Evaluation of the Time-dependent Electric Charge in the Series of RLC Circuit Loop Under the Time-dependent Voltage Cosine Function

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Abstract

In this work, we developed a model of mathematics and physics for the series of the RLC circuit loop. The purpose of this study is to evaluate for finding the time-dependent electric charge that is a consequence of time-dependent voltage force. Which the voltage force is in the cosine function. We calculated by using the second-order non-homogeneous ordinary differential equation and integration by part technique. We can find that the time-dependent electric charge corresponds to capacitance but is inversely proportional to induction. The time-dependent electric charge is in contrast to the charge. If δ and σ have slightly different values the time-dependent electric charge behaves like an oscillation wave group.

Keyword : Time-dependent Electric Charge, Time-dependent Voltage Force, Wave Group

INTRODUCTION

In one loop of RLC circuit or second-order circuit is from the including of resistor, inductor and capacitor where in this case, we analyze where the voltage depends on time and it is in cosine function. Mohazzab J. *et al*. (2008) study RLC circuit response and analysis using state space method. So he easily fined the response and stability of the RLC circuit or second-order circuit and also with the help of the response of the

RLC is examined from different input functions by using Matlab. The analysis of an RLC circuit becomes too simpler. Sonam (2015) can use Matlab for analyzing the free natural angular frequency repercussion of the second-order circuit, time repercussion of the circuit. To analyse other standard second-order circuit conformation such as low pass and high pass RLC network, we use interactive GUI. In GUI you can change the RLC or second-order circuit parameter and see the fructification on the time and free natural angular frequency repercussion in real time.

Dino (2019) study a low frequency hook with positive imaginary part in the impedance spectrum can be explained by several phenomena and it occurs in a number of photo-electrochemical system. Ahammodullah (2019) have successfully applied the Kirchhoff's voltage rule (KVL) modified into non-homogeneous second-order differential equation to series circuits loop containing an electromotive force, resistor, inductor and capacitor. The purpose of this paper is solving the series of RLC circuits or second-order circuit (Sokol *et al*., 2013) in the driven force dependent on time in cosine function form by using Wronskian's method form of the non-homogeneous second-order differential equation. As the result gives 2 cases of charge parameter that the electric charge value corresponds to the capacitance but inversely proportional to induction, anywhere if the value of the parameter δ is defined as ratio between resistance with double inductance and the free natural angular frequency (σ) have more different values, electric charge would have the behavior of themselves like wave group.

MATERIALS AND METHODS

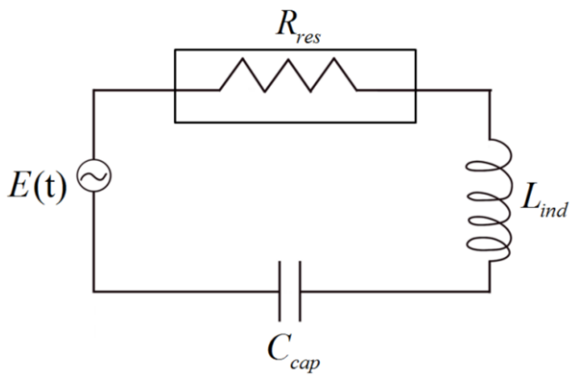


Figure 1: Representation the series of RLC circuit with time-dependent voltage.

The electrical system was made up of a capacitor$\left(C_{Cap}\right)$, resistance$\left(R_{res}\right)$, inductance$\left(L_{ind}\right)$ and applied voltage force$\left(E\left(t\right)\right)$ is one of the most important resonate systems in figure 1 (Kishore, 2008). In this work-study, the voltage force depends on time and in cosine form. Thus the sum of the voltages encountered in going around the whole circuit must be zero. Let us analyze the circuit of this case by applying Kirchhoff's rule, we get

$$E(t) = E_R + E_L + E_C \qquad (1)$$

From the article above, the time-dependent applied voltage force is cosine function form $E(t) = E_0 e^{-\eta^2 t}\cos^2(\sigma t)$ (Hutem & Masoongnoen,. 2021) which E_0 is the initial voltage force, η is the coefficient of damping of the voltages force and σ is a constant and is called the free natural angular frequency. After substituting $E(t)$ into Equation (1), we obtain

$$E_0 e^{-\eta^2 t}\cos^2(\sigma t) = E_R + E_L + E_C \qquad (2)$$

We may write the voltage drop across the resistance $E_R = I_{sys}R_{res}$, across the capacitor it is $E_C = q/C_{cap}$ and across the inductance it is $E_L = L_{ind}\left(dI_{sys}/dt\right)$ (Atamp , 1990) and in the new form where $I_{sys} = dq/dt$ (Goldstein and Safko, 2002), we get

$$\frac{d^2q}{dt^2} + \frac{R_{res}}{L_{ind}}\frac{dq}{dt} + \frac{1}{L_{ind}C_{cap}}q = \frac{E_0}{L_{ind}}e^{-\eta^2 t}\cos^2(\sigma t) \qquad (3)$$

