Some properties of new multiplicative pulsating 3-Fibonacci sequence

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Abstract

We know that the Fibonacci sequence is usually reloaded to pattern in nature. For example, it has been known that the arrangement of leaves or flowers around the stem is formed Fibonacci sequence which is called phyllotactic formations. In this paper, we studied about the Fibonacci-like sequence which have three dimensional. We found the new ideas for construction them. Then we described the basic concepts using for construction the new multiplicative pulsating 3-Fibonacci sequences. Moreover, some identities of them were proved.

Keywords: 3-Fibonacci sequence, Pulsating, Multiplicative, Number theory

Introduction

In the recent years, the coupled recurrence relations are popularized. They are the new sequences which are the ordinary recursive sequences with using two sequences of integers for construction. Then we got some result from the considering of two sequences identical. On the other hand, Fibonacci sequence is usually reloaded to pattern in nature. For example, it has been known that the arrangement of leaves or flowers around the stem is formed Fibonacci sequence which is called phyllotactic formations [1]. Thus, to understand the mathematical properties of the Fibonacci and the related sequences are important also from the pattern formation point of view. Moreover, the first researcher who introduced the idea of 2-Fibonacci (sometimes we called coupled Fibonacci) sequence is Atanassov [2]. He showed some curious properties in [3-6]. He explained the new construction of Fibonacci sequence which were defined and presented in many different ways for the generating coupled sequences. One of them is pulsating Fibonacci sequence. Then the new types of Fibonacci sequence, which is called the multiplicative pulsating Fibonacci sequence, was explained by Suvarnamani and Koyram [7] in 2015. They found that the explicit formulas for the form of its members.

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That is $\alpha_{2k} = a^{\left(\frac{3^{k-1}+(-1)^k}{2}\right)} b^{\left(\frac{3^{k-1}+(-1)^{k-1}}{2}\right)} c^{\left(3^{k-1}\right)}$, $\beta_{2k} = a^{\left(\frac{3^{k-1}+(-1)^{k-1}}{2}\right)} b^{\left(\frac{3^{k-1}+(-1)^k}{2}\right)} c^{\left(3^{k-1}\right)}$ and

$\alpha_{2k+1} = \beta_{2k+1} = a^{\left(3^{k-1}\right)} b^{\left(3^{k-1}\right)} c^{2\left(3^{k-1}\right)}$ for every integer number $k \geq 0,$ where the ordered pair of $\{\alpha_n\}$ and $\{\beta_n\}$ is a multiplicative pulsating 2-Fibonacci sequences with $\alpha_0 = a, \beta_0 = b, \alpha_1 = \beta_1 = c, \alpha_{2n} = \alpha_{2n-1} \cdot \beta_{2n-2}, \beta_{2n} = \beta_{2n-1} \cdot \alpha_{2n-2}, \alpha_{2n+1} = \beta_{2n+1} = \alpha_{2n} \cdot \beta_{2n}$ for every natural number n. In 2016, some identities of type of the multiplicative pulsating 3-Fibonacci sequence were shown by Suvarnamani and Tatong [8]. After that Suvarnamani [9] defined the Fibonacci sequence which is the type of the multiplicative pulsating n-Fibonacci sequence and showed some results of them in 2017. Moreover, the complex pulsating Fibonacci sequence and some results of them were shown by Halıcı and Karatas [10]. So, the study of new ideas for the generalization of Fibonacci sequences in different ways to generate coupled sequences is very interesting. Therefore, we study about them in this paper. We describe the basic concepts for using to construction the multiplicative pulsating 3-Fibonacci sequences. The explicit formulas for the form of its members be showed. Then some fundamental properties of them are proved.

Material and Methods

We have to introduce some theoretical background for studying the properties of multiplicative pulsating 3-Fibonacci sequences. There are three topics: the multiplicative pulsating 3-Fibonacci sequences, the geometric sequence and the geometric series. The details are as follows.

