	23.384
	0.05	88	28	3	1	1.4892	0.7088	0.3485	0.9569	0.0981	97.223
	0.1	87	8	3	1	1.6434	0.5739	0.4482	0.9597	0.0899	90.962
	0.15	65	10	3	1	1.9069	0.5533	0.3411	0.9841	0.0986	69.980
	0.2	3	2	0	1	1.7360	0.6857	0.6389	0.9509	0.0970	3.089
0.03	0.06	85	29	6	1	1.7523	0.6760	0.6758	0.9528	0.0140	88.805
	0.09	51	10	5	2	1.8641	1.2112	0.7495	0.9527	0.0859	52.565
	0.12	26	20	3	2	1.6674	0.7959	0.7418	0.9609	0.0988	29.882
	0.15	6	5	1	1	1.9411	0.5674	0.5655	0.9589	0.0634	6.590
	0.3	3	2	1	1	1.9371	0.5351	0.4253	0.9940	0.0433	3.219
0.05	0.1	69	13	6	2	1.5937	0.6574	0.6570	0.9533	0.0598	72.894
	0.15	50	11	6	1	1.8444	0.4414	0.4350	0.9548	0.0037	53.578
	0.2	4	4	1	1	1.8270	0.5144	0.4840	0.9583	0.0949	4.390
	0.25	3	3	1	2	1.8916	1.3382	0.6665	0.9526	0.2761	3.761
	0.5	3	1	1	1	0.7884	0.7664	0.1796	0.9503	0.0024	3.082

Table 3 Optimal parameters of the AVS-MDSR for the asymmetric case $P_L = \frac{p}{4}, P_U = \frac{3p}{4}$ where α = 0.05 and β = 0.1.

p_1	p_2	n_1	n_2	c	m	k_a	k_d	k_r	$P(p_1)$	$P(p_2)$	ASN
0.001	0.002	174	31	1	1	1.6445	1.1300	1.0474	0.9802	0.0438	174.952
	0.003	165	37	1	1	1.7583	1.1236	1.0208	0.9787	0.0917	166.861
	0.004	150	33	1	1	1.8262	1.0146	0.9920	0.9666	0.0503	152.174
	0.006	126	54	2	1	1.7812	1.0771	0.9397	0.9993	0.0735	127.110
	0.008	122	35	1	1	1.3551	0.7414	0.5885	0.9772	0.0918	126.588
	0.01	111	37	1	1	1.5728	0.7792	0.6096	0.9778	0.0586	116.744
	0.015	76	19	1	1	1.7027	0.6923	0.3845	0.9896	0.0210	79.027
	0.02	61	14	0	4	1.0257	0.3968	0.2600	0.9587	0.0920	66.373
0.0025	0.005	152	28	2	3	1.7162	1.1115	0.9571	0.9979	0.0686	152.675
	0.01	120	41	1	1	1.0453	0.7438	0.5803	0.9528	0.0982	127.677
	0.015	67	24	1	1	1.7939	0.6738	0.4539	0.9508	0.0775	70.342
	0.02	61	10	1	1	1.9048	0.6237	0.5360	0.9610	0.0968	62.779
	0.03	48	20	1	1	1.8769	0.7220	0.4870	0.9746	0.0947	52.307
	0.05	16	11	1	1	1.6857	0.7823	0.7562	0.9975	0.0861	17.045
0.005	0.01	142	22	2	1	1.6853	1.0038	0.9056	0.9810	0.0463	144.262
	0.015	122	31	4	3	1.8736	0.9614	0.8340	0.9993	0.0998	122.586
	0.02	115	5	2	1	1.3199	0.6069	0.5558	0.9521	0.0997	116.063
	0.03	42	2	1	2	1.3466	0.8635	0.6842	0.9500	0.0988	42.379
	0.04	30	10	1	3	1.2995	1.1022	0.9152	0.9558	0.0462	31.744
	0.05	27	4	1	3	1.7011	0.5217	0.1434	0.9803	0.0967	27.804
0.01	0.02	145	31	4	1	1.7051	0.8377	0.8160	0.9555	0.0981	147.824
	0.03	134	38	4	1	1.3662	0.6795	0.5315	0.9560	0.0154	141.347
	0.04	63	32	3	3	1.2711	0.5335	0.3969	0.9859	0.0986	66.972
	0.05	41	15	2	1	1.1018	0.5754	0.1877	0.9778	0.0939	43.589
	0.1	22	21	2	3	1.6984	1.0409	0.8856	0.9846	0.0847	26.004
	0.15	17	6	1	1	1.7832	0.6098	0.1425	0.9611	0.0994	19.280
	0.2	6	4	1	1	1.6201	0.3050	0.1291	0.9954	0.0699	6.692
0.03	0.06	85	47	6	1	1.5300	0.7181	0.7030	0.9616	0.0997	91.167
	0.09	65	23	5	1	1.6457	0.5737	0.4224	0.9523	0.0036	71.323
	0.12	27	15	3	2	1.4818	0.7770	0.7445	0.9566	0.0742	30.133
	0.15	13	4	2	0	1.5830	1.2642	0.7230	0.9529	0.0996	13.628
	0.3	4	1	1	1	1.6953	0.8130	0.5156	0.9776	0.0693	4.177
0.05	0.1	*	*	*	*	*	*	*	*	*	*
	0.15	*	*	*	*	*	*	*	*	*	*
	0.2	6	5	1	3	0.7556	0.6483	0.5255	0.9501	0.0938	6.944
	0.25	*	*	*	*	*	*	*	*	*	*
	0.5	3	2	2	1	1.8176	0.5019	0.2641	0.9997	0.0998	3.125

*The optimal sampling plan did not appear

Table 4 $ASNs$ of the AVS-MDSR and the existing mixed sampling plan for symmetric and asymmetric cases