Therefore, we can set the new form where R_{res}/L_{ind} is defined as 2δ ,$1/L_{ind}C_{cap}$ is defined as σ^2 and E_0/L_{ind} is defined as ε_0. Figure 1. represents an analogous electrical driven oscillator with an applied voltage force emf source given by $E(t) = E_0 e^{-\eta^2 t}\cos^2(\sigma t)$. We can rewrite Equation (3) as

$$\frac{d^2q}{dt^2} + 2\delta\frac{dq}{dt} + \sigma^2 q = \varepsilon_0 e^{-\eta^2 t}\cos^2(\sigma t) \qquad (4)$$

Equation 4 is called a non-homogeneous second-order differential equation (Tikjha *et al*., 2018) that can give the solution by using the summation of two part as

$$q(t) = q_C(t) + q_P(t)\ , \qquad (5)$$

where $q_P(t)$ is the time-dependent electric charge particular solution of an inhomogeneous differential equation and the time-dependent electric charge of complementary function $q_C(t)$ is solution of the corresponding homogeneous differential equation (that is equation (4) with the right side equal to zero number), then equation (5) is also a solution of the non-homogeneous second-order differential equation. $q_C(t)$ is the time-dependent electric charge of complementary function solution of the homogeneous second-order differential equation (Riley & Hobson , 2006)

$$\frac{d^2q}{dt^2}+2\delta\frac{dq}{dt}+\sigma^2 q=0 \qquad (6)$$

Let us solve the auxiliary equation as $m^2+2\delta m+\sigma^2=0$. The auxiliary equation has the roots

$$m=-\delta\pm\sqrt{\delta^2-\sigma^2} \qquad (7)$$

For this case, we can select to analyze the underdamped for the convenience to substitution where $\delta<\sigma$. We must have $m=-\delta\pm i\lambda$ and $\lambda=\sqrt{\sigma^2-\delta^2}$. Thus, the complementary function solution of the homogeneous differential equation of Equation (6) is

$$q_C(t)=e^{-\delta t}\left(\alpha\sin(\lambda t)+\beta\cos(\lambda t)\right). \qquad (8)$$

We can find the value of α is a constant and β is a constant by setting the boundary conditions $q(0)=q_0$ (the initial electric charge) and $dq(0)/dt=0$. Thus, we consider the charge depends on time when $t=0$ as

$$q_C(0)=0+\beta\cos(0), \Rightarrow \beta=q_0 \qquad (9)$$

where $\beta=q_0$ is the initial charge. Next, we can find the derivative of charge depends on time term as

$$\frac{dq_C(t)}{dt}=e^{-\delta t}\left(\alpha\lambda\cos(\lambda t)-\beta\sin(\lambda t)\right)-\delta\,e^{-\delta t}\left(\alpha\lambda\sin(\lambda t)+\beta\cos(\lambda t)\right)$$

From the initial condition of derivative part, we must have

$$\alpha=\frac{\delta q_0}{\lambda}=\frac{\delta q_0}{\sqrt{\sigma^2-\delta^2}}, \qquad (10)$$

where $R_{res}/2L_{ind}$ is defined as δ, q_0 is the initial electric charge. Thus, the homogeneous of Equation (6) gives the complementary function solution as

$$q_C(t) = e^{-\delta t}\left(\frac{\delta q_0}{\lambda}\sin(\lambda t) + q_0\cos(\lambda t)\right) \qquad (11)$$

Then, let us seek the particular solution of Equation (5) by using Wronskian method (Susan, 2004). We try the trigonometry and exponential solution of the form that

$$y_1(t) = e^{-\delta t}\sin(\lambda t) \quad , \qquad y_2(t) = e^{-\delta t}\cos(\lambda t)$$

$$y_1'(t) = \lambda e^{-\delta t}\cos(\lambda t) - \delta e^{-\delta t}\sin(\lambda t) \qquad y_2'(t) = -\lambda e^{-\delta t}\sin(\lambda t) - \delta e^{-\delta t}\cos(\lambda t)$$