1. The multiplicative pulsating 3-Fibonacci sequences

Next, we will introduce some types of Fibonacci-like sequence. If a, b, c and d are four fixed real numbers. We can construct the multiplicative pulsating 3-Fibonacci sequences in the following different ways:

Scheme 1: For the natural number n,

$$\alpha_n = \begin{cases} \alpha_{n-1} \cdot \beta_{n-1} \cdot \gamma_{n-1}, & \text{where } n \text{ is odd number} \\ \alpha_{n-1} \cdot \beta_{n-2} \cdot \gamma_{n-2}, & \text{where } n \text{ is even number} \end{cases},$$

$$\beta_n = \begin{cases} \alpha_{n-1} \cdot \beta_{n-1} \cdot \gamma_{n-1}, & \text{where } n \text{ is odd number} \\ \alpha_{n-2} \cdot \beta_{n-1} \cdot \gamma_{n-2}, & \text{where } n \text{ is even number} \end{cases},$$

$$\gamma_n = \begin{cases} \alpha_{n-1} \cdot \beta_{n-1} \cdot \gamma_{n-1}, & \text{where } n \text{ is odd number} \\ \alpha_{n-2} \cdot \beta_{n-2} \cdot \gamma_{n-1}, & \text{where } n \text{ is even number} \end{cases},$$

with conditions . $\alpha_0 = a, \beta_0 = b, \gamma_0 = c$

Scheme 2: For the natural number $n>1$,

$$\alpha_n = \begin{cases} \alpha_{n-1}\cdot\beta_{n-2}\cdot\gamma_{n-2}, & \text{where } n \text{ is even number} \\ \alpha_{n-1}\cdot\beta_{n-1}\cdot\gamma_{n-1}, & \text{where } n \text{ is odd number} \end{cases},$$

$$\beta_n = \begin{cases} \alpha_{n-2}\cdot\beta_{n-1}\cdot\gamma_{n-2}, & \text{where } n \text{ is even number} \\ \alpha_{n-1}\cdot\beta_{n-1}\cdot\gamma_{n-1}, & \text{where } n \text{ is odd number} \end{cases},$$

$$\gamma_n = \begin{cases} \alpha_{n-2}\cdot\beta_{n-2}\cdot\gamma_{n-1}, & \text{where } n \text{ is even number} \\ \alpha_{n-1}\cdot\beta_{n-1}\cdot\gamma_{n-1}, & \text{where } n \text{ is odd number} \end{cases},$$

with conditions . $\alpha_0 = a, \beta_0 = b, \gamma_0 = c,\ \alpha_1 = \beta_1 = \gamma_1 = d$

The ordered pair of three sequences of each scheme is called a multiplicative pulsating 3-Fibonacci sequence. Next, we introduced a new way for constructing the multiplicative pulsating 3-Fibonacci sequences. The details are as follows.

Scheme 3: For the natural number n,

$$\alpha_n = \begin{cases} \alpha_{n-1}\cdot\beta_{n-1}\cdot\gamma_{n-1}, & \text{where } n \text{ is odd number} \\ \alpha_{n-2}\cdot\beta_{n-1}\cdot\gamma_{n-1}, & \text{where } n \text{ is even number} \end{cases},$$

$$\beta_n = \begin{cases} \alpha_{n-1}\cdot\beta_{n-1}\cdot\gamma_{n-1}, & \text{where } n \text{ is odd number} \\ \alpha_{n-1}\cdot\beta_{n-2}\cdot\gamma_{n-1}, & \text{where } n \text{ is even number} \end{cases},$$

$$\gamma_n = \begin{cases} \alpha_{n-1}\cdot\beta_{n-1}\cdot\gamma_{n-1}, & \text{where } n \text{ is odd number} \\ \alpha_{n-1}\cdot\beta_{n-1}\cdot\gamma_{n-2}, & \text{where } n \text{ is even number} \end{cases},$$

with conditions . $\alpha_0 = a, \beta_0 = b, \gamma_0 = c$

2. The geometric sequence

A sequence $\{a_k\}$ is called a geometric sequence if it has a common ratio r of each two consecutive terms. That means $\frac{a_{k+1}}{a_k} = r$ for k is a positive integer and all elements of $\{a_k\}$ are not zero.

3. The geometric series

A geometric series $\sum a_k$ is a series which $\{a_k\}$ is a geometric sequence with the common ratio r and the condition $a_1 = a$. Moreover, we have

(1) $$\sum_{k=1}^{n} a_k = \frac{a\left(1-r^{n+1}\right)}{1-r} .$$

(2) $\sum_{k=1}^{\infty} a_k$ is convergent and $\sum_{k=1}^{\infty} a_k = \frac{a}{1-r}$ If $|r| \geq 1$.