Symmetric case				**Asymmetric case**			
p_1	p_2	**Existing mixed sampling plan**	**AVS-MDSR**	p_1	p_2	**Existing mixed sampling plan**	**AVS-MDSR**
0.001	**0.004**	**187.823**	**178.679**	**0.001**	0.004	266.229	162.575
	0.008	141.385	134.489		**0.008**	**127.585**	**115.465**
	0.015	177.936	81.298		0.015	64.339	63.321
0.0025	0.01	167.647	163.607	0.0025	0.01	115.704	85.795
	0.015	126.043	124.596		0.015	73.959	63.868
	0.03	123.894	123.511		0.05	18.853	15.429
0.005	0.03	143.879	118.320	0.005	0.03	38.732	34.634
	0.05	66.508	50.120		0.04	27.951	24.767
0.01	0.03	175.880	151.156		0.05	21.630	19.499
	0.04	160.208	149.958	0.01	0.03	49.558	39.149
0.03	0.2	154.493	136.084		0.04	37.183	23.384
	0.15	73.006	69.212		0.2	4.469	3.089
	0.3	31.415	14.285	0.03	0.15	8.863	6.590
0.05	0.2	54.294	43.614		0.3	3.589	3.219
	0.25	25.688	22.346	0.05	0.2	7.482	4.390
	0.5	5.755	3.254		0.25	5.756	3.761
					0.5	4.988	3.082

3.3 Comparison between the AVS-MDSR and the existing mixed sampling plan

This section evaluates the efficiency of the proposed AVS-MDSR in comparison with the existing mixed acceptance sampling plan introduced by (16). The comparison is performed under the assumption of normally distributed quality data. The efficiency of the acceptance sampling plans can be compared by using the ASN, given a specified producer's risk $\alpha = 0.05$ and consumer's risk $\beta = 0.01$. As shown in Table 4, the AVS-MDSR consistently demonstrates a lower ASN compared to the existing mixed sampling plan for both symmetric and asymmetric cases. For instance, in the symmetric scenario with $p_1 = 0.001$ and $p_2 = 0.004$, the ASN of the existing plan is 187.823, whereas the AVS-MDSR achieves a reduced ASN of 178.679. Similarly, in the asymmetric scenario with $p_1 = 0.001$ and $p_2 = 0.008$, the ASN decreases from 127.585 under the existing plan to 115.465 with AVS-MDSR. These results highlight the enhanced sampling efficiency offered by the proposed AVS-MDSR approach.

4. Conclusions

This study proposed an innovative attribute–variable sampling plan based on a multiple dependent state repetitive framework (AVS-MDSR) to overcome the limitations of traditional acceptance sampling plans. The AVS-MDSR integrates the strengths of attribute and variable inspection within the MDSR framework, utilizing the PCI as a central quality indicator. This combination allows for flexibility and dependability, especially when the results of the initial inspection are unclear or borderline. The study found that the AVS-MDSR can reduce the average sample size, which is a significant advantage when compared to the existing mixed sampling plan. This results in the ability to lower inspection costs without sacrificing the efficiency of quality assessment for the final product. Furthermore, the plan helps to make the decision-making process more accurate, especially when the results of the first inspection are inconclusive. This reduces the risk of accepting non-standard products or rejecting products of good quality.

The AVS-MDSR is well-suited for industries that require strict quality control and need to reduce inspection costs. When comparing the efficiency of the AVS-MDSR against the existing mixed sampling plan under the same parameter specifications, it is found that the AVS-MDSR can reduce the ASN more than the existing mixed sampling plan. Therefore, it can be concluded that the AVS-

MDSR is more flexible and efficient than the existing mixed sampling plan. The AVS-MDSR can be applied to the inspection of quality characteristics that follow a normal distribution. Both symmetric and asymmetric cases of fraction nonconforming have been considered in the selection of the most suitable parameters for the proposed sampling plan. In addition, Tables of optimal parameter values have been created by solving nonlinear equations for different quality levels, which also considers the risks for both the producer and the consumer. Furthermore, a case study from a real-world scenario has been presented to demonstrate the application and actual implementation of this AVS-MDSR.

The AVS-MDSR's dependence on previous lots might mean that decisions take longer to make, but the plan's structure makes this worry almost go away. The reliance on previous lot data only starts when the findings are unclear or on the border (i.e., when $k_d \leq \hat{C}_{pk} < k_a$). Most lots are chosen right away during the first attribute $(d \leq c)$ or variable $\left(\hat{C}_{pk} \geq k_a\right)$ inspection stage. Also, the evolutionary algorithm optimization always finds that the number of dependent lots (m) is quite small, usually 1, 2, or 3. This little step in the process is a strategic trade-off that makes the plan's main advantage possible: a big drop in the ASN. This increase in efficiency lowers the cost of inspections and speeds up the total lot disposition compared to current mixed sampling plans, which speeds up production instead of slowing it down.

The study and development of this AVS-MDSR can be adapted for use with various production processes and product types, especially in industries with strict quality requirements. This allows for the determination of optimal parameters suitable for different scenarios and reduces the average sample size effectively without affecting the accuracy of the final product quality inspection. It is important to acknowledge that the proposed AVS-MDSR plan, as developed in this study, is specifically applicable to inspection processes where the quality characteristic follows a normal distribution. This assumption is a primary limitation. Future research may extend this framework to accommodate non-normal or multivariate quality characteristics, employ Bayesian or machine learning techniques for adaptive decision-making, and evaluate its performance in automated inspection environments. Such extensions would further strengthen the robustness and practical utility of the AVS-MDSR, advancing the development of intelligent and responsive quality assurance systems for next-generation manufacturing.

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Declaration of Conflicting Interests

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