We can find the solution of Wronskian method by the form below

$$W_{Wr} = \begin{vmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{vmatrix}$$

Thus, we get

$$W_{Wr} = \begin{vmatrix} e^{-\delta t}\sin(\lambda t) & e^{-\delta t}\cos(\lambda t) \\ \lambda e^{-\delta t}\cos(\lambda t) - \delta e^{-\delta t}\sin(\lambda t) & -\lambda e^{-\delta t}\sin(\lambda t) - \delta e^{-\delta t}\cos(\lambda t) \end{vmatrix}$$

$$W_{Wr} = -\lambda e^{-2\delta t} \qquad (12)$$

Consider the first Wronskian method and we satisfy the solution as

$$W_{W1} = \begin{vmatrix} 0 & e^{-\delta t}\cos(\lambda t) \\ \varepsilon_0 e^{-\eta^2 t}\cos^2(\sigma t) & -\left(\lambda e^{-\delta t}\sin(\lambda t) + \delta e^{-\delta t}\cos(\lambda t)\right) \end{vmatrix}$$

$$W_{W1} = -\varepsilon_0 e^{-(\eta^2+\delta)t}\cos^2(\sigma t)\cos(\lambda t) \qquad (13)$$

Consider the second Wronskian method and we satisfy the solution as

$$W_{W2} = \begin{vmatrix} e^{-\delta t}\sin(\lambda t) & 0 \\ \lambda e^{-\delta t}\cos(\lambda t) - \delta e^{-\delta t}\sin(\lambda t) & \varepsilon_0 e^{-\eta^2 t}\cos^2(\sigma t) \end{vmatrix}$$

$$W_{W2} = \varepsilon_0 e^{-(\eta^2+\delta)t}\cos^2(\sigma t)\sin(\lambda t) \qquad (14)$$

We can satisfy the derivative equation for Wronskian method (Sadri ,1991) as

$$\frac{du_1}{dt} = \frac{W_{W1}}{W_{Wr}} = \frac{-\varepsilon_0 e^{-(\eta^2+\delta)t}\cos^2(\sigma t)\cos(\lambda t)}{-\lambda e^{-2\delta t}} = \frac{\varepsilon_0}{\lambda}e^{(\delta-\eta^2)t}\cos(\lambda t)\cos^2(\sigma t)$$

So we set the parameter of $\chi = (\delta - \eta^2)$ as

$$\frac{du_1}{dt} = \frac{\varepsilon_0}{\lambda}e^{\chi t}\cos^2(\sigma t)\cos(\lambda t)$$

$$\frac{du_1}{dt} = \frac{\varepsilon_0}{\lambda}e^{\chi t}\left(\frac{1+\cos(2\sigma t)}{2}\right)\cos(\lambda t)$$

$$u_1 = \frac{\varepsilon_0}{2\lambda}\int e^{\chi t}\left(1+\cos(2\sigma t)\right)\cos(\lambda t)\,dt \qquad (15)$$

Thus,

$$\therefore u_1(t) = \frac{\varepsilon_0}{2\lambda}\left(\int e^{\chi t}\cos(\lambda t)dt + \int e^{\chi t}\cos(\lambda t)\cos(\sigma 2t)dt\right) \tag{16}$$

We can find the first integral function by using integration by part technique for estimation the right hand of Equation (16).

$$\int e^{\chi t}\cos(\lambda t)dt = e^{\chi t}\left(\frac{\lambda\sin(\lambda t) + \chi\cos(\lambda t)}{(\lambda^2 + \chi^2)}\right) \tag{17}$$

Then consider the second integral function by using integration by part technique for estimation the right-hand side of Equation (16).

$$\int e^{\chi t}\cos(\lambda t)\cos(2\sigma t)dt = \frac{1}{2}\left[\frac{e^{\chi t}\left((2\sigma+\lambda)\sin((2\sigma+\lambda)t) + \chi\cos((2\sigma+\lambda)t)\right)}{((2\sigma+\lambda)^2 + \chi^2)} + \frac{e^{\chi t}\left((\lambda-2\sigma)\sin((\lambda-2\sigma)t) + \chi\cos((\lambda-2\sigma)t)\right)}{((\lambda-2\sigma)^2 + \chi^2)}\right] \tag{18}$$