(3) $\sum_{k=1}^{\infty} a_k$ is divergent If $|r| < 1$.

Results and Discussion

Now we have some knowledge sufficiency for proving about properties of the multiplicative pulsating 3-Fibonacci sequences. Next, we will prove some properties of the multiplicative pulsating 3-Fibonacci sequences in form of scheme 3.

Theorem 1. If the ordered pair of $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ is a multiplicative pulsating 3-Fibonacci sequences of scheme 3 for every natural number n. Then we get $\alpha_{2n-1} = \beta_{2n-1} = \gamma_{2n-1} = a^{7^{n-1}} b^{7^{n-1}} c^{7^{n-1}}$,

$$\alpha_{2n} = a^{1+2\sum_{i=0}^{n-1} 7^i} b^{2\sum_{i=0}^{n-1} 7^i} c^{2\sum_{i=0}^{n-1} 7^i}, \ \beta_{2n} = a^{2\sum_{i=0}^{n-1} 7^i} b^{1+2\sum_{i=0}^{n-1} 7^i} c^{2\sum_{i=0}^{n-1} 7^i} \text{ and } \gamma_{2n} = a^{2\sum_{i=0}^{n-1} 7^i} b^{2\sum_{i=0}^{n-1} 7^i} c^{1+2\sum_{i=0}^{n-1} 7^i}.$$

Proof. Let the ordered pair of $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ be a multiplicative pulsating 3-Fibonacci sequences of scheme 3 for every natural number n. Then we can prove it by using the mathematical induction. First, let the statement $P(n)$: $\alpha_{2n-1} = \beta_{2n-1} = \gamma_{2n-1} = a^{7^{n-1}} b^{7^{n-1}} c^{7^{n-1}}$, $\alpha_{2n} = a^{1+2\sum_{i=0}^{n-1} 7^i} b^{2\sum_{i=0}^{n-1} 7^i} c^{2\sum_{i=0}^{n-1} 7^i}$, $\beta_{2n} = a^{2\sum_{i=0}^{n-1} 7^i} b^{1+2\sum_{i=0}^{n-1} 7^i} c^{2\sum_{i=0}^{n-1} 7^i}$ and $\gamma_{2n} = a^{2\sum_{i=0}^{n-1} 7^i} b^{2\sum_{i=0}^{n-1} 7^i} c^{1+2\sum_{i=0}^{n-1} 7^i}$ for every natural number n.

Consider the statement $P(n)$ when $n = 1$. Then we have

$$\alpha_{2(1)-1} = \beta_{2(1)-1} = \gamma_{2(1)-1} = \alpha_0 \cdot \beta_0 \cdot \gamma_0 = abc = a^{7^{1-1}} b^{7^{1-1}} c^{7^{1-1}},$$

$$\alpha_{2(1)} = \alpha_0 \cdot \beta_1 \cdot \gamma_1 = (a)(abc)(abc) = a^3 b^2 c^2 = a^{1+2\cdot 7^{1-1}} b^{2\cdot 7^{1-1}} c^{2\cdot 7^{1-1}},$$

$$\beta_{2(1)} = \alpha_1 \cdot \beta_0 \cdot \gamma_1 = (abc)(b)(abc) = a^2 b^3 c^2 = a^{2\cdot 7^{1-1}} b^{1+2\cdot 7^{1-1}} c^{2\cdot 7^{1-1}},$$

$$\gamma_{2(1)} = \alpha_1 \cdot \beta_1 \cdot \gamma_0 = (abc)(abc)(c) = a^2 b^2 c^3 = a^{2\cdot 7^{1-1}} b^{2\cdot 7^{1-1}} c^{1+2\cdot 7^{1-1}}.$$

Thus $P(1)$ is true.

Next, we consider $P(n)$ for some natural number k. Then we assume that the statement $P(k)$ is true. That is,

$$\alpha_{2k-1}=\beta_{2k-1}=\gamma_{2k-1}=a^{7^{k-1}}b^{7^{k-1}}c^{7^{k-1}},$$

$$\alpha_{2k}=a^{1+2\sum_{i=0}^{k-1}7^i}b^{2\sum_{i=0}^{k-1}7^i}c^{2\sum_{i=0}^{k-1}7^i},$$

$$\beta_{2k}=a^{2\sum_{i=0}^{k-1}7^i}b^{1+2\sum_{i=0}^{k-1}7^i}c^{2\sum_{i=0}^{k-1}7^i},$$

$$\gamma_{2k}=a^{2\sum_{i=0}^{k-1}7^i}b^{2\sum_{i=0}^{k-1}7^i}c^{1+2\sum_{i=0}^{k-1}7^i}.$$

Then we will show that $P(k+1)$ is true. So, we have

$$\alpha_{2(k+1)-1}=\beta_{2(k+1)-1}=\gamma_{2(k+1)-1}=\alpha_{2(k+1)-2}\beta_{2(k+1)-2}\gamma_{2(k+1)-2}=a^{1+6\sum_{i=0}^{k-1}7^i}b^{1+6\sum_{i=0}^{k-1}7^i}c^{1+6\sum_{i=0}^{k-1}7^i}=a^{7^{k-1}}b^{7^{k-1}}c^{7^{k-1}}$$

$$\alpha_{2(k+1)}=\alpha_{2k}\beta_{2k-1}\gamma_{2k-1}=a^{\frac{7^k+2}{3}}b^{\frac{7^k-1}{3}}c^{\frac{7^k-1}{3}}=a^{1+2\sum_{i=0}^{(k+1)-1}7^i}b^{2\sum_{i=0}^{(k+1)-1}7^i}c^{2\sum_{i=0}^{(k+1)-1}7^i}$$

$$\beta_{2(k+1)}=\alpha_{2k-1}\beta_{2k}\gamma_{2k-1}=a^{\frac{7^k-1}{3}}b^{\frac{7^k+2}{3}}c^{\frac{7^k-1}{3}}=a^{2\sum_{i=0}^{(k+1)-1}7^i}b^{1+2\sum_{i=0}^{(k+1)-1}7^i}c^{2\sum_{i=0}^{(k+1)-1}7^i}$$

$$\gamma_{2(k+1)}=\alpha_{2k-1}\beta_{2k-1}\gamma_{2k}=a^{\frac{7^k-1}{3}}b^{\frac{7^k-1}{3}}c^{\frac{7^k+2}{3}}=a^{2\sum_{i=0}^{(k+1)-1}7^i}b^{2\sum_{i=0}^{(k+1)-1}7^i}c^{1+2\sum_{i=0}^{(k+1)-1}7^i}$$

So, the statement $P(k+1)$ is true.

By mathematical induction, the statement $P(n)$ is true when n is a natural number.

Theorem 2. If the ordered pair of $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ is a multiplicative pulsating 3-Fibonacci sequences of scheme 3, then $\prod_{i=1}^{n}\alpha_{2i-1}=\prod_{i=1}^{n}\beta_{2i-1}=\prod_{i=1}^{n}\gamma_{2i-1}=(abc)^{\frac{7^n-1}{6}}$ for every natural number n.

Proof. Let the ordered pair of $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ be a multiplicative pulsating 3-Fibonacci sequences of scheme 3. Then we can prove it by using the mathematical induction.

First, let the statement $P(n)$: $\prod_{i=1}^{n}\alpha_{2i-1}=\prod_{i=1}^{n}\beta_{2i-1}=\prod_{i=1}^{n}\gamma_{2i-1}=(abc)^{\frac{7^n-1}{6}}$ for every natural number n. Consider the statement $P(n)$ when $n=1$. Then we have

$$\prod_{i=1}^{1}\alpha_{2i-1}=\prod_{i=1}^{1}\beta_{2i-1}=\prod_{i=1}^{1}\gamma_{2i-1}=abc=(abc)^{\frac{7^1-1}{6}}$$

Next, we consider the statement $P(n)$ for some natural number k. Then we assume that $P(k)$ is true. That is,

$$\prod_{i=1}^{k}\alpha_{2i-1}=\prod_{i=1}^{k}\beta_{2i-1}=\prod_{i=1}^{k}\gamma_{2i-1}=(abc)^{\frac{7^k-1}{6}}$$

Then we will show that $P(k+1)$ is true.

$$\prod_{i=1}^{k+1}\alpha_{i}=\prod_{i=1}^{k+1}\beta_{2i-1}=\prod_{i=1}^{k+1}\gamma_{2i-1}=(abc)^{7^k}(abc)^{\frac{7^k-1}{6}}=(abc)^{\frac{7^{k+1}-1}{6}}.$$

So, the statement $P(k+1)$ is true.