We will get the solution of derivative equation for Wronskian method as

$$u_1(t) = \frac{\varepsilon_0 e^{\chi t}}{2\lambda}\left(\frac{\left(\lambda\sin(\lambda t) + \chi\cos(\lambda t)\right)}{(\lambda^2 + \chi^2)} + \frac{\left((2\sigma+\lambda)\sin((2\sigma+\lambda)t) + \chi\cos((2\sigma+\lambda)t)\right)}{2((2\sigma+\lambda)^2 + \chi^2)} + \frac{(\lambda-2\sigma)\sin((\lambda-2\sigma)t) + \chi\cos((\lambda-2\sigma)t)}{2((\lambda-2\sigma)^2 + \chi^2)}\right) \tag{19}$$

The derivative for the second Wronskian method can be satisfied as

$$\frac{du_2}{dt} = \frac{W_2}{W} = -\frac{\varepsilon_0 e^{-(\eta^2+\delta)t}\cos^2(\sigma t)\sin(\lambda t)}{\lambda e^{-2\delta t}} = -\frac{\varepsilon_0}{2\lambda}e^{\chi t}(1+\cos(2\sigma t))\sin(\lambda t)$$

Therefore,

$$u_2(t) = -\frac{\varepsilon_0}{2\lambda}\left(\int e^{\chi t}\sin(\lambda t)dt + \int e^{\chi t}\sin(\lambda t)\cos(2\sigma t)dt\right) \tag{20}$$

We can find the first derivative in the right hand of Equation (20) by using integration by part technique for estimation.

$$\int e^{\chi t}\sin(\lambda t)\,dt = \frac{e^{\chi t}\left(\chi\sin(\lambda t) - \lambda\cos(\lambda t)\right)}{(\chi^2 + \lambda^2)} \tag{21}$$

Then, we also use integration by part technique for estimate the second integral function in the right hand of Equation (20) as

$$\int e^{\chi t}\cos(2\sigma t)\sin(\lambda t)dt = \frac{e^{\chi t}\left(\chi\sin((\lambda-2\sigma)t) - (\lambda-2\sigma)\cos((\lambda-2\sigma)t)\right)}{2(\chi^2 + (\lambda-2\sigma)^2)} + \frac{e^{\chi t}\left(\chi\sin((\lambda+2\sigma)t) - (\lambda+2\sigma)\cos((\lambda+2\sigma)t)\right)}{2(\chi^2 + (\lambda+2\sigma)^2)} \tag{22}$$

Thus, we get the solution of derivative of the second Wronskian method as

$$u_2(t)=-\frac{\varepsilon_0 e^{\chi t}}{2\lambda}\left(\frac{\left(\chi\sin(\lambda t)-\lambda\cos(\lambda t)\right)}{(\chi^2+\lambda^2)}+\frac{\left(\chi\sin((\lambda-2\sigma)t)-(\lambda-2\sigma)\cos((\lambda-2\sigma)t)\right)}{2(\chi^2+(\lambda-2\sigma)^2)}+\frac{\left(\chi\sin((\lambda+2\sigma)t)-(\lambda+2\sigma)\cos((\lambda+2\sigma)t)\right)}{2(\chi^2+(\lambda+2\sigma)^2)}\right) \quad (23)$$

The particular of a non-homogeneous, the second order, linear equation in Equation (4) with the constant C is

$$q_P(t)=\frac{\varepsilon_0 e^{-\eta^2 t}}{2\lambda}\sin(\lambda t)\left(\frac{\left(\lambda\sin(\lambda t)+(\delta-\eta^2)\cos(\lambda t)\right)}{(\lambda^2+(\delta-\eta^2)^2)}+\frac{\left((2\sigma+\lambda)\sin((2\sigma+\lambda)t)+(\delta-\eta^2)\cos((2\sigma+\lambda)t)\right)}{2((2\sigma+\lambda)^2+(\delta-\eta^2)^2)}+\frac{(\lambda-2\sigma)\sin((\lambda-2\sigma)t)+(\delta-\eta^2)\cos((\lambda-2\sigma)t)}{2((\lambda-2\sigma)^2+(\delta-\eta^2)^2)}\right)-\frac{\varepsilon_0 e^{-\eta^2 t}}{2\lambda}\cos(\lambda t)\left(\frac{\left((\delta-\eta^2)\sin(\lambda t)-\lambda\cos(\lambda t)\right)}{((\delta-\eta^2)^2+\lambda^2)}+\frac{\left((\delta-\eta^2)\sin((\lambda-2\sigma)t)-(\lambda-2\sigma)\cos((\lambda-2\sigma)t)\right)}{2((\delta-\eta^2)^2+(\lambda-2\sigma)^2)}+\frac{\left((\delta-\eta^2)\sin((\lambda+2\sigma)t)-(\lambda+2\sigma)\cos((\lambda+2\sigma)t)\right)}{2((\delta-\eta^2)^2+(\lambda+2\sigma)^2)}\right)+C \quad (24)$$

We can find the parameter of C by using the initial condition as $q_p(0)=q_0$.

$$q_p(0)=-\frac{\varepsilon_0}{2\lambda}\left(-\frac{\lambda}{(\lambda^2+(\delta-\eta^2)^2)}-\frac{(\lambda-2\sigma)}{2((\lambda-2\sigma)^2+(\delta-\eta^2)^2)}-\frac{(\lambda+2\sigma)}{2((\lambda+2\sigma)^2+(\delta-\eta^2)^2)}\right)+C$$

$$C=q_0-\frac{\varepsilon_0}{2\lambda}\left(\frac{\lambda}{(\lambda^2+(\delta-\eta^2)^2)}+\frac{(\lambda-2\sigma)}{2((\lambda-2\sigma)^2+(\delta-\eta^2)^2)}+\frac{(\lambda+2\sigma)}{2((\lambda+2\sigma)^2+(\delta-\eta^2)^2)}\right) \quad (25)$$

Therefore, we get the particular solution of a non-homogeneous, second order, differential equation as

$$q_P(t)=\frac{\varepsilon_0 e^{-\eta^2 t}}{2\lambda}\sin(\lambda t)\left(\frac{\left(\lambda\sin(\lambda t)+(\delta-\eta^2)\cos(\lambda t)\right)}{(\lambda^2+(\delta-\eta^2)^2)}+\frac{\left((2\sigma+\lambda)\sin((2\sigma+\lambda)t)+(\delta-\eta^2)\cos((2\sigma+\lambda)t)\right)}{2((2\sigma+\lambda)^2+(\delta-\eta^2)^2)}+\frac{(\lambda-2\sigma)\sin((\lambda-2\sigma)t)+(\delta-\eta^2)\cos((\lambda-2\sigma)t)}{2((\lambda-2\sigma)^2+(\delta-\eta^2)^2)}\right)-\frac{\varepsilon_0 e^{-\eta^2 t}}{2\lambda}\cos(\lambda t)\left(\frac{\left((\delta-\eta^2)\sin(\lambda t)-\lambda\cos(\lambda t)\right)}{((\delta-\eta^2)^2+\lambda^2)}+\frac{\left((\delta-\eta^2)\sin((\lambda-2\sigma)t)-(\lambda-2\sigma)\cos((\lambda-2\sigma)t)\right)}{2((\delta-\eta^2)^2+(\lambda-2\sigma)^2)}+\frac{\left((\delta-\eta^2)\sin((\lambda+2\sigma)t)-(\lambda+2\sigma)\cos((\lambda+2\sigma)t)\right)}{2((\delta-\eta^2)^2+(\lambda+2\sigma)^2)}\right)+q_0-\frac{\varepsilon_0}{2\lambda}\left(\frac{\lambda}{(\lambda^2+(\delta-\eta^2)^2)}+\frac{(\lambda-2\sigma)}{2((\lambda-2\sigma)^2+(\delta-\eta^2)^2)}+\frac{(\lambda+2\sigma)}{2((\lambda+2\sigma)^2+(\delta-\eta^2)^2)}\right) \quad (26)$$

Thus, we get the parameter of the time-dependent electric charge in Equation (5) as

$$q(t)=e^{-\delta t}\left(\frac{\delta q_0}{\lambda}\sin(\lambda t)+q_0\cos(\lambda t)\right)+\frac{\varepsilon_0 e^{-\eta^2 t}}{2\lambda}\sin(\lambda t)\left(\frac{\left(\lambda\sin(\lambda t)+(\delta-\eta^2)\cos(\lambda t)\right)}{(\lambda^2+(\delta-\eta^2)^2)}+\frac{\left((2\sigma+\lambda)\sin((2\sigma+\lambda)t)+(\delta-\eta^2)\cos((2\sigma+\lambda)t)\right)}{2((2\sigma+\lambda)^2+(\delta-\eta^2)^2)}+\frac{(\lambda-2\sigma)\sin((\lambda-2\sigma)t)+(\delta-\eta^2)\cos((\lambda-2\sigma)t)}{2((\lambda-2\sigma)^2+(\delta-\eta^2)^2)}\right)$$