By mathematical induction, the statement $P(n)$ is true when n is a natural number.

Theorem 3. If the ordered pair of $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ is a multiplicative pulsating 3-Fibonacci sequences of scheme 3, then

1) $$\prod_{i=1}^{n}\alpha_{2i}=\alpha_2\cdot\alpha_4\cdot\ldots\cdot\alpha_{2n}<a^{n+\frac{n(7^{n-1}-1)}{3}}b^{\frac{n(7^{n-1}-1)}{3}}c^{\frac{n(7^{n-1}-1)}{3}}$$

2) $$\prod_{i=1}^{n}\beta_{2i}=\beta_2\cdot\beta_4\cdot\ldots\cdot\beta_{2n}<a^{\frac{n(7^{n-1}-1)}{3}}b^{n+\frac{n(7^{n-1}-1)}{3}}c^{\frac{n(7^{n-1}-1)}{3}}$$

3) $$\prod_{i=1}^{n}\gamma_{2i}=\gamma_2\cdot\gamma_4\cdot\ldots\cdot\gamma_{2n}<a^{\frac{n(7^{n-1}-1)}{3}}b^{\frac{n(7^{n-1}-1)}{3}}c^{n+\frac{n(7^{n-1}-1)}{3}}$$

for every natural number n.

Proof. Let the ordered pair of $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ be a multiplicative pulsating 3-Fibonacci sequences of scheme 3. We have

1) $$\prod_{i=1}^{n}\alpha_{2k}=\alpha_2\cdot\alpha_4\cdot\ldots\cdot\alpha_{2n}=\left(a^{1+2\sum_{i=0}^{0}7^i}b^{2\sum_{i=0}^{0}7^i}c^{2\sum_{i=0}^{0}7^i}\right)\left(a^{1+2\sum_{i=0}^{1}7^i}b^{2\sum_{i=0}^{1}7^i}c^{2\sum_{i=0}^{1}7^i}\right)\cdot\ldots\cdot\left(a^{1+2\sum_{i=0}^{n-1}7^i}b^{2\sum_{i=0}^{n-1}7^i}c^{2}\right.$$

$$=a^{n+2\left(n+7(n-1)+7^2(n-2)+\ldots+7^{n-1}\right)}b^{2\left(n+7(n-1)+7^2(n-2)+\ldots+7^{n-1}\right)}c^{2\left(n+7(n-1)+7^2(n-2)+\ldots+7^{n-}\right.}$$

$$<a^{n+\frac{n(7^{n-1}-1)}{3}}b^{\frac{n(7^{n-1}-1)}{3}}c^{\frac{n(7^{n-1}-1)}{3}}.$$

2) $$\prod_{i=1}^{n}\beta_{2i}=\beta_2\cdot\beta_4\cdot\ldots\cdot\beta_{2n}=\left(a^{2\sum_{i=0}^{0}7^i}b^{1+2\sum_{i=0}^{0}7^i}c^{2\sum_{i=0}^{0}7^i}\right)\left(a^{2\sum_{i=0}^{1}7^i}b^{1+2\sum_{i=0}^{1}7^i}c^{2\sum_{i=0}^{1}7^i}\right)\cdot\ldots\cdot\left(a^{2\sum_{i=0}^{n-1}7^i}b^{1+2\sum_{i=0}^{n-1}7^i}c^{2\sum_{i=0}^{n-1}7^i}\right)$$

$$=a^{2\left(n+7(n-1)+7^2(n-2)+\ldots+7^{n-1}\right)}b^{n+2\left(n+7(n-1)+7^2(n-2)+\ldots+7^{n-1}\right)}c^{2\left(n+7(n-1)+7^2(n-2)+\ldots+7^{n-1}\right)}$$

$$<a^{\frac{n(7^{n-1}-1)}{3}}b^{n+\frac{n(7^{n-1}-1)}{3}}c^{\frac{n(7^{n-1}-1)}{3}}.$$