$$-\frac{\varepsilon_0 e^{-\eta^2 t}}{2\lambda}\cos(\lambda t)\left(\frac{\left((\delta-\eta^2)\sin(\lambda t)-\lambda\cos(\lambda t)\right)}{((\delta-\eta^2)^2+\lambda^2)}+\frac{\left((\delta-\eta^2)\sin((\lambda-2\sigma)t)-(\lambda-2\sigma)\cos((\lambda-2\sigma)t)\right)}{2((\delta-\eta^2)^2+(\lambda-2\sigma)^2)}\right.$$
$$\left.+\frac{\left((\delta-\eta^2)\sin((\lambda+2\sigma)t)-(\lambda+2\sigma)\cos((\lambda+2\sigma)t)\right)}{2((\delta-\eta^2)^2+(\lambda+2\sigma)^2)}\right)+q_0-\frac{\varepsilon_0}{2\lambda}\left(\frac{\lambda}{(\lambda^2+(\delta-\eta^2)^2)}\right.$$
$$\left.+\frac{(\lambda-2\sigma)}{2((\lambda-2\sigma)^2+(\delta-\eta^2)^2)}+\frac{(\lambda+2\sigma)}{2((\lambda+2\sigma)^2+(\delta-\eta^2)^2)}\right) \quad (27)$$

From Equation (27) is the charge depends on time show in program. Putting this into program Mathematica for plotting graph.

Case1:the time-dependent electric charge where δ and σ have more different value. $(\sigma \gg \delta)$

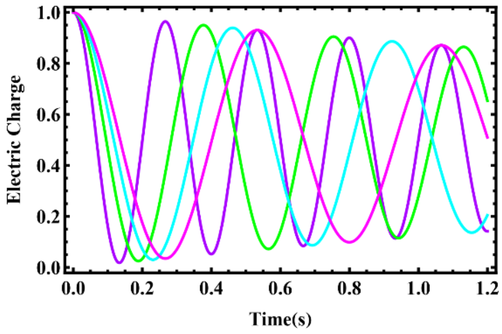
The time-dependent electric charge can behave like the underdamp wave. (Teoh & Rahifa, 2018).

Case2: the time-dependent electric charge where δ and σ have slightly different value. $(\sigma > \delta)$

The time-dependent electric charge can behave like the wave group.

RESULTS

We can explain of numerical and result of the time-dependent of electric charge in Equation (27) which effects by cosine voltage force as time-dependent charge as figure (2), figure (3), figure (4) and figure (5)



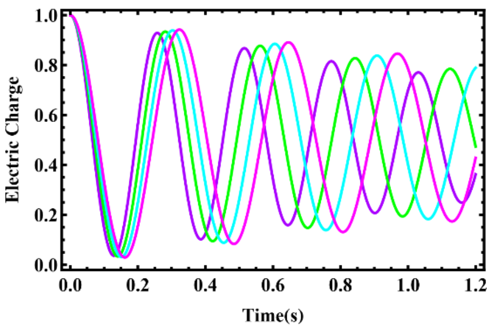
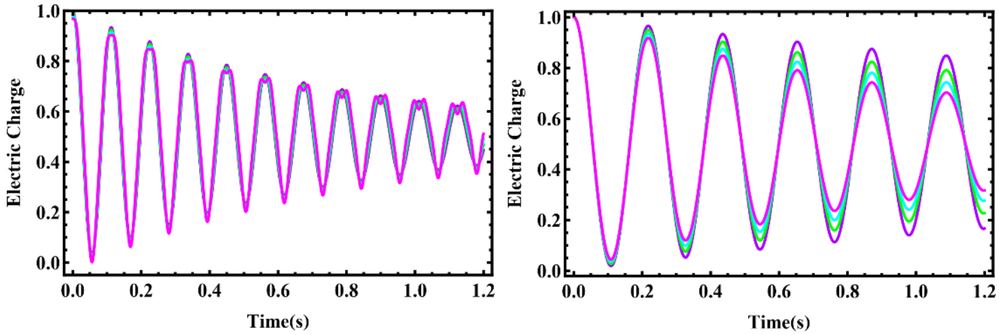
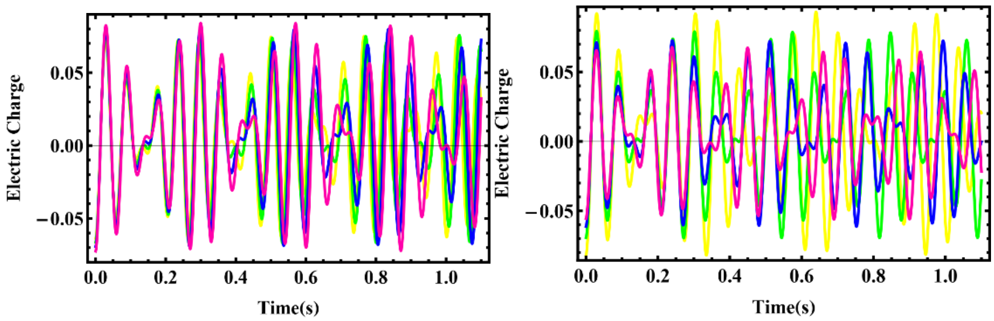


Figure 2: Illustration showing the relation parameter between charges depends on time where δ and σ have more different value. (a) Representation the relation between the charge and time when the capacitor value is unable (The purple solid line is $C_{Cap1} = 4\mu F$. The green solid line is $C_{Cap2} = 8\mu F$. The light blue solid line is $C_{Cap3} = 12\mu F$ The pink solid line is $C_{Cap4} = 16\mu F$), (b) Representation the relation between the charge and time when the Inductor value is unable (The purple solid line is $L_{ind1} = 210H$. The green solid line is $L_{ind2} = 250H$. The light blue solid line is $L_{ind3} = 290H$ The pink solid line is $L_{ind4} = 330H$).



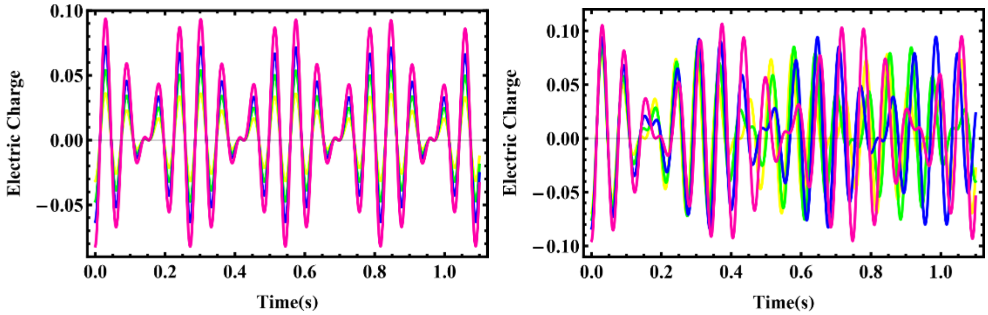
(a) (b)

Figure 3: Illustration showing the relation parameter between charges depends on time where δ and σ have more different value. (a) Representation the relation between the electric charge and time when the initial applied voltage force (E_0) value is unable (The purple solid line is $E_{01} = 0.1\,Vol$. The green solid line is $E_{02} = 8\ kV$. The light blue solid line is $E_{03} = 16\ kV$ The pink solid line is $E_{04} = 26\ kV$), (b) Representation the relation between the electric charge and time when the resistance (R_{res}) value is unable (The purple solid line is $R_{res1} = 100$ W. The green solid line is $R_{res2} = 150$ W. The light blue solid line is $R_{res3} = 200$ W The pink solid line is $R_{res4} = 250$ W).



(a) (b)

Figure 4: Illustration showing the relation parameter between charges depends on time where δ and σ have slightly different value $(\sigma > \delta)$. (a) Representation the relation between the charge and time when the capacitor value is unable (The yellow solid line is $C_{Cap1} = 410\mu F$. The green solid line is $C_{Cap2} = 412\mu F$. The blue solid line is $C_{Cap3} = 414\mu F$ The pink solid line is $C_{Cap4} = 416\mu F$), (b) Representation the relation between the charge and time when the inductor value is unable (The yellow solid line is $L_{ind1} = 1.10H$. The green solid line is $L_{ind2} = 1.12H$. The blue solid line is $L_{ind3} = 1.14H$ The pink solid line is $L_{ind4} = 1.16H$).



(a) (b)

Figure 5: Illustration showing the relation parameter between charges depends on time where δ and σ have slightly different value $(\sigma > \delta)$. (a) Representation the relation between the time-dependent electric charge and time when the initial applied voltage force (E_0) value is unable, (b) Representation the relation between the time-dependent electric charge and time when the resistance (R_{res}) value is unable.