3) $$\prod_{i=1}^{m}\gamma_{2i}=\gamma_2\cdot\gamma_4\cdot\ldots\cdot\gamma_{2n}=\left(a^{2\sum_{i=0}^{0}7^i}b^{2\sum_{i=0}^{0}7^i}c^{1+2\sum_{i=0}^{0}7^i}\right)\left(a^{2\sum_{i=0}^{1}7^i}b^{2\sum_{i=0}^{1}7^i}c^{1+2\sum_{i=0}^{1}7^i}\right)\cdot\ldots\cdot\left(a^{2\sum_{i=0}^{n-1}7^i}b^{2\sum_{i=0}^{n-1}7^i}c^{1+2\sum_{i=0}^{n-1}7^i}\right)$$

$$=a^{2\left(n+7(n-1)+7^2(n-2)+\ldots+7^{n-1}\right)}b^{2\left(n+7(n-1)+7^2(n-2)+\ldots+7^{n-1}\right)}c^{n+2\left(n+7(n-1)+7^2(n-2)+\ldots+7^{n-1}\right)}$$

$$<a^{\frac{n\left(7^{n-1}-1\right)}{3}}b^{\frac{n\left(7^{n-1}-1\right)}{3}}c^{n+\frac{n\left(7^{n-1}-1\right)}{3}}.$$

Next, we compare the different of each paper which is the similarity topic. First, Suvarnamani and Koyram [7] proved some properties of the multiplicative pulsating 2-Fibonacci sequences in 2015. Then Suvarnamani and Tatong [8] explained the multiplicative pulsating 3-Fibonacci sequences in 2016. So, we consider the multiplicative pulsating 3-Fibonacci sequences and its properties. We found that the results difference with previous works. That is, we showed some properties of the multiplicative pulsating 3-Fibonacci sequences with the new scheme in this paper. For future research, we can start from the finding of other properties of the multiplicative pulsating 3-Fibonacci sequences.

Conclusion

The formulas of multiplicative pulsating 3-Fibonacci sequences were proved in this paper. For scheme 3, if n is a positive integer and $\alpha_0=a,\beta_0=b,\gamma_0=c$, we get $\alpha_{2n-1}=\beta_{2n-1}=\gamma_{2n-1}=a^{7^{n-1}}b^{7^{n-1}}c^{7^{n-1}}$, $\alpha_{2n}=a^{1+2\sum_{i=0}^{n-1}7^i}b^{2\sum_{i=0}^{n-1}7^i}c^{2\sum_{i=0}^{n-1}7^i}$, $\beta_{2n}=a^{2\sum_{i=0}^{n-1}7^i}b^{1+2\sum_{i=0}^{n-1}7^i}c^{2\sum_{i=0}^{n-1}7^i}$ and $\gamma_{2n}=a^{2\sum_{i=0}^{n-1}7^i}b^{2\sum_{i=0}^{n-1}7^i}c^{1+2\sum_{i=0}^{n-1}7^i}$.

Moreover, we get $\prod_{i=1}^{n}\alpha_{2i-1}=\prod_{i=1}^{n}\beta_{2i-1}=\prod_{i=1}^{n}\gamma_{2i-1}=(abc)^{\frac{7^n-1}{6}}$, $\prod_{i=1}^{n}\alpha_{2i}<a^{n+\frac{n\left(7^{n-1}-1\right)}{3}}b^{\frac{n\left(7^{n-1}-1\right)}{3}}c^{\frac{n\left(7^{n-1}-1\right)}{3}}$, $\prod_{i=1}^{n}\beta_{2i}<a^{\frac{n\left(7^{n-1}-1\right)}{3}}b^{n+\frac{n\left(7^{n-1}-1\right)}{3}}c^{\frac{n\left(7^{n-1}-1\right)}{3}}$ and $\prod_{i=1}^{n}\gamma_{2i}<a^{\frac{n\left(7^{n-1}-1\right)}{3}}b^{\frac{n\left(7^{n-1}-1\right)}{3}}c^{n+\frac{n\left(7^{n-1}-1\right)}{3}}$.

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