DISCUSSION

From figure 2 (a), we set $q_0 = 0.5\,\mu C$, $\eta = 0.1$, $L_{ind} = 450\,H$, $E_0 = 350\,Vol$, $R_{res} = 250\text{W}$ are the control variable. But the capacitor is an independent variable and the time-dependent electric charge is a dependent variable. If the capacitance is higher, the wavelength of the vibrating electric charge is longer, but the frequency of the vibrating electric charge is lower. From figure 2 (b), we set $q_0 = 0.5\,\mu C$, $\eta = 0.1$, $C_{Cap} = 8\mu F$, $E_0 = 350\,Vol$, $R_{res} = 250\text{W}$ is control variable. The inductance is an independent variable and the time-dependent electric charge is a dependent variable. If the inductance is higher, the amplitude of the vibrating electric charge is higher, and the wavelength of it is longer. From Figure 3 (a), we set $q_0 = 0.5\,\mu C$, $\eta = 0.1$, $L_{ind} = 40\,H$, $C_{Cap2} = 8\mu F$, $R_{res} = 100\text{W}$ as the control variable. The initial voltage force E_0 is the control variable and the time-dependent electric charge is the dependent variable. If E_0 higher, the vibrating of electric charge is smaller, more of the amplitude of the vibrating electric charge is split. From figure 3 (b), we set $q_0 = 0.5\,\mu C$, $\eta = 0.1$, $L_{ind} = 150\,H$, $C_{Cap2} = 8\mu F$, $E_0 = 250Vol$ as the control variable. The resistance R_{res} is the independent variable and the time-dependent electric charge is the dependent variable. If the resistance is higher, the amplitude of the vibrating electric charge is lower. Thus, we can compare the resistance as the damping coefficient of the electric charge. From figure 4 (a), we set $q_0 = 0.5\,\mu C$, $\eta = 0.1$, $L_{ind} = 1.102\,H$, $E_0 = 220Vol$, $R_{res} = 100\text{W}$ is control variable. The capacitance is the independent variable and the time-dependent electric charge is the dependent variable. If δ and σ have slightly different, the time-dependent electric charge behaves itself like wave group. If the capacitance is higher, the amplitude of the vibrating electric charge is higher. From figure 4 (b), we set $q_0 = 0.5\,\mu C$, $\eta = 0.1$, $C_{Cap} = 420\mu F$, $E_0 = 220Vol$, $R_{res} = 100\text{W}$ as control variable. The inductance is the independent variable and the time-dependent electric charge is the dependent variable. If the inductance is higher, the amplitude of the vibrating electric charge is lower. From figure 5 (a), we set $q_0 = 0.5\,\mu C$, $\eta = 0.1$, $L_{ind} = 1.12\,H$, $C_{Cap2} = 420\mu F$, $R_{res} = 100\text{W}$ as the control variable. The initial voltage force E_0 is the independent variable and the time-dependent electric charge is the dependent variable. If E_0 is higher, the amplitude of the vibrating electric charge is higher. From figure 5 (b) we set $q_0 = 0.5\,\mu C$, $\eta = 0.1$, $L_{ind} = 1.12\,H$, $C_{Cap2} = 420\mu F$, $E_0 = 220Vol$ is the control variable. The resistance R_{res} is the independent variable and the time-dependent electric charge is the dependent variable. If resistance is higher, the amplitude of the vibrating electric charge is higher.

We had explicated the series of RLC loop circuit of resonant system that can be used to effectually substantiated, in a very visual way, the phase relation between the voltages across reactive and resistive elements (Sokol *et al.*, 2013). We have explicated the series of RLC loop circuit of resonant system that can be used to effectually substantiate new soft ferromagnetic magnetic materials made possible the use of the magnetic amplifier technology in designing competitive electric-power engineering supplies (Eloısa & Romeo, 2004). We can see the time-dependent electric charge perturb via applied voltage force emf has the behavior like wave group but the time-dependent electric charge in paper of title application of linear differential equation in an analysis transient and steady response for second order RLC closed series circuit of Ahammodullah Hasan has not behavior like wave group, if we add the time, the amplitude slowly decreases. A modeling approach of a magnetic amplifier based on the magnetic hysteresis loop of the core soft ferromagnetic material is presented here for a common amorphous magnetic alloy.

CONCLUSIONS

From equation (27), we known that if the value of the parameter of δ is defined as ratio between resistance with double inductance and the free natural angular frequency σ have more different values, the electric charge have their behavior like underdamped. If the value of the parameter of δ is defined as ratio between resistance with double inductance and the free natural angular frequency σ have slightly different values, the electric charge will have their behavior like wave group.